Frequently Asked Questions

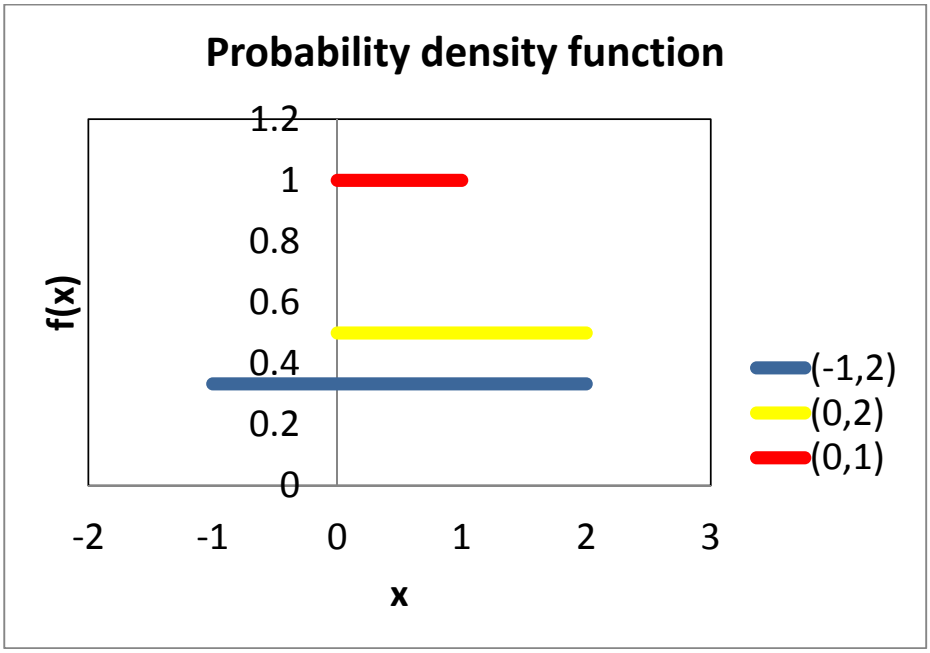
1. Sketch the pdf of uniform distribution with parameters (a, b) and comment.

Answer:

The pdf of uniform distribution is given by,

$f(x)=\frac{1}{[b-a]}$, a<x<b

For different values of (a, b) the curve of pdf is given by,

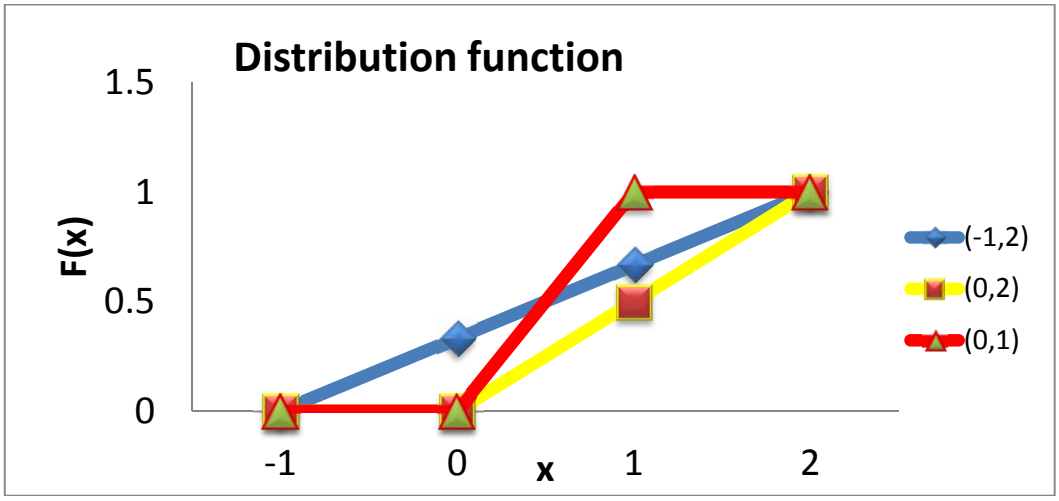


Observe that as the difference between the two parameters decreases, the height increases, that is, the line of density function moves away from the centre.

