Frequently Asked Questions

1. Explain marginal probability mass function.

Answer:

Let (X, Y) be a discrete bivariate random variable which takes up countable number of values (x_i, y_i). Then, the probability distribution of X is determined as follows.

$p_X(x_i) = P(X=x_i) = P(X=x_i \cap Y=y_1)+P(X=x_i \cap Y=y_2)+\ldots+P(X=x_i \cap Y=y_m)=p_{i1}+p_{i2}+\ldots+p_{im}$ and is known as marginal probability mass function or discrete marginal density function of X

2. Verify whether $\sum_{i=1}^{n} p_{i.} = 1$

Answer:

Consider, $\sum_{i=1}^{n} p_{i.} = p_{1.} + p_{2.} + \ldots + p_{n.} = \sum_{i=1}^{n}\sum_{j=1}^{m} p(x_i, y_j) = 1$

3. Define conditional probability function.

Answer:

Let (X, Y) be a discrete bivariate random variable. Then the conditional discrete density function or the conditional probability mass function of X, given Y=y denoted by $p_{X|Y}(x|y)$, is defined as $p_{X|Y}(x \mid y) = \frac{P(X = x, Y = y)}{P(Y = y)}$ provided $P(Y=y)\neq 0$

4. Show that conditional probability function is also a mass function.

Answer:

For a fixed y,

$$\sum_i \frac{P(X = x_i, Y = y)}{P(Y = y)} = \frac{1}{P(Y = y)} \sum_i P(X = x_i, Y = y) = \frac{1}{P(Y = y)} P(Y = y) = 1$$

It follows that the conditional mass function $p_{X|Y}(x|y)$ is a mass function, when considered as a function of the values of X.

5. Write the condition for independence of two discrete random variables.

Answer:

A necessary and sufficient condition for the discrete random variable X and Y to be independent is,

P(X=xi, Y=yi)=P(X=xi)P(Y=yj) for all values of (xi, yj) of (X, Y) i.e. the joint probability mass function can be expressed as the product of the conditional probability mass function.

6. Define marginal distribution function for a bivariate random variable (X, Y).

Answer:

From the knowledge of joint distribution function $F_{XY}(x, y)$, it is possible to obtain the individual distribution functions $F_X(x)$ and $F_Y(y)$ which are termed as marginal distribution function of X and Y respectively with respect to the joint distribution function $F_{XY}(x, y)$.

$$F_X(x) = P(X \le x) = P(X \le x, Y < \infty) = \lim_{y \to \infty} F_{XY}(x, y) = F_{XY}(x, \infty)$$

$$F_y(y) = P(Y \le y) = P(X < \infty, Y \le y) = \lim_{x \to \infty} F_{XY}(x, y) = F_{Xy}(\infty, y)$$

7. Can we find the joint distribution of any two random variables from their marginal?

Answer:

From the marginal distributions of two jointly distributed random variables, we cannot determine the joint distribution of these two random variables.

Suppose the variables are independent, then we can write joint pdf (pmf) as the product of their marginal pdfs (pmfs)

8. Explain conditional distribution function.

Answer:

For a bivariate random variable (X, Y), the joint distribution function $F_{XY}(x,y)$ for any real numbers x and y is given by

$F_{XY}(x, y)=P(X\leq x, Y\leq y)$.

The conditional distribution function $F_{Y|X}(y|x)$ denotes the distribution function of Y when X has already assumed the particular value x. Hence

$F_{Y|X}(y|x)=P(Y\leq y|X=x)=P(A|X=x)$

9. Write the expression of joint pdf in terms of conditional distribution function.

Answer:

Using the expression $F_{Y|X}(y|x)=P(Y\leq y|X=x)=P(A|X=x)$, the joint distribution function $F_{XY}(x, y)$ may be expressed in terms of the conditional distribution function as follows.

$$F_{XY}(x,y)=\int_{-\infty}^{x}F_{Y|X}(y\mid x)dF_X(x), or$$

$$F_{XY}(x,y)=\int_{-\infty}^{y}F_{X|Y}(x\mid y)dF_Y(y)$$

10. Show that if $f_X(x)>0$, then $f_{Y|X}(y\mid x)=\dfrac{f_{XY}(x,y)}{f_X(x)}$.

Answer:

We have $F_{XY}(x,y)=\int_{-\infty}^{x}f_{Y|X}(y\mid x)dF_X(x)=\int_{-\infty}^{x}f_{Y|X}(y\mid x)f_X(x)dx$

Differentiating w.r.t x, $\dfrac{\partial}{\partial x}F_{XY}(x,y)=F_{Y|x}(y\mid x)f_X(x)$

Differentiating w.r.t. y we get,

$$\frac{\partial}{\partial y}\left[\frac{\partial}{\partial x}F_{XY}(x,y)\right]=f_{Y|x}(y\mid x)f_X(x)$$

$$\Rightarrow f_{XY}(x,y)=f_{Y|x}(y\mid x)f_X(x)$$

$$\therefore f_{Y|x}(y\mid x)=\frac{f_{XY}(x,y)}{f_X(x)}$$

11. Write an expression for conditional probability using differentials.

Answer:

In terms of the differentials, we have

$$P(x<X\leq x+dx\mid y<Y\leq y+dy)=\frac{P(x<X\leq x+dx,y<Y\leq y+dy)}{P(y<Y\leq y+dy)}$$

$$=\frac{f_{XY}(x,y)dxdy}{f_Y(y)dy}=f_{X|Y}(x\mid y)dx$$

12. Two discrete random variables X and Y have the joint probability mass function $p_{XY}(x,y) = \dfrac{\lambda^x e^{-\lambda} p^y (1-p)^{x-y}}{y!(x-y)!}$ $y = 0,1,2\ldots x; x = 0,1,2\ldots$ where λ and p are constants with λ>0 and 0<p<1. Find the marginal probability mass functions of X and Y and identify them.

