Frequently Asked Questions

1. Write the expression for marginal expectation of discrete bivariate data.

Answer:

For discrete bivariate data, marginal expectation is given by,

$E(X)=\Sigma x.p(x)$, where $p(x)$ is marginal distribution of X

$E(Y)=\Sigma y.p(y)$, where $p(y)$ is marginal distribution of Y.

2. Write the expression for finding the marginal variance of the bivariate distribution.

Answer:

$V(x)=E(X)^2-[E(X)]^2$ and $V(Y)=E(Y)^2-[E(Y)]^2$, E(X) and E(Y) are marginal expectations.

3. Give an expression for Conditional expectation of discrete random variable X given $Y=y_j$

Answer:

Conditional expectation of discrete random variable X given $Y=y_j$ is given by,

$$E[X \mid Y = y_j] = \sum_{i=1}^{\infty} x_i P(X = x_i \mid Y = y_j)$$

4. Give an expression for conditional expectation of discrