Frequently Asked Questions

1. Define bivariate normal distribution.

Answer:

If (X,Y)~BVN($\mu_1,\mu_2,\sigma_1^2,\sigma_2^2,\rho$), its pdf is given by,

$$f_{XY}(x,y) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{(1-\rho^2)}} e^{-\frac{1}{2(1-\rho^2)}\left(\frac{(x-\mu_1)^2}{\sigma_1^2} - 2\rho\frac{(x-\mu_1)(y-\mu_2)}{\sigma_1\sigma_2} + \frac{(y-\mu_2)^2}{\sigma_2^2}\right)}$$

, Range of the distribution is, $-\infty<(x,y)<\infty$, $-\infty<(\mu_1, \mu_2)<\infty$, $(\sigma_1, \sigma_2)>0$ $-1<\rho<1$

2. How will you draw samples from bivariate normal distribution?

Answer:

The following algorithm can be used to generate sample from the bivariate normal distribution:

Let z_1 and z_2 be independent draws from the standard normal distribution, $N(0,1)$.

Then x and y, calculated as follows will have a joint bivariate normal distribution with parameters ($\mu_1,\mu_2,\sigma_1^2,\sigma_2^2,\rho$):

$x=\mu_1+\sigma_1 z_1$

$y=\mu_2+\sigma_2[z_1\rho+z_2\sqrt{(1-\rho^2)}]$

3. Write the expression writing random samples from bivariate normal distribution.

Answer:

The expressions used from writing samples from bivariate normal distribution are, $x=\mu_1+\sigma_1 z_1$ and $y=\mu_2+\sigma_2[z_1\rho+z_2\sqrt{(1-\rho^2)}]$

4. Write the expression writing random samples from bivariate normal distribution with parameters $\mu_1=30$, $\mu_2=47$, $\sigma_1^2=49$, $\sigma_2^2=81$, $\rho=0.75$.

Answer:

$x=\mu_1+\sigma_1 z_1=30+7z_1$ *and*

$y=\mu_2+\sigma_2[z_1\rho+z_2\sqrt{(1-\rho^2)}] = 47+9((z_1*0.75)+(z_2\sqrt{(1-0.75^2)}))$

5. Write the expression writing random samples from bivariate normal distribution with parameters $\mu_1=30$, $\mu_2=47$, $\sigma_1^2=49$, $\sigma_2^2=81$, $\rho=0$.

Answer:

$x=\mu_1+\sigma_1 z_1=30+7z_1$ *and*

$y=\mu_2+\sigma_2[z_1\rho+z_2\sqrt{(1-\rho^2)}] = 47+9((z_1*0)+(z_2\sqrt{(1-0^2)})) = 47+9z_2$

6. Write the expression writing random samples from bivariate normal distribution with parameters $\mu_1=0$ $\mu_2=0$, $\sigma_1=1$ $\sigma_2=1$ and $\rho=0.5$

Answer

$x=\mu_1+\sigma_1 z_1=z_1$ *and*

$y=\mu_2+\sigma_2[z_1\rho+z_2\sqrt{(1-\rho^2)}] = (z_1*0.5)+(z_2\sqrt{(1-0.5^2)})$

7. What can you say while generating random samples from bivariate random samples, $\rho=0$.

Answer:

When $\rho=0$, we get the following expression for x and y.

$x=\mu_1+\sigma_1 z_1$ *and* $y=\mu_2+\sigma_2[z_1\rho+z_2\sqrt{(1-\rho^2)}] = \mu_2+\sigma_2 z_2$, that is bivariate random samples become two independent random samples from $N(\mu_1, \sigma_1^2)$ and $N(\mu_2,\sigma_2^2)$

8. Write the expression writing random samples from bivariate normal distribution with parameters $\mu_1=0$ $\mu_2=0$, $\sigma_1=1$ $\sigma_2=1$ and $\rho=1$and comment.

Answer:

The expressions for x and y are given by,

$x=\mu_1+\sigma_1 z_1 = z_1$ *and* $y=\mu_2+\sigma_2[z_1\rho+z_2\sqrt{(1-\rho^2)}] = z_1$ observe that in this case the bivariate samples are identical.