2. Sketch the distribution function of uniform distribution with parameters (a, b) and comment.

Answer:

The distribution function of uniform distribution is $F(x)=\frac{(x-a)}{(b-a)}$, a<x<b

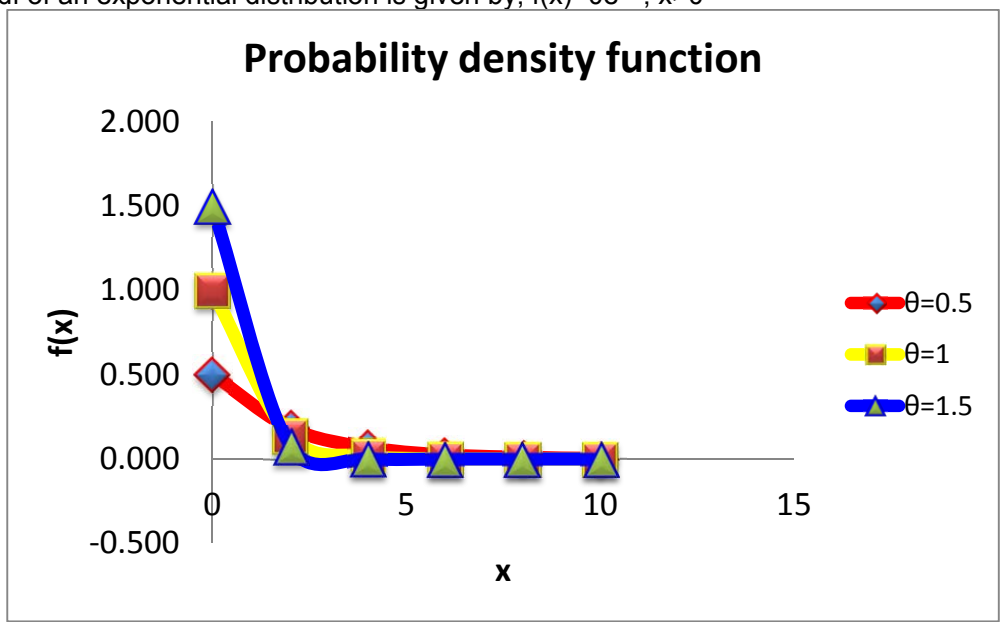


As the difference between the two parameter decreases, the slope of the curve of distribution function also decreases. That is the lines are steeper.

3. Sketch the pdf of exponential distribution with parameters θ and comment.

Answer:

The pdf of an exponential distribution is given by, $f(x)=\theta e^{-x\theta}$, $x>0$

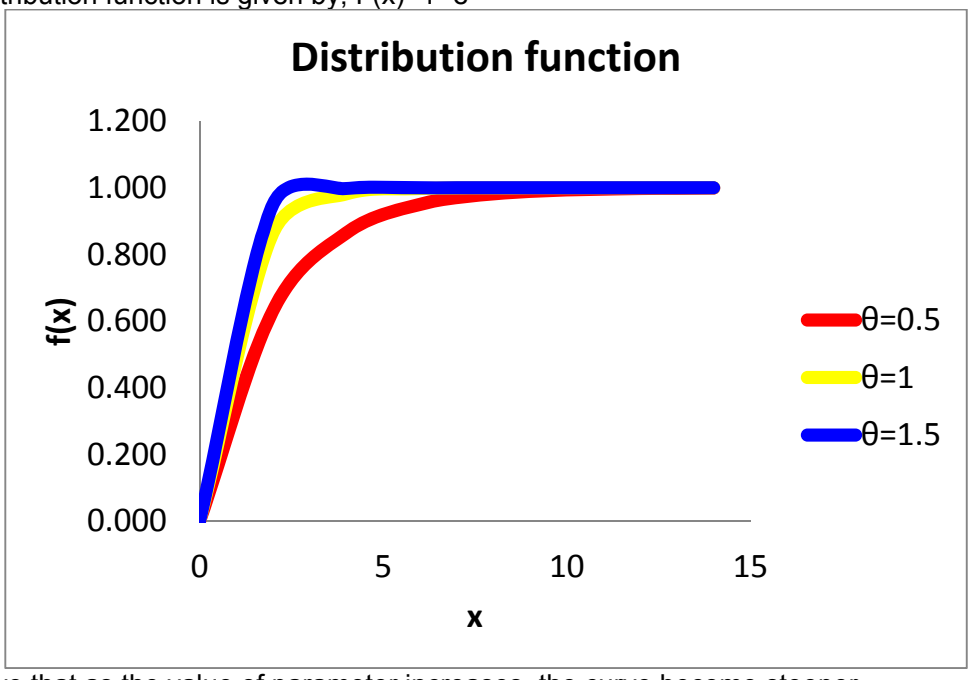


Observe that the pdf of an exponential distribution is strictly decreasing function and as the value of parameter increases, the curve become steeper

4. Sketch the distribution function of exponential distribution with parameters θ and comment.

Answer:

The distribution function is given by, $F(x)=1- e^{-x\theta}$



Observe that as the value of parameter increases, the curve become steeper.

