Frequently Asked Questions

1. Define two dimensional or bivariate random variable.

Answer:

Let X and Y be two random variables defined on the same sample space S, then the function (X, Y) that assigns a point $R^2(=R\times R)$, is called a two dimensional random variable.

2. When can you say that the random variable is discrete?

Answer:

A two dimensional random variable is said to be discrete if it takes at most a countable number of points in R^2

3. When can you say that the random variable is continuous?

Answer:

A two dimensional random variable is said to be continuous if it assumes all the values in R^2.

4. When is it said that the 2 random variables are jointly distributed?

Answer:

Two random variables X and Y are said to be jointly distributed if they are defined on the same probability space. The sample points consist of 2-tuples.

5. Define probability mass function.

Answer:

If (X, Y) is a two dimensional discrete random variable, then the joint discrete function of X, Y, also called joint probability mass function of X, Y denoted by $p_{X,Y}$ is defined as, $p_{XY}(xi, yi)=p(X=x_i, Y=y_i)$ for a value (x_i, y_i) of (X, Y) and $p_{XY}(x_i, y_i)=0$, otherwise.

6. Define distribution function.

Answer:

The distribution of two dimensional random variable (X, Y) is a real valued function F define for all real x and y by the relation $F_{XY}(x, y)=P(X\le x, Y\le y)$

7. Write the properties of distribution function.

Answer:

1. For the real numbers a_1, b_1, a_2 and b_2,
   $P(a_1<X\le b_1, a_2<X\le b_2) =F_{XY}(b_1, b_2)+ F_{XY}(a_1, a_2)-F_{XY}(a_1, b_2)- F_{XY}(b_1, a_2)$
   Also F(x, y) is monotonic non decreasing function
2. $F(-\infty, y)=0=F(x, -\infty)$, $F(+\infty, +\infty)=1$
3. If the density function f(x, y) is continuous at (x, y), $\frac{\partial^2 F}{\partial x\partial y} = f(x,y)$

8. Show that F(x, y) is monotonic non decreasing function.

Answer:

Let $a_1<a_2$, $b_1<b_2$ then

$(X\le a_1, Y\le a_2)+(a_1<X\le b_1, Y\le a_2)=(X\le b_1, Y\le a_2)$ and the events on LHS are mutually exclusive.

$\therefore F(a_1, a_2)+P(a_1<X\le b_1, Y\le a_2)=F(b_1, a_2)$

$\Rightarrow F(b_1, a_2)-F(a_1, a_2)=P(a_1<X\le b_1, Y\le a_2)$

$\therefore F(b_1, a_2)\ge F(a_1, a_2)$ $[\because P(a_1<X\le b_1, Y\le a_2)\ge 0)$

Similarly, it follows that $F(a_1, b_2)-F(a_1, a_2)=P(X\le a_1, a_2<Y\le b_2)$

$\therefore F(a_1, b_2) \geq F(a_1, a_2)$, which shows that F(x, y) is monotonic non decreasing function.

9. Define probability density function using joint distribution function.

Answer:

From the joint distribution function, $F_{XY}(x, y)$ of bivariate continuous random variable, we get the joint probability density function by differentiation as follows.

$$f(x,y) = \frac{\partial^2 F}{\partial x \partial y} = \lim_{\delta x \to 0, \delta y \to 0} \frac{P(x \leq X \leq x + \delta x, y \leq Y \leq y + \delta y)}{\delta x \delta y}$$

10. Define probability density function in terms of probability function.

Answer:

The probability that the point (x, y) will lie in the infinitesimal rectangular region, of area dx dy is given by,

$P(x-\frac{1}{2}dx \leq X \leq x+\frac{1}{2}dx,\ y-\frac{1}{2}dy \leq Y \leq y+\frac{1}{2}dy) = d\,F_{XY}(x,\ y)$ and is denoted by $f_{xy}(x,\ y)dx\ dy$, where $f_{XY}(x, y)$ is called the joint probability density function of X and Y.

11. When is it said that a given function is a probability mass function?

Answer:

To verify whether the given function is probability mass function or not, we check the following 2 conditions.

$p(x, y) \geq 0, \forall x, y$ and $\Sigma_{x,y} p(x, y) = 1$

12. When is it said that a given function is a probability mass function?

Answer:

To verify whether the given function is probability density function or not, we check the following 2 conditions.

$f(x, y) \geq 0, \forall x, y$ and $\int_{x,y} f(x, y)dx\, dy = 1$

13. In the random placement of 3 balls in three cells, describe the possible outcomes of the experiment. Let X_i denote the number of balls in cell i, i=1, 2, 3; and N, the number of cells occupied. Obtain the joint distribution of (X_1, N).

Answer:

a. Let the three balls be denoted by a, b and c. Then the possible outcomes of placing the three balls in three cells are as follows.

