Frequently Asked Questions

1. What do you mean by moments?
   Answer:
   The characteristics of a frequency distribution are described by its moments.
   The r^{th} moment of a set of values about any constant is the mean of the r^{th} powers of the deviations of the values from the constant.

2. Define r^{th} raw moment.
   Answer:
   If X is a continuous random variable, then r^{th} raw moment is given by,
   Mr'=E (X^r)
   If the continuous random variable X has pdf f(x), then $\mu_r' = \int_x x^r f(x)dx$

3. Define r^{th} central moment.
   Answer:
   If X is a continuous random variable, then r^{th} central moment is given by,
   $\mu_r = E[X - E(X)]^r = \int_x [x - E(X)]^r f(x)dx$

4. Write the conditions for existence of moments.
   Answer:
   E(|X|) always exists in the extended real numbers $R \equiv R\cup\{\infty\}\cup\{-\infty\}$ and E(|X|) ∈ [0,∞]; i.e., either E(|X|) is a non-negative real number or E(|X|) = ∞.
   - *E(X) exists and is finite⇔ E (|X|) < ∞.*
   - *E (|X|) < ∞ ⇒ |E(X)| ≤ E (|X|) < ∞.*
   - *If 1≤ r ≤ s, then E ($|X|^s$) <∞⇒E ($|X|^r$) < ∞.*

5. If E ($|X|^s$) <∞ *then prove that E ($|X|^r$) < ∞.*
   Answer:
   Consider $\int_{-\infty}^{\infty} |x|^r f(x)dx = \int_{-1}^{1} |x|^r f(x)dx + \int_{|x|>1} |x|^r f(x)dx$
   If r<s, then $|x|^r < |x|^s$, for |x|>1
   Therefore $\int_{-\infty}^{\infty} |x|^r f(x)dx \le \int_{-1}^{1} |x|^r f(x)dx + \int_{|x|>1} |x|^s f(x)dx$
   $\le \int_{-1}^{1} f(x)dx + \int_{|x|>1} |x|^s f(x)dx$
   Since for -1<x<1, $|x|^r < 1$
   $\int_{-\infty}^{\infty} |x|^r f(x)dx \le 1 + \int_{|x|>1} |x|^s f(x)dx < \infty$
   ⇒E ($|X|^r$) exists for all 1≤r≤s

6. Show that converse of the result *If 1≤ r ≤ s, then E ($|X|^s$) <∞⇒E ($|X|^r$) < ∞ is not true.*
   Answer:
   We prove this with an example.
   Let X be a random variable with pdf
   F(x) $=2/x^3$, x≥1
   = 0 x<1

We have $= 2\int_1^\infty x^{-2}dx = \left|\frac{-2}{x}\right|_1^\infty = 2$

$E(X^2) = \int_1^\infty x^2 f(x)dx = 2\int_1^\infty x^{-1}dx = \left|\log x\right|_1^\infty = \infty$

Thus for the above distribution, 1st order moment exists but 2nd order moment does not exist.

7. Give the relationship between raw and central moments.
   Answer:
   The relation between raw and central moments is given by the expression,
   $Mr = \mu_r' - {}^rC_1\, \mu_{r-1}'\, \mu_1' + {}^rC_2\, \mu_{r-2}'\, \mu_1'^2 - \ldots + (-1)^r \mu_1'^r$

8. Write 2nd, 3rd and 4th central moments in terms of raw moments.
   Answer:
   $\mu_2 = \mu_2' - \mu_1'^2$
   $\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$
   $\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$

9. Write the uses of first 4 moments.
   Answer:
   Uses of first four moments are,
   - The first moment about zero is the arithmetic mean
   - The second central moment is the variance of the distribution
   - The third central moment is a measure of skewness
   - The fourth central moment is a measure of kurtosis.

10. Write the measure of skewness and how to interpret it?
    Answer:
    The measure of skewness is given by, $\beta_1 = \frac{\mu_3^2}{\mu_2^3}$

    As β_1 is always positive we use the sign of μ_3 to decide whether the distribution is positively skewed or negatively skewed. If μ_3 is positive, the distribution is positively skewed and if μ_3 is negative, the distribution is negatively skewed and if $\mu_3 = 0$ then the distribution is symmetric.

11. Write the measure of kurtosis and how to interpret it?
    Answer:
    The measure of kurtosis is given by, $\beta_2 = \frac{\mu_4}{\mu_2^2}$

    β_2 gives the peak or flatness of the distribution. That is if $\beta_2 < 3$, the distribution is platykurtic, if $\beta_2 > 3$ the distribution is leptokurtic and if $\beta_2 = 3$ then the distribution has normal curve.

12. What do you mean by quantiles?
    Answer:

Some *q*-quantiles have special names
- The 2-quantile is called the median

- The 3-quantiles are called tertiles or terciles → T
- The 4-quantiles are called quartiles → Q
- The 5-quantiles are called quintiles → QU
- The 6-quantiles are called sextiles → S
- The 10-quantiles are called deciles → D
- The 12-quantiles are called duo-deciles → Dd
- The 20-quantiles are called vigintiles → V
- The 100-quantiles are called percentiles → P
- The 1000-quantiles are called permiles → Pr

13. How to find median of a distribution?
Answer:
If a continuous random variable X has probability mass function f(x) and if median of the distribution is taken as M, then $\int_{-\infty}^{M} f(x)dx = \int_{M}^{\infty} f(x)dx = \frac{1}{2}$
Thus solving $\int_{-\infty}^{M} f(x)dx = \frac{1}{2}$ or $\int_{M}^{\infty} f(x)dx = \frac{1}{2}$ for M, we get the value of median.

14. How to find quartiles of a distribution?
Answer:
Since quartiles divide the distribution into 4 equal parts, we have 3 quartiles, Q_1, Q_2 and Q_3 and are given by the equations,
$\int_{-\infty}^{Q_1} f(x)dx = \frac{1}{4}$, $\int_{-\infty}^{Q_2} f(x)dx = \frac{1}{2}$ *and* $\int_{-\infty}^{Q_3} f(x)dx = \frac{3}{4}$

15. How to find deciles and percentiles?
Answer:
As deciles divide the distribution in to 10 equal parts, we have 9 deciles and i^{th} decile is given by the equation, $\int_{-\infty}^{D_i} f(x)dx = \frac{i}{10}, i = 1,2,\ldots,9$
Similarly percentiles divide the distribution into 100 equal parts, we have 99 percentiles and i^{th} percentile is given by the equation,
$\int_{-\infty}^{P_i} f(x)dx = \frac{i}{100}, i = 1,2,\ldots,99$