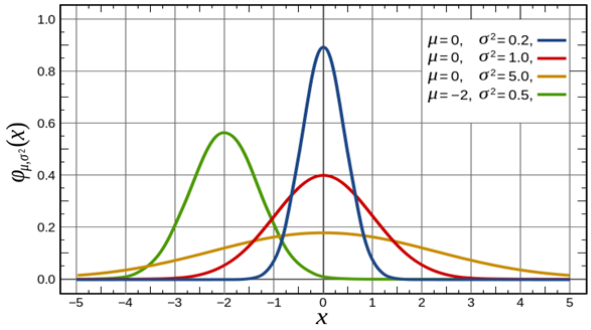
5. Sketch the pdf of Normal distribution and comment.

Answer:

The pdf of normal distribution is given by,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad -\infty < x < \infty ; -\infty < \mu < \infty$$

$$\sigma > 0$$



As σ^2 decreases, the values taken by the variable are in the smaller range and the curve of probability density function flattens. On the other hand, as σ^2 increases, the peakedness of the curve also increases.

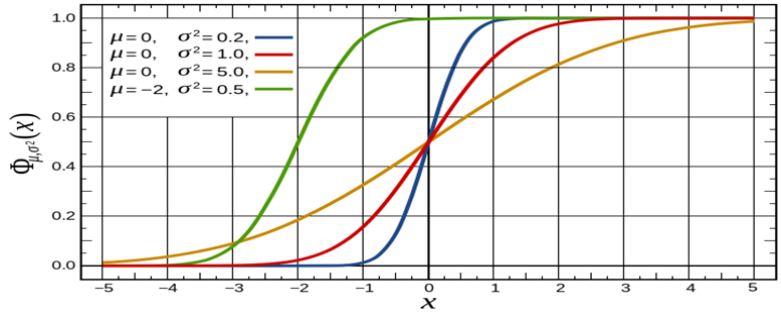
As the value of the parameter μ changes, the curve of probability density function shifts according with the value and is always symmetric about this value.

6. Sketch the distribution function of Normal distribution and comment.

Answer:

Distribution function of normal distribution is given by,

$$F(x) = \Phi_{\mu,\sigma^2}(x) = \int_{-\infty}^{x} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$



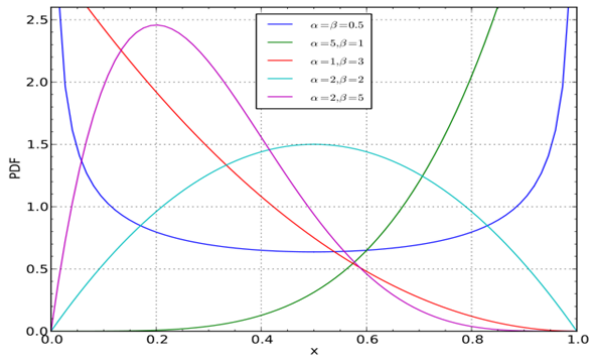
Observe that as the value of σ^2 decreases, the slope of the curve of distribution function increases. It becomes flatter, whereas, as the value of the parameter μ changes, the position of the curve changes.

7. Sketch the probability density function of beta distribution with parameters α and β

Answer:

The pdf of beta distribution of first kind is given by,

$$f(x) = \frac{1}{B(\alpha,\beta)} x^{\alpha-1}(1-x)^{\beta-1}, 0 < x < 1$$

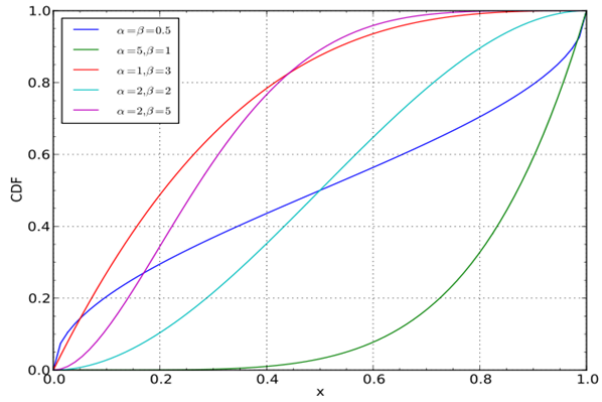


8. Sketch cumulative distribution function of beta distribution with parameters α and β.

Answer:

Distribution function of beta distribution of first kind is given by,

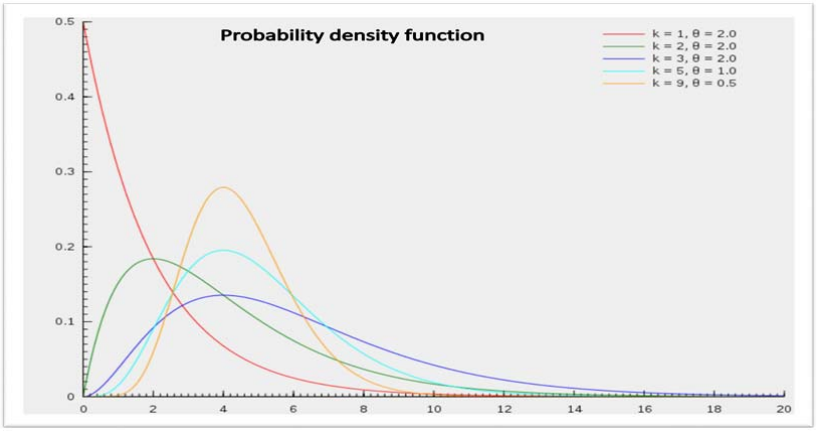
$$cdf = \frac{1}{B(\alpha, \beta)} \int_0^x x^{\alpha-1}(1-x)^{\beta-1} dx$$



9. Let X~Gamma(k, θ). Sketch pdf of the distribution for different values of the parameters and comment.

Answer:

For gamma distribution, pdf is given by, $f(x) = \frac{\theta^k}{\Gamma k} e^{-\theta x} x^{k-1}, x > 0$



Observe that when k=1, the curve is similar to that of exponential curve. Hence exponential distribution is a particular case of gamma distribution that is when k = 1.

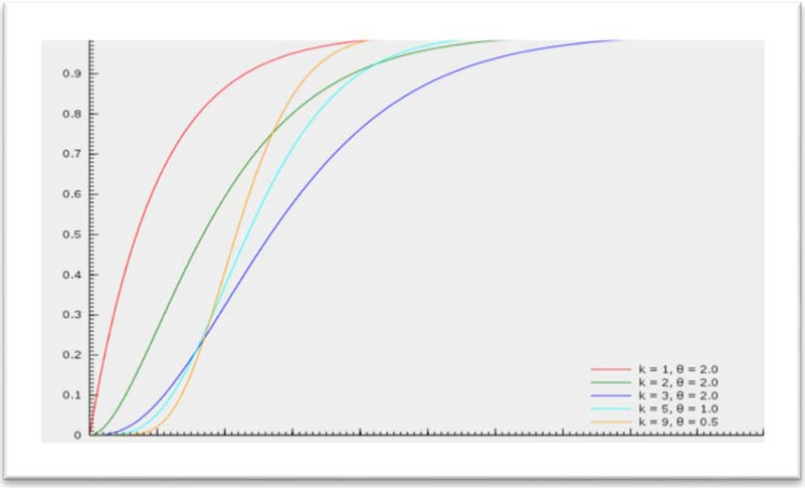
For smaller values of k, the distribution is positively skewed. As k increases, we can notice that the curve become more and more symmetric. Hence for large values of k, gamma distribution tends to normal distribution.

10. Let X ~ Gamma (k, θ). Sketch pdf of the distribution for different values of the parameters.

Answer:

Distribution function is given by,

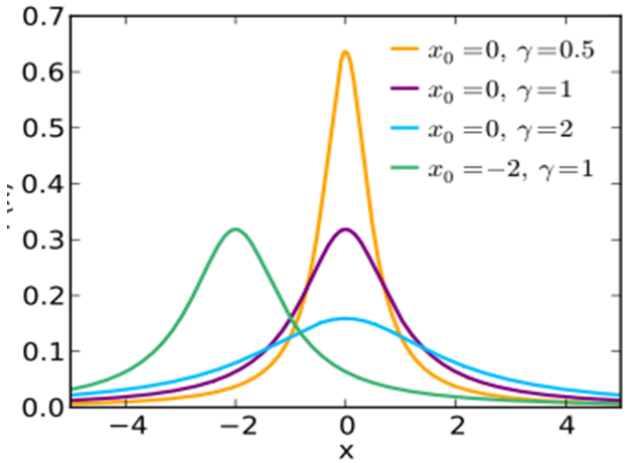
$$F(x) = \frac{\theta^k}{\Gamma k}\int_0^x e^{-\theta x}x^{k-1}dx$$