9. Write the expression writing random samples from bivariate normal distribution with parameters $\mu_1=30$, $\mu_2=47$, $\sigma_1^2=49$, $\sigma_2^2=81$, $\rho=1$ and comment

Answer:

$x=\mu_1+\sigma_1 z_1=30+7z_1$ *and*

$y=\mu_2+\sigma_2[z_1\rho+z_2\sqrt{(1-\rho^2)}] =47+9((z_1)$

observe that in this case we need not consider two sets of random numbers. It is enough if we consider only one random number and get two samples from that.

10. How will you get convert the random numbers into probabilities?

Answer:

To convert given random numbers, first we see how many digits are given. Suppose two digit random numbers are given, then they are divided by 100. Suppose 3 digit random numbers are given then they are divided by 1000 and if 4 digit random numbers are given then they are divided by 10000.

11. Can we have any number of digits in random numbers in a problem of drawing samples using random numbers?

Answer:

For a given problem, we should have same number of digits in the random numbers. Suppose we have given 3 digit random numbers and a number with 2 digits appears, then we put a zero before the number so that the value of the given number doesn't change and we will have 3 digit random number.

12. Draw 10 random samples from Bivariate normal distribution with parameters ($\mu_1=30$, $\mu_2=47$, $\sigma_1^2=49$, $\sigma_2^2=81$, $\rho=0.75$) using following 20 random numbers. 268, 166, 838, 280, 455, 465, 696, 020, 469, 797, 346, 974, 418, 615, 159, 659, 157, 628, 465, 705.

Answer:

Since we need to draw 10 bivariate random samples, let us consider 20 random samples. These 20 random samples are taken pair wise to get bivariate sample. Since we have 3 digit random numbers, first we convert random numbers into probability by dividing them by 1000.

Now let us find corresponding points to these probabilities, which are named as z_1 and z_2. then using the relation, $x=\mu_1+\sigma_1 z_1=30+7z_1$ *and* $y=\mu_2+\sigma_2[z_1\rho+z_2\sqrt{(1-\rho^2)}]=47+9((z_1*0.75)+(z_2\sqrt{(1-0.75^2)}))$, we obtain the random samples.

The calculations are done in the following table.

Random variables	$P(Z<z_1)$	$P(Z<z_2)$	z_1	z_2	$x=30+7z_1$	$y=47+9((z_1*0.75)+(z_2\sqrt{(1-$

268	166	0.268	0.166	-0.62	-0.97	25.66	37.04
838	280	0.838	0.280	0.99	-0.58	36.93	50.23
455	465	0.455	0.465	-0.11	-0.09	29.23	45.72
696	20	0.696	0.020	0.51	-2.05	33.57	38.24
469	797	0.469	0.797	-0.09	0.83	29.37	51.33
346	974	0.346	0.974	-0.40	2.00	27.20	56.21
418	615	0.418	0.615	-0.21	0.29	28.53	47.31
159	656	0.159	0.656	-1.00	0.40	23.00	42.63
157	628	0.157	0.628	-1.01	0.33	22.93	42.15
465	705	0.465	0.705	-0.09	0.54	29.37	49.61

13. Draw 10 random samples from Bivariate normal distribution with parameters ($\mu_1=0$ $\mu_2=0$, $\sigma_1=1$ $\sigma_2=1$ and $\rho=0.5$) using following 20 random numbers. 2682, 1663, 8389, 2805, 4557, 4658, 6960, 0020, 4695, 7971, 3468, 9744, 4185, 6154, 1592, 6599, 1573, 6288, 4650, 7053.

Answer:

Since we need to draw 10 bivariate random samples, let us consider 20 random samples. These 20 random samples are taken pair wise to get bivariate sample. Since we have 4 digit random numbers, first we convert random numbers into probability by dividing them by 10000.

Now let us find corresponding points to these probabilities, which are named as z_1 and z_2. then using the relation, $x=\mu_1+\sigma_1 z_1=z_1$ *and* $y=\mu_2+\sigma_2[z_1\rho+z_2\sqrt{(1-\rho^2)}]=(z_1*0.5)+(z_2\sqrt{(1-0.5^2)})$, we obtain the random samples.

The calculations are done in the following table.

