Summary

- A two dimensional random variable is said to be discrete if it takes at most a countable number of points in R^2 and a two dimensional random variable is said to be continuous if it assumes all the values in R^2.
- Two random variables X and Y are said to be jointly distributed if they are defined on the same probability space. The sample points consist of 2-tuples. If the joint probability function is denoted by $P_{XY}(x, y)$ then the probability of certain event E is given by, $P_{XY}(X, Y)=P[(X, Y)\in E]$
- If (X, Y) is a two dimensional discrete random variable, then the joint discrete function of X, Y, also called joint probability mass function of X, Y denoted by $p_{X,Y}$ is defined as, $p_{XY}(xi, yi)=p(X=x_i, Y=y_i)$ for a value (x_i, y_i) of (X, Y) and $p_{XY}(x_i, y_i)=0$, otherwise.
- The distribution of two dimensional random variable (X, Y) is a real valued function F defined for all real x and y by the relation $F_{XY}(x, y)=P(X\le x, Y\le y)$
- From the joint distribution function, $F_{XY}(x, y)$ of bivariate continuous random variable, we get the joint probability density function by differentiation as follows.

$$f(x,y)=\frac{\partial^2 F}{\partial x \partial y}=\lim_{\delta x\to 0,\delta y\to 0}\frac{P(x\le X\le x+\delta x, y\le Y\le y+\delta y)}{\delta x\,\delta y}$$

- To verify whether the given probability function is pmf/pdf or not, we check for following two conditions.
    - $p(x, y)\ge 0, \forall x, y$ (for discrete bivariate distribution)
      $f(x, y)\ge 0, \forall x, y$ (for continuous bivariate distribution)
    - $\Sigma_{x,y}p(x, y)=1$ (for discrete bivariate distribution)
      $\int_{x,y}f(x, y)dx\,dy=1$ (for continuous bivariate distribution)