11. Let X~Cauchy(x0,γ). Sketch pdf of the distribution for different values of the parameters and comment.

Answer:

The pdf of Cauchy distribution is given by, $f(x) = \frac{\gamma}{\pi\left[\gamma^2 + (x - x_0)^2\right]}, -\infty < x_0 < \infty; \gamma > 0$



Observe that the curve of Cauchy distribution has bell shaped and symmetric about the parametric value x_0.

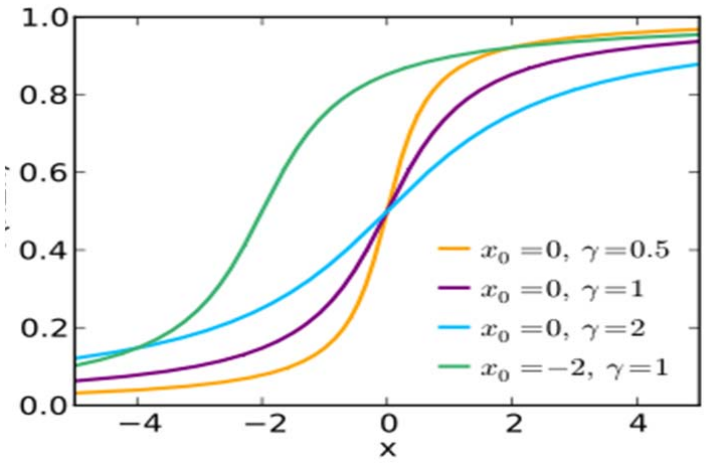
As the value of gamma increases, the curve becomes flatter than the smaller values of gamma.

As the value of the parameter x_0 changes, the curve shifts its position accordingly.

12. Let X~Cauchy(x0,γ). Sketch cumulative distribution function of the distribution for different values of the parameters.

Answer:

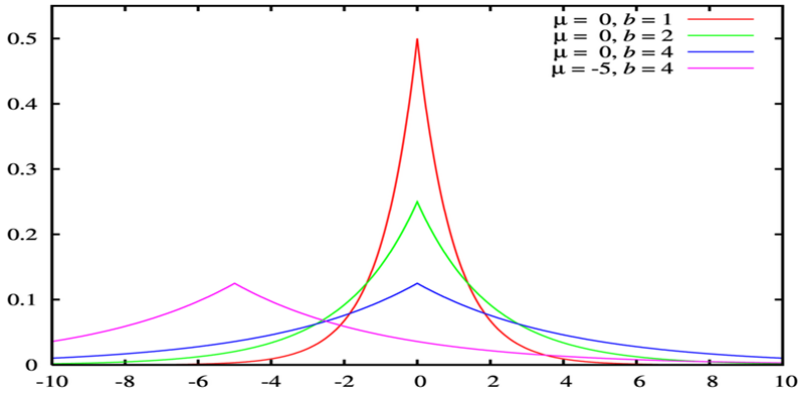
The distribution function is given by, $F(x) = \frac{1}{\pi}\int_{-\infty}^{x} \frac{\gamma}{\left[\gamma^2 + (x - x_0)^2\right]} dx$



13. Let X ~ Laplace (μ, b). Sketch pdf of the distribution for different values of the parameters and comment.

Answer:

Pdf of Laplace distribution is given by, $f(x) = \frac{1}{2b} e^{-\frac{|x-\mu|}{b}}, \quad -\infty < x < \infty \quad b > 0$

