Summary

- The bivariate normal distribution is the generalisation of a normal distribution for a single variate. Let X and Y be two normally correlated variables with correlation coefficient of ρ and $E(X)=\mu_1$, $Var(X)=\sigma_1^2$; $E(Y)=\mu_2$, $Var(Y)=\sigma_2^2$. In deriving the bivariate normal distribution we make the following three assumptions.
  - The regression of Y on X is linear. Since the mean of each array is on the line of regression $Y=\rho(\sigma_2/\sigma_1)X$, the mean or expected value of Y is $\rho(\sigma_2/\sigma_1)X$, for different values of X.
  - The arrays are homoscedastic, i.e. variance in each array is same. The common variance of estimate of Y in each array is then given by $\sigma_2^2(1-\rho^2)$, ρ being the correlation coefficient between variables X and y and is independent of X.
  - The distribution of Y in different arrays is normal.
- The pdf of bivariate normal distribution is given by, $f_{XY}(x,y)=\frac{1}{2\pi\sigma_1\sigma_2\sqrt{(1-\rho^2)}}e^{-\frac{1}{2(1-\rho^2)}\left(\frac{(x-\mu_1)^2}{\sigma_1^2}-2\rho\frac{(x-\mu_1)(y-\mu_2)}{\sigma_1\sigma_2}+\frac{(y-\mu_2)^2}{\sigma_2^2}\right)}$, where μ_1, μ_2, $\sigma_1(>0)$, $\sigma_2(>0)$ and $\rho(-1<\rho<1)$ are the five parameters of the distribution.
- The curve z=f(x, y) which is the equation of a surface in three dimension is called the 'Normal Correlation Surface'.
- Mgf of Bivariate normal distribution is given by, $e^{t_1\mu_1+t_2\mu_2+\frac{1}{2}\left(t_1^2\sigma_1^2+t_2^2\sigma_2^2+2\rho t_1t_2\sigma_1\sigma_2\right)}$
- Let $(X, Y)\sim BVN(\mu_1, \mu_2, \sigma_{12}, \sigma_{22}, \rho)$. Then, X and Y are independent if and only if $\rho=0$.
- (X, Y) possesses a bivariate normal distribution if and only if every linear combination of X and Y namely, $aX+bY$, $a\neq 0$, $b\neq 0$, is a normal variate.
- The marginal distribution of random variable X is given by, $f_X(x)=\frac{1}{\sigma_1\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu_1}{\sigma_1}\right)^2}$

  and $f_Y(y)=\frac{1}{\sigma_2\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu_2}{\sigma_2}\right)^2}$