Frequently Asked Questions

1. Write the situations, where binomial distribution can be used.

Answer:

Binomial distribution can be used, when considering entities which have two states, boy-girl, head-tail, working-broken and so on which are classified as success and failure.

2. Give any 3 examples of trinomial variate.

Answer:

1. A bicycle has three principal states: Parked, Ridden or Pushed.
2. The traffic lights can be Red, Green or Changing.
3. There was brief period when ternary computers were thought worth exploring: voltage would have been positive, zero or negative.

3. Differentiate between binomial and trinomial distribution.

Answer:

Binomial distribution has 2 outcomes, which can be termed as success and failure. For example, tossing a coin a head turns up or a tail turns up. Whereas, as the name suggests, the trinomial distribution has 3 outcome, like the traffic lights can be Red, Green or Changing.

4. Define trinomial distribution.

Answer:

Suppose we define the sample space consists of all sequences of length n such that $\omega=(i_1, i_2, \ldots, i_n)$ where each i_j take values 1, 0 and -1 with probabilities $P(i_j=1)=p$, $P(i_j=0)=\theta$, $P(i_j=-1)=1-p-\theta$.

If a specific sequence of ω has x 'successes' (1's), and y 'failures' (0's)

Let X be the number of trails where 1 occurs, and Y be the number of trials where 0 occurs.

The joint distribution of pare (X, Y) is called the trinomial distribution.

5. Define trinomial distribution using pmf.

Answer:

Three random variables X, Y are said to have trinomial distribution with parameters, n, p_1, and p_2 if its pmf is given by,

$$P(X=x,Y=y)=\frac{n!}{x!\,y!\,z!}p_1^x p_2^y (1-p_1-p_2)^{n-x-y}$$

6. Obtain joint pmf of a trinomial distribution.

Answer:

The sample space consists of all sequences of length n such that $\omega=(i_1, i_2, \ldots, i_n)$ where each i_j take values 1, 0 and -1 with probabilities $P(i_j=1)=p$, $P(i_j=0)=\theta$, $P(i_j=-1)=1-p-\theta$. If a specific sequence of ω has x 'successes'(1's), and y 'failures' (0's) then $P(\omega)=p^x\theta^y(1-p-\theta)^{n-x-y}$

There are $\binom{n}{x}\binom{n-x}{y}=\frac{n!}{x!\,y!\,(n-x-y)!}$ different sequences with x successes and y failures. Hence $P(X=x,Y=y)=\frac{n!}{x!\,y!\,(n-x-y)!}p^x\theta^Y(1-p-\theta)^{n-x-y}$

7. Why the distribution is known as trinomial distribution?

Answer:

The name of the distribution comes from the trinomial expansion

$(a+b+c)^n = [a+(b+c)]^n$

$$= \sum_{x=0}^{n} \binom{n}{x} a^x (b+c)^{n-x} = \sum_{x=0}^{n} \sum_{y=0}^{n-x} \binom{n}{x} \binom{n-x}{y} a^x b^y c^{n-x-y}$$

$$= \sum_{x=0}^{n} \sum_{y=0}^{n-x} \frac{n!}{x!y!(n-x-y)!} a^x b^y c^{n-x-y}$$

8. Write mean if (X,Y)~trinomial (n, p, θ)

Answer:

The marginal distributions of X and Y are just X~ Binomial(n,p) and Y~ Binomial(n,θ). This follows the fact that X is the number of 'successes' in n independent trials with p being the probability of 'successes' in each trial. Similar argument works with Y

Therefore, E(X)=np, E(Y)=nθ

9. Write variance if (X,Y)~trinomial (n, p, θ).

Answer:

The marginal distributions of X and Y are just X~ Binomial (n,p) and Y~ Binomial(n,θ). This follows the fact that X is the number of 'successes' in n independent trials with p being the probability of 'successes' in each trial. Similar argument works with Y.

Therefore, V(X)=np(1-p) and V(Y)=nθ(1-θ)

10. If Y = y, then show that the conditional distribution of X|(Y = y) is Binomial (n-1, p/1-θ)

Answer:

$$P(X = x \mid Y = y) = \frac{P(X = x, Y = y)}{P(Y = y)}$$

$$= \frac{\frac{n!}{x!y!(n-x-y)!} p^x \theta^y (1-p-\theta)^{n-x-y}}{\frac{n!}{y!(n-y)!} \theta^y (1-\theta)^{n-y}}$$

$$= \binom{n-1}{x} \left(\frac{p}{1-\theta}\right)^x \left(1 - \frac{p}{1-\theta}\right)^{n-y-x}$$

For x=0, 1, 2, …, n-y. Hence *X|(Y = y) is Binomial* (n-1, $^p/_{1\text{-}\theta}$)

11. Obtain covariance of trinomial distribution.

Answer:

Let (X,Y)~trinomial (n, p, θ)

