Summary

- Conditional mean value or expectation of a continuous function g(X, Y) given that $Y=y_j$, is defined by $E[g(X,Y) \mid Y = y_j] = \sum_{i=1}^{\infty} g(x_i, y_j) P(X = x_i \mid Y = y_j)$

$$= \frac{\sum_{i=1}^{\infty} g(x_i, y_j) P(X = x_i \cap Y = y_j)}{P(Y = y_j)}$$

- The conditional variance of X may be defined as,

$$V(X|Y=y)=E[\{X-E(X|Y=y)\}^2 \,|Y=y]$$

- The expected value of X is equal to the expectation of the conditional expectation of X given Y. i.e., E(X)=E{E(X|Y)}

- The variance of X can be regarded as consisting of two parts, the expectation of the conditional variance and the variance of the conditional expectation.

  i.e. V(X)=E[V(X|Y)]+V[E(X|Y)]

- If A and B be two mutually exclusive events, then,

$$E(X \mid AUB) = \frac{P(A)E(X \mid A) + P(B)E(X \mid B)}{P(AUB)}$$

- Given two variables X and Y with joint density function f(x,y), prove that conditional mean of Y given X coincide with unconditional mean only if X and Y are independent