1.Introduction and Basic Terminologies

Welcome to the series of E-learning modules on Statistics. In this session we are going to cover the concept of probability and a brief introduction to the basic terminologies of Probability. We will also be covering the module Approaches to Probability in this session.

By the end of this session, you will be able to:

- Explain the concept of Probability
- Explain the following Basic Terminologies in Probability, with the help of Examples:
    - Random Experiment
    - Sample Space
    - Events and types of Event
- Define Probability
- Explain the approaches to Probability

Probability is a commonly used word in the day to day conversation of the people. Look at the few examples:

1) It may *probably* rain today
2) It is *possible* that life exists on other planets
3) It is *likely* that they may not come for lunch today

If we observe we can see that all the terms probably, possible & likely leads to one common element uncertainty. Probability is the mathematical framework for describing (modelling) uncertainty. It is a numerical measure of the likelihood that a specific event will occur.

Therefore, we can define the term probability as the likelihood or chance of occurring of a particular event.

In a real life situation let us take a business firm, this firm works in an uncertain situation and needs to take a lot of decisions for its forward planning. A wrong decision can lead to a lot of risk or even huge loss to the business, Thus the management needs to take care in making decisions and the theory of probability is used as a guide in making decisions.

Here the business firm operates in uncertain environment and the Probability Theory helps in decision making**.**

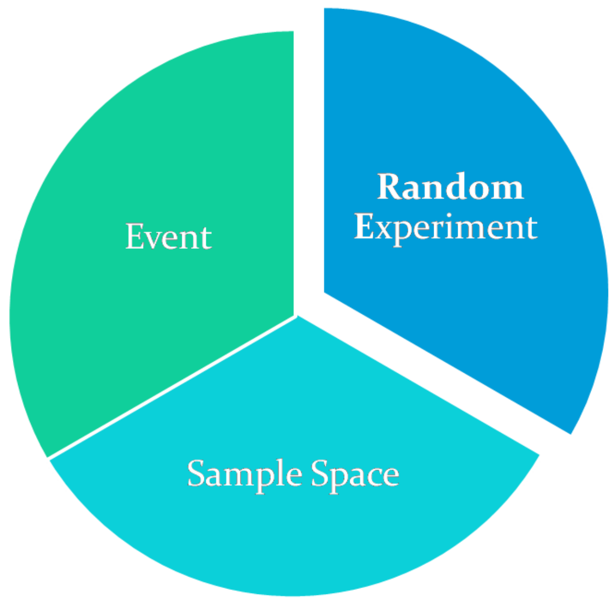
Theory of probability thus helps in making appropriate decisions or generalizations from limited observations which becomes necessary to confront problems.

The theory of probability has been applied as a tool in making investment decisions, setting targets, quality control, etc.

In order to understand various aspects of probability, it is necessary for us to be familiar with some basic terminologies frequently used in any probability discussion:

- Random experiment
- Sample Space &
- Events

Figure 1



The game of cricket has world-wide appeal and is enjoyed by people of all ages. We all are familiar that the game starts with a tossing of the coin.

1. Tossing of a single coin is Random Experiment where we can only predict the outcome.
2. The possible outcomes of the experiment are the "head" or "a tail", which we call as the possible disjoint outcomes.
3. Finally, the outcome of this action that is "a head" in this case is the actual result.

This entire process can be termed as a random experiment.

A **random** phenomenon is a situation in which we know what outcomes could happen, but we don't know which particular outcome will happen.
Experiment is a process of observation that leads to a single outcome that cannot be predicted with certainty.

Thus, the term Random Experiment is an activity that results in one outcome out of disjoint outcomes, where an outcome cannot be predicted with certainty.

Let us continue with the same random experiment and this time we will toss two coins, then the outcomes would be

- head in both the coins
- it could be head in the first coin and tail in the second coin
- it could be tail in the first coin and head in the second coin

  or
- it could be tails in both the coins

Summing up of this outcome is called a sample space and each element in this sample space is termed as a sample point. In this case we have four sample points. S= (H, H), (H, T), (T, H), (T, T).

