

Summary

- Let (X, Y) be a discrete bivariate random variable. Then, the conditional discrete density function or the conditional probability mass function of X , given $Y=y$ denoted by $p_{X|Y}(x|y)$, is defined as $p_{X|Y}(x|y) = \frac{P(X=x, Y=y)}{P(Y=y)}$ provided $P(Y=y) \neq 0$
- A necessary and sufficient condition for the discrete random variable X and Y to be independent is $P(X=x_i, Y=y_j) = P(X=x_i)P(Y=y_j)$ for all values of (x_i, y_j) of (X, Y) i.e. the joint probability mass function can be expressed as the product of the conditional probability mass function.
- From the knowledge of joint distribution function $F_{XY}(x, y)$, it is possible to obtain the individual distribution functions $F_X(x)$ and $F_Y(y)$ which are termed as marginal distribution function of X and Y respectively with respect to the joint distribution function $F_{XY}(x, y)$.
- If we know the joint pdf (pmf) $f_{XY}(x, y)$ of two random variables X and Y , we can obtain the individual distribution of X and Y in the form of their marginal pdfs (pmfs) $f_X(X)$ and $f_Y(y)$. However, the converse is not true. That is, from the marginal distributions of two jointly distributed random variables, we cannot determine the joint distribution of these two random variables.
- Suppose the variables are independent, then we can write joint pdf (pmf) as the product of their marginal pdfs (pmfs)
- For a bivariate random variable (X, Y) , the joint distribution function $F_{XY}(x, y)$ for any real numbers x and y is given by $F_{XY}(x, y) = P(X \leq x, Y \leq y)$.
- The conditional probability density function of Y given X for bivariate random variable (X, Y) which are jointly continuously distributed is defined as follows, for two real numbers x and y $f_{Y|X}(y|x) = \frac{\partial}{\partial y}[F_{Y|X}(y/x)]$