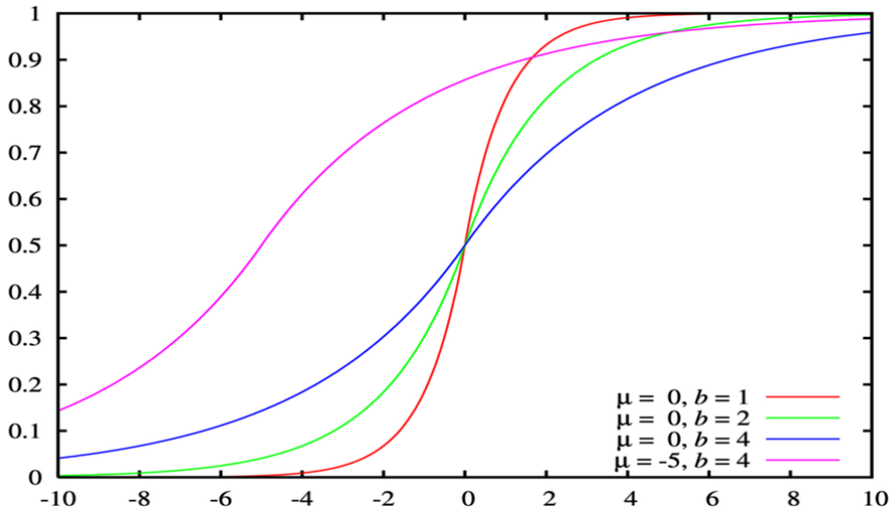


Note that the curve of Laplace distribution is symmetric about μ. It is not bell shaped but has inverted v shape. As the value of the parameter b increases, the curve becomes flatter and as the value of μ changes, the curve shifts its position accordingly.

14. Let X~Laplace(μ, b). Sketch cumulative distribution function of the distribution for different values of the parameters and comment.

Answer:

The distribution function is given by, $F(x) = \frac{1}{2b}\int_{-\infty}^{x} e^{-\frac{|x-\mu|}{b}} dx$



15. Sketch pdf and cumulative distribution function of Laplace distribution for different values of the parameters α and b.

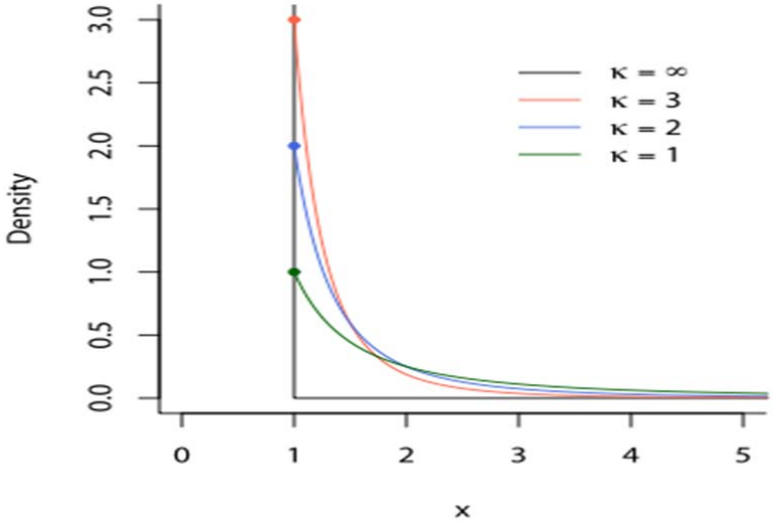
Answer:

We know that pdf of the distribution is $f(x) = \frac{\alpha b^{\alpha}}{x^{\alpha+1}}; x \geq b, \alpha > 0$

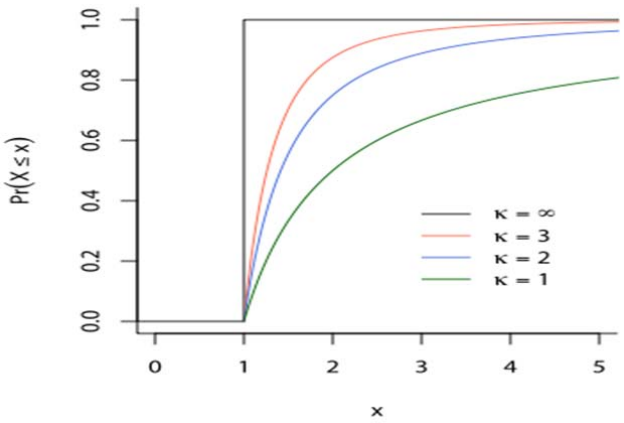
Cumulative distribution function is given by, $F(x) = 1 - \frac{b^{\alpha}}{x^{\alpha}}$

To draw the graph, we have taken b=1 and α=k.

Graph of Pdf



Graph of cumulative distribution function


Pr(X ≤ x)
0.0
0.2
0.4
0.6
0.8
1.0
0
1
2
3
4
5
x
κ = ∞
κ = 3
κ = 2
κ = 1