We know that for any two random variables, E[XY]=E[Y.E(X|Y)]. And according to the property 2, X|(Y = y) is Binomial (n-1, $^p/_{1\text{-}\theta}$) and hence E[X|Y=y]= (n-y).[$^p/_{1\text{-}\theta}$] and thus E[X|Y]=(n-Y).[$^p/_{1\text{-}\theta}$] . Hence

$$E[XY] = E\left[Y \times (n-Y)\frac{p}{1-\theta}\right] = \frac{p}{1-\theta} E(nY - Y^2)$$

$$= \frac{p}{1-\theta}(n^2\theta - n\theta(1-\theta) - n^2\theta^2) = \frac{p \times n\theta}{1-\theta}[n - 1 - \theta(n-1)]$$

$$= \frac{p \times n\theta}{1-\theta}[(1-\theta)(n-1)] = n(n-1)p\theta$$

Therefore,

Cov(X,Y)=E(XY)-E(X)E(Y)=n(n-1)pθ-n^{2}pθ=-npθ

12. Find correlation coefficient of Trinomial distribution by assuming covariance and variances.

Answer:

Let (X,Y)~trinomial (n, p, θ). We know that E(X)=np(1-p) and V(Y)=nθ(1-θ). Also Cov(X,Y)=-npθ

Hence $\rho_{XY} = \dfrac{Cov(X,Y)}{\sqrt{V(X)V(Y)}} = \dfrac{-np\theta}{\sqrt{n^2 p(1-p)\theta(1-\theta)}} = -\left(\dfrac{p\theta}{(1-p)(1-\theta)}\right)^{1/2}$

13. If (X,Y)~Trinomial(n, p, θ), then find coefficient of correlation of the distribution.

Answer:

We know that $\rho_{XY} = \dfrac{Cov(X,Y)}{\sqrt{V(X)V(Y)}}$

The marginal of x and Y are nothing by Binomial variates. Hence E(X)=np(1-p) and V(Y)=nθ(1-θ).

We know that for any two random variables, E[XY]=E[Y.E(X|Y)]. And according to the property 2, X|(Y = y) is Binomial (n-1, $^p/_{1-\theta}$) and hence

E[X|Y=y]= (n-y).[$^p/_{1-\theta}$] and thus E[X|Y]=(n-Y).[$^p/_{1-\theta}$] . Hence

$$E[XY] = E\left[Y \times (n-Y)\frac{p}{1-\theta}\right] = \frac{p}{1-\theta}E(nY - Y^2)$$

$$= \frac{p}{1-\theta}(n^2\theta - n\theta(1-\theta) - n^2\theta^2) = \frac{p \times n\theta}{1-\theta}[n-1-\theta(n-1)]$$

$$= \frac{p \times n\theta}{1-\theta}[(1-\theta)(n-1)] = n(n-1)p\theta$$

Therefore,

Cov(X,Y)=E(XY)-E(X)E(Y)=n(n-1)pθ-n^2pθ=-npθ

Hence $\rho_{XY} = \dfrac{Cov(X,Y)}{\sqrt{V(X)V(Y)}} = \dfrac{-np\theta}{\sqrt{n^2 p(1-p)\theta(1-\theta)}} = -\left(\dfrac{p\theta}{(1-p)(1-\theta)}\right)^{1/2}$

14. Write the generalized form of trinomial distribution.

Answer:

Now suppose that there are k outcomes possible at each of the n independent trials. Denote the outcomes A_1, A_2, … , A_k and the corresponding probabilities p_1, … ,p_k where $\Sigma_j p_j=1$, j=1, 2, …,k. Let X_j count the number of A_j occurs. Then

$P(X_1 = x_1, \ldots, X_{k-1} = x_{k-1}) = \dfrac{n!}{x_1!\ldots x_{k-1}!(n-\sum_{j=1}^{k-1} x_j)!} p_1^{x_1} p_2^{x_2} \ldots p_{k-1}^{x_{k-1}} p_k^{n-\sum_{j=1}^{k-1} x_j}$, where x_1, x_2, …, x_{k-1} are non-negative integers with $\Sigma_j x_j \le n$

15. In a recent three-way election for a large country, candidate A received 20% of the votes, candidate B received 30% of the votes, and candidate C received 50% of the votes. If six voters are selected randomly, what is the probability that there will be exactly one supporter for candidate A, two supporters for candidate B and three supporters for candidate C in the sample?

Answer:

Since we're assuming that the voting population is large, it is reasonable and permissible to think of the probabilities as unchanging once a voter is selected for the sample.

Hence, we can use trinomial distribution to find the probability. i.e., we can write the pmf as,

$$P(X=x, Y=y, Z=z) = \frac{n!}{x!\,y!\,z!} p_1^x p_2^y p_3^z$$, where x+y+z=n and $p_1+p_2+p_3=1$.

Here $p_1=0.2$, $p_2=0.3$ and $p_3=0.5$

$$P(X=1, Y=2, Z=3) = \frac{6!}{1!2!3!}(0.2)^1(0.3)^2(0.5)^3 = 0.135$$