Random Numbers		$P(Z<z_1)$	$P(Z<z_2)$	z_1	z_2	x	Y
2682	1663	0.2682	0.1663	-0.62	-0.97	-0.62	-0.78
8389	2805	0.8389	0.2805	0.99	-0.58	0.99	1.24
4557	4658	0.4557	0.4658	-0.11	-0.09	-0.11	-0.14
6960	0020	0.6960	0.0020	0.51	-2.05	0.51	0.64
4695	7971	0.4695	0.7971	-0.08	0.83	-0.08	-0.10
3468	9744	0.3468	0.9744	-0.40	2.00	-0.40	-0.50
4185	6154	0.4185	0.6154	-0.21	0.29	-0.21	-0.26
1592	6569	0.1592	0.6569	-1.00	0.40	-1.00	-1.25
1573	6288	0.1573	0.6288	-1.01	0.33	-1.01	-1.26
4650	7053	0.4650	0.7053	-0.09	0.54	-0.09	-0.11

14. Draw 10 random samples from Bivariate normal distribution with parameters ($\mu_1=30$, $\mu_2=47$, $\sigma_1^2=49$, $\sigma_2^2=81$, $\rho=0$) using following 20 random numbers. 268, 166, 838, 280, 455, 465, 696, 020, 469, 797, 346, 974, 418, 615, 159, 659, 157, 628, 465, 705.

Answer:

Since we need to draw 10 bivariate random samples, let us consider 20 random samples. These 20 random samples are taken pair wise to get bivariate sample.

Since we have 3 digit random numbers, first we convert random numbers into probability by dividing them by 1000.
Now let us find corresponding points to these probabilities, which are named as z_1 and z_2. then using the relation, $x=\mu_1+\sigma_1 z_1=30+7z_1$ *and* $y=\mu_2+\sigma_2[z_1\rho+z_2\sqrt{(1-\rho^2)}]=47+9((z_1*0)+(z_2\sqrt{(1-0^2)}))= 47+9z_2$, we obtain the random samples.
The calculations are done in the following table.

Random Numbers		$P(Z<z_1)$	$P(Z<z_2)$	z_1	z_2	x	Y
268	166	0.268	0.166	-0.62	-0.97	25.66	38.27
838	280	0.838	0.280	0.99	-0.58	36.93	41.78
455	465	0.455	0.465	-0.11	-0.09	29.23	46.19
696	020	0.696	0.020	0.51	-2.05	33.57	28.55
469	797	0.469	0.797	-0.09	0.83	29.37	54.47
346	974	0.346	0.974	-0.40	2.00	27.20	65.00
418	615	0.418	0.615	-0.21	0.29	28.53	49.61
159	656	0.159	0.656	-1.00	0.40	23.00	50.60
157	628	0.157	0.628	-1.01	0.33	22.93	49.97
465	705	0.465	0.705	-0.09	0.54	29.37	51.86

15. Draw 10 random samples from Bivariate normal distribution with parameters ($\mu_1=0$ $\mu_2=0$, $\sigma_1=1$ $\sigma_2=1$ and $\rho=1$) using following 20 random numbers. 2682, 1663, 8389, 2805, 4557, 4658, 6960, 0020, 4695, 7971, 3468, 9744, 4185, 6154, 1592, 6599, 1573, 6288, 4650, 7053.

Answer:
Since we need to draw 10 bivariate random samples, let us consider 20 random samples. These 20 random samples are taken pair wise to get bivariate sample.
Since we have 4 digit random numbers, first we convert random numbers into probability by dividing them by 10000.
Now let us find corresponding points to these probabilities, which are named as z_1 and z_2. then using the relation, $x=\mu_1+\sigma_1 z_1=z_1$ *and* $y=\mu_2+\sigma_2[z_1\rho+z_2\sqrt{(1-\rho^2)}]=z_1$, we obtain the random samples.
The calculations are done in the following table.

Random Numbers		$P(Z<z_1)$	$P(Z<z_2)$	z_1	z_2	x	Y
2682	1663	0.2682	0.1663	-0.62	-0.97	-0.62	-0.62
8389	2805	0.8389	0.2805	0.99	-0.58	0.99	0.99
4557	4658	0.4557	0.4658	-0.11	-0.09	-0.11	-0.11
6960	0020	0.6960	0.0020	0.51	-2.05	0.51	0.51
4695	7971	0.4695	0.7971	-0.08	0.83	-0.08	-0.08
3468	9744	0.3468	0.9744	-0.40	2.00	-0.40	-0.40
4185	6154	0.4185	0.6154	-0.21	0.29	-0.21	-0.21
1592	6569	0.1592	0.6569	-1.00	0.40	-1.00	-1.00
1573	6288	0.1573	0.6288	-1.01	0.33	-1.01	-1.01
4650	7053	0.4650	0.7053	-0.09	0.54	-0.09	-0.09

Observe that both the samples are identical.