We can thus define a sample space and a sample point as following:

Sample Space is all possible outcomes of a random experiment.
Members of a sample space are known as **Sample Points**.
And is denoted with the alphabet **"S".**

All events of interest which is a possible outcome of the experiment can be considered as an event and is a subset of the sample space **"S".**

In the random experiment of tossing a coin the sample space is, S= (H, H), (H, T), (T, H), (T, T) and if the interest is on head appearing in the first coin then the event sub set are E= {(H, H), (H, T)}.

For a better understanding of the term in a real life situation let us take the example of a survey on employee satisfaction which directly links with the financial result of the firm. A firm would be interested in knowing the Employees with higher job satisfaction as they typically believe that their organization will be productive in the long run. Thus, conducting of the survey is a random experiment, which could have outcomes Highly Satisfied, Satisfied, Neutral, Dissatisfied, Highly Dissatisfied in the sample space the interested outcome is higher satisfaction thus the event is {(Highly Satisfied), (Satisfied)}.

2. Events and types of Events

Events: When we say "Event" we mean one (or more) outcomes.

Example Events: Getting a Tail when tossing a coin is an event, Rolling a "5" is an event.

An event can include several outcomes: Choosing a "King" from a deck of cards (any of the 4 Kings) is also an event,   Rolling an "even number" (2, 4 or 6) is an event.

Depending on the relativity between the events we have classified the events as Compound event, Independent & Dependent Event, Mutually exclusive Event, Collective Exhaustive Event, Equally likely event and Complementary event.

Figure 2

Compound Event: is a collection of more than one outcome for an experiment.

When 2 coins are tossed the event of obtaining one head and one tail is a compound event.

When 2 balls are picked randomly from a bag containing 3 red balls and 3 white balls the event of pulling one red and one white ball is a compound event.

Independent Events: Events can be "Independent", meaning each event is not affected by other events.

When tossing a coin, say three times, a head comes up each time. The chance of a head on the first toss, does not affect the possibility of obtaining a head on the second toss. What it did in the past will not affect the current toss.

Some events can be "dependent" ... which means they can be affected by previous events.

Example: Drawing 2 Cards from a Deck, After taking one card from the deck there are fewer cards available, so the probabilities change!

Let's say you are interested in the chances of getting a King. For the 1st card the chance of drawing a King is 4 out of 52, But for the 2nd card: If the 1st card was a King, then the 2nd card is less likely to be a King, as only 3 of the 51 cards left are Kings. If the 1st card was not a King, then the 2nd card is slightly more likely to be a King, as 4 of the 51 cards left are King. This is because you are removing cards from the deck.

Replacement: When you put each card back after drawing it the chances don't change, as the events are independent. Without Replacement: The chances will change, and the events are dependent.

Mutually Exclusive: Mutually Exclusive means you cannot get both events at the same time. It is either one or the other, but not both.

Examples:
Turning left or right is Mutually Exclusive (you can't do both at the same time)
Heads and Tails are Mutually Exclusive
Kings and Aces are Mutually Exclusive
What is not Mutually Exclusive: Kings and Hearts are not Mutually Exclusive, because you can have a King of Hearts.

A list consisting of all the possible outcomes of an experiment is called collective exhaustive events.

For example, when rolling a six-sided die, the outcomes 1, 2, 3, 4, 5, and 6 are collectively exhaustive, because they encompass the entire range of possible outcomes.

Two or more events are said to be equally likely if each has an equal chance of occurrence. When a coin is tossed, the outcomes head and tail have an equal chance of occurrence.

Two events are said to be complementary when one event occurs if and only if the other does not occur.

In any event 'E' we define the new event 'E^c' as a complementary event, if it consists of all the outcomes in the sample space S that are not in E. If two dice are thrown and

E= {(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)} where the sum of the dice is equal to 7. Then E^c will occur only when the sum of the dice is not equal to 7**.**

3.Approaches to Probability and limitations of Classical Approach

There are three approaches to probability. They are: Classical Approach, Relative Frequency Approach and Subjective Approach.
Let us take a quick recap of what does probability mean. "Probability is the likelihood or chance of occurring of a particular event."