Answer:

The marginal pmf of X is given by,

$$p_X(x) = \sum_{y=0}^{x} p(x,y) = \sum_{y=0}^{x} \frac{\lambda^x e^{-\lambda} p^y (1-p)^{x-y}}{y!(x-y)!}$$
$$= \frac{\lambda^x e^{-\lambda}}{x!} \sum_{y=0}^{x} \frac{x!\, p^y (1-p)^{x-y}}{y!(x-y)!} = \frac{\lambda^x e^{-\lambda}}{x!} \sum_{y=0}^{x} \binom{x}{y} p^y (1-p)^{x-y}$$
$$= \frac{\lambda^x e^{-\lambda}}{x!} [p + (1-p)]^x = \frac{\lambda^x e^{-\lambda}}{x!}; x = 0,1,2,\ldots$$

which is the probability function of a Poisson distribution with parameter λ.
The marginal pmf of Y is given by,

$$p_y(y) = \sum_{x=y}^{\infty} p(x,y) = \sum_{x=y}^{\infty} \frac{\lambda^x e^{-\lambda} p^y (1-p)^{x-y}}{y!(x-y)!}$$
$$= \frac{(\lambda p)^y e^{-\lambda}}{y!} \sum_{x-y=0}^{\infty} \frac{[\lambda(1-p)]^{x-y}}{(x-y)!} = \frac{(\lambda p)^y e^{-\lambda}}{y!} \sum_{t=0}^{\infty} \frac{[\lambda(1-p)]^t}{(t)!}, (t = x-y)$$
$$= \frac{(\lambda p)^y e^{-\lambda}}{y!} e^{\lambda(1-p)} = \frac{(\lambda p)^y e^{-\lambda p}}{y!}; y = 0,1,2,\ldots$$

which is the probability mass function of Poisson distribution with parameter λp

13. Two discrete random variables X and Y have the joint probability mass function $p_{XY}(x,y) = \dfrac{\lambda^x e^{-\lambda} p^y (1-p)^{x-y}}{y!(x-y)!}$ $y = 0,1,2\ldots x; x = 0,1,2\ldots$ where λ and p are constants with λ>0 and 0<p<1. Find the conditional distribution of Y for a given X.

Answer:

The marginal pmf of X is given by,

$$p_X(x) = \sum_{y=0}^{x} p(x,y) = \sum_{y=0}^{x} \frac{\lambda^x e^{-\lambda} p^y (1-p)^{x-y}}{y!(x-y)!}$$
$$= \frac{\lambda^x e^{-\lambda}}{x!} \sum_{y=0}^{x} \frac{x!\, p^y (1-p)^{x-y}}{y!(x-y)!} = \frac{\lambda^x e^{-\lambda}}{x!} \sum_{y=0}^{x} \binom{x}{y} p^y (1-p)^{x-y}$$
$$= \frac{\lambda^x e^{-\lambda}}{x!} [p + (1-p)]^x = \frac{\lambda^x e^{-\lambda}}{x!}; x = 0,1,2,\ldots$$

Conditional distribution of Y given X is given by,

$$p(y/x) = \frac{p(x,y)}{p(x)} = \frac{\lambda^x e^{-\lambda} p^y (1-p)^{x-y}\, x!}{y!(x-y)!\,\lambda^x e^{-\lambda}}$$
$$= \frac{p^y (1-p)^{x-y}\, x!}{y!(x-y)!} = \binom{x}{y} p^y (1-p)^{x-y}, x \ge y, i.e. y = 0,1,2,\ldots x$$

14. The joint probability density function of a two-dimensional random variable (X, Y) is given by, f(x,y)=2;0<x<1, 0<y<x,
    Find marginal distribution of X and Y and conditional distribution of X given Y=y. Also, verify whether X and Y are independent.

Answer:

The marginal distribution of X is given by,

$$f(x) = \int_{-\infty}^{\infty} f(x, y)dy = \int_0^x 2dy = 2x, 0 < x < 1$$

To find marginal distribution, range for Y is given as follows. We have 0<x<1, 0<y<x. Combining both we get, 0<y<x<1

$$f(y) = \int_{-\infty}^{\infty} f(x, y)dx = \int_y^1 2dx = 2(1 - y), 0 < y < 1$$

Conditional distribution of X given Y is equal to y is given by,

$$f(x \mid y) = \frac{f(x, y)}{f(y)} = \frac{2}{2(1-y)} = \frac{1}{(1-y)}, y < x < 1$$

15. The joint probability density function of a two-dimensional random variable (X, Y) is given by, f(x,y)=2;0<x<1, 0<y<x. Test whether X and Y are independent.

Answer:

X and Y are independent if f(x,y)=f(x).f(y)

Hence first let us find the marginal distribution of X and Y.

The marginal distribution of X is given by,

$$f(x) = \int_{-\infty}^{\infty} f(x, y)dy = \int_0^x 2dy = 2x, 0 < x < 1$$

To find marginal distribution, range for Y is given as follows. We have 0<x<1, 0<y<x. Combining both we get, 0<y<x<1

$$f(y) = \int_{-\infty}^{\infty} f(x, y)dx = \int_y^1 2dx = 2(1 - y), 0 < y < 1$$

Consider f(x).f(y)=2x.[1/(1-y)]≠f(x, y)

Therefore, X and Y are not independent.