1.	a	b	c	10.	ac	b	-	19.	b	ca	-
2.	a	c	b	11.	ac	-	b	20.	b	-	ca
3.	b	a	c	12.	-	ac	b	21.	-	b	ca
4.	b	c	a	13.	bc	a	-	22.	c	ab	-
5.	c	a	b	14.	bc	-	a	23.	c	-	ab
6.	c	b	a	15.	-	bc	a	24.	-	c	ab
7.	ab	c	-	16.	a	bc	-	25.	abc	-	-
8.	ab	-	c	17.	a	-	bc	26.	-	abc	-
9.	-	ab	c	18.	-	a	bc	27.	-	-	abc

Each of these arrangements represents a sample event, i.e., a sample point. The sample space contains 27 points.

Let N denote the number of occupied cells. The favorable cases for N=1 are at numbers 25, 26 and 27, i.e., 3; for N=2 are at numbers 7 to 24, i.e., 18 and for N=3 are at numbers 1 to 6, i.e., 6. Accordingly the probability distribution of N is

$P(N=1)={}^3/_{27}$; $P(N=2)={}^{18}/_{27}$ & $P(N=3)={}^6/_{27}$

Let X_1 denote the number of balls placed in the first cell. Then form the above table of sample point X1 can take values, 0, 1, 2 and 3.

$P(X_1=0)={}^8/_{27}$; $P(X_1=1)={}^{12}/_{27}$; $P(X_1=2)={}^6/_{27}$ and $P(X_1=3)={}^1/_{27}$.

The joint distribution of N and X_1 can be obtained as follows:

$P(N=1, X_1=0)={}^2/_{27}$; $P(N=1, X_1=1)=0$; $P(N=1, X_1=2)=0$; $P(N=1, X_1=3)={}^1/_{27}$;

$P(N=2, X_1=0)={}^6/_{27}$; $P(N=2, X_1=1)={}^6/_{27}$; $P(N=2, X_1=2)={}^6/_{27}$; $P(N=2, X_1=3)=0$;
$P(N=3, X_1=0)=0$; $P(N=3, X_1=1)={}^6/_{27}$; $P(N=3, X_1=2)=0$; $P(N=3, X_1=3)=0$;
The above probabilities are written in a tabular form to get joint distribution of N and X_1 as follows.

X_1 \ N	1	2	3	Distribution of X_1
0	${}^2/_{27}$	${}^6/_{27}$	0	${}^8/_{27}$
1	0	${}^6/_{27}$	${}^6/_{27}$	${}^{12}/_{27}$
2	0	${}^6/_{27}$	0	${}^6/_{27}$
3	${}^1/_{27}$	0	0	${}^1/_{27}$
Distribution of N	${}^3/_{27}$	${}^{18}/_{27}$	${}^6/_{27}$	1

14. For the following bivariate probability distribution of X and Y find, $P(X=1, Y=1)$, $P(X\le 1,Y=2)$ and $P(X<3,Y\le 4)$.

X \ Y	1	2	3	4	5	6
0	0	0	${}^1/_{32}$	${}^2/_{32}$	${}^2/_{32}$	${}^3/_{32}$
1	${}^1/_{16}$	${}^1/_{16}$	${}^1/_8$	${}^1/_8$	${}^1/_8$	${}^1/_8$
2	${}^1/_{32}$	${}^1/_{32}$	${}^1/_{64}$	${}^1/_{64}$	0	${}^2/_{64}$

Answer:
$P(X=1, Y=1)={}^1/_{16}$
$P(X\le 1,Y=2) =P(X=0, Y=2)+ P(X=1,Y=2)= 0+{}^1/_{16} ={}^1/_{16}$
$P(X<3,Y\le 4)=P(X=0,Y\le 4)+P(X=1,Y\le 4)+P(X=2,Y\le 4)$
$=(0+0+{}^1/_{32}+{}^2/_{32})+({}^1/_{16}+{}^1/_{16}+{}^1/_8+{}^1/_8)+({}^1/_{32}+{}^1/_{32}+{}^1/_{64}+{}^1/_{64})={}^9/_{16}$

15. If X and Y are two random variables having joint density function $f(x, y)= \frac{1}{8}(6-x-y)$; $0\le x<2$, $2\le y<4$. Find $P(X<1\cap Y<3)$ and $P(X+Y<3)$.

Answer:
We can write,

$$P(X<1\cap Y<3)=\int_{-\infty}^{1}\int_{-\infty}^{3} f(x,y)dxdy = \frac{1}{8}\int_0^1\int_2^3 (6-x-y)dxdy = \frac{3}{8}$$

To find $P(X+Y<3)$, let us find the limits for X and Y. $X+Y<3 \Rightarrow Y<3-X$.
The minimum value taken by Y=2. Hence X cannot be greater than 1.

$$P(X+Y<3)=\int_0^1\int_2^{3-x} f(x,y)dxdy = \frac{1}{8}\int_0^1\int_2^3 (6-x-y)dxdy = \frac{5}{24}$$