The classical approach is the oldest and simplest approach to probability. It is also called as 'PRIORI' probability as we can state the answer in advance.

JAMES BERNOULLI is the first man to obtain a quantitative measure for uncertainty. According to JAMES "If a Random Experiment results in 'N' Exhaustive, Mutually Exclusive and Equally likely Outcomes (cases) out of which 'm' are favourable to the happening of an event 'A', then the probability of occurrence of 'A', usually denoted by p(A) and is given by"

Example: When a coin is tossed the probability of getting a head in a toss of unbiased coin is N is equal to 2, m = 1 and P (A) is m/N or ½.

Here, A is the Event of getting a head.

P (A) is the probability of occurrence.

M is the number of times that the event occurs.

N is the number of times the experiment is performed.

P (A) = m/N = favorable number of cases to A /Exhaustive number of cases.

Now take one more example where when we draw a card from a shuffled pack of cards the probability of getting a Ace is: 1/13. Let us see how:

Here, N is equal to 52 (52 is the total number of cards)

M is equal to 4 (4 is the number of ace cards in the deck of cards)

Thus, P (A) is m/N or 4/52 or 1/13.

The number of cases favourable to the complementary event A^{l} i.e., the non happening of the event A are (N-m).

The probability that A will not happen is denoted by $A^{|}$.

Hence the probability of non occurrence $A^{|}$ is given by $P(A^{|}) = 1 - P(A)$, which also means the $P(A) + P(A^{|}) = 1$.

N & m are non negative integers, thus $P(A) \geq 0$.

Further the favorable cases are always less than 1 or equal to N $\therefore m \leq N$.

We have $P(A) \leq 1$.

Hence probability of any event is a number laying between 0 & 1 $\therefore 0 \leq P(A) \leq 1$.

The probability of the event A happening is known as success and denoted as p and the probability of the event not happening is called failure and is denoted as q.

Thus we get $p + q = 1$.

- The Classical Approach does NOT need any actual experiment for computation
- It is not based on previous experience
- It is obtained by logical reasoning

There are certain limitations of classical approach:

1. If N, the exhaustive number of outcomes of the random experiment is infinite.

Let us take example of tossing a coin.

Toss a coin until a head appears. In this Random Experiment the possibility of outcomes is infinite. Thus, we cannot apply the classical approach.

2. If the various outcomes of the random experiment are not equally likely.

For example: if a person jumps from the top of tall building, then the probability of his survival will not be 50% , because the outcomes survival & death the two mutually exclusive & exhaustive outcomes are not equally likely.

3. If the actual value of N is not known.

For example: Let us assume that there is a bag with 2 colours of balls (red & White) and the numbers is unknown. Now if we draw the balls from the bag we will be able to draw some conclusion of the red to white balls. In the absence of any such experiment we cannot draw any conclusion regarding the probability of drawing a red or a white ball from the bag.

4.Relate Frequency Approach and Empirical Approach

Relate Frequency Approach is the next approaches to Probability.

The relative frequency approach is an approach based on the statistical data.

This method uses the relative frequencies of past occurrence as the basis for computing present probabilities.

Assigning probabilities is based on cumulated relative frequencies. Thus past data is used to predict future probability

The relative frequency approach is defined by VON MISES as "if an experiment is performed repeatedly under essentially homogeneous & identical conditions then, the limiting value of the ratio of the number of times the event occurs to the number of trials, as the number of trials becomes indefinitely large, is called the probability of happening of the event."

It is being assumed that the limit is finite and unique.

An event occurs m times in N repetition of a random experiment. Then the ratio m/N gives the relative frequency of the event A and it will not vary appreciably from one trial to another. In the limiting case when N becomes sufficiently large, it more or less settles to a number which is called the probability of A.

Symbolically,

- $P(A) = \lim m/N$
    - $N \to \infty$

Example:

You are watching a birdfeeder, and you notice a lot of cardinals coming for dinner.

You want to find the probability that the next bird that comes to the feeder is a cardinal.

You can estimate this probability by counting the number of birds that come to your feeder over a period of time and noting how many cardinals you see.

If you count 100 bird visits, and 27 of the visitors are cardinals, you can say that for the period of time you observe, 27 out of 100 visits – or 27 percent, the relative frequency – were made by cardinals.

Relative Frequency Approach is also known as Empirical Approach. Relative frequency approach depends on probability obtained objectively by repetitive empirical observation and is known as Empirical Probability.

The empirical probability provides validity to the classical theory of a probability.

For example: If an unbiased coin is tossed 20 times then there should be 10 heads according to the classical probability, however in reality it is not true. In fact in a 20 toss of the coin we may not get a head.

However, empirical probability suggest that if a coin is tossed a large number of times then expected outcome will be 50%, say 500 times we should on the average expect 50% heads & tails.

As remarked by the empirical probability we can attempt only a close estimate by making sufficiently large experiments. However the following limitations are found.

The experimental condition may not remain the same in a large number of repetitions.

The relative frequency m/N may not attain a unique value, no matter however large N may be.

For example: On collecting your birdfeeder data, when you offered sunflower seeds, and when you offer thistle seed (which is mainly loved by smaller birds), your probability of seeing a cardinal changes.

Also, if you look at the feeder only at 5 p.m. each day, when cardinals are more likely to be out than any other bird, your predictions work only at that same time period, not at noon when all the finches are also out.

Here, in this table, on 1st day, at 5 p.m. 30 cardinals were found in the sunflower seeds feeder, whereas only 10 other birds were found at the same time or same day in the thistle seeds feeder.

On 2nd day, at 12 p.m. 6 cardinals were found in the sunflower seeds feeder and at the same 25 other birds were found in the thistle seeds feeder.

Thus, the probability of seeing a cardinal changes depending on the seeds offered in the feeder. Also, the prediction will only work at the same time period.

Formula for relative frequency is :

Probability of an event = Relative Frequency = f/n.

Where, f is the frequency of an event.

N is the sample size.

5. Law of large numbers

Law of large numbers in relative frequency.

In the long run, as the sample size increases and increases, the relative frequencies of outcomes get closer and closer to the theoretical (or actual) probability value.

Law of large numbers is the reason that many businesses can exist and make profits, like:

- Health Insurance
- Automobile Insurance and
- Gambling Casinos

We will take Automobile insurance as an example:

In Automobile Insurance Business –

If number of policies is less, then the possibility of claim will be high.

If number of policies increases, the Relative Frequency outcome gets closer to the probability value or the possibility of claims.

See this graph; here for 1000 policies the possibility of claim is very high whereas when the number of policies reaches 5000 then the outcome starts getting closer to the probability value.

Subjective Approach is the third Approach in probability.

The subjective approach is based on the intuition of an individual. It is based on the accumulation of knowledge, understanding & experience of an individual.

A strong interaction is there between the intuition, environment, experience and talent in predicting the future.

It is the degree of confidence that a rational person has on a specific outcome of an event.

The approach is highly flexible & can be applied in any situation

Professor Savage uses the term Personalistic instead of Subjective.

According to Professor Savage:

"Personalistic views hold the probability measures the confidence that a particular individual has in the truth of a particular proposition."

For example: A sales manager wants to promote one of his subordinates out of the three.

Three subordinates are equal in one or the other like efficiency, potential, behaviour, sales and relationship with customers as shown in the graph.

In this kind of situation the sales manager has to rely on his intuition only through which he can promote one out of the three as sales executive.

Subjective Approach also has limitations.

This approach is totally based on a person's assessment of the situation or environment which differs from person to person.

Conclusion:

We have learnt the following in this session:

The basic concept of probability and the basic terminologies used in probability theory, random experiment, sample space and event.

Approaches to probability and their limitations

The three approaches are complementary to each other, when one approach fails the other becomes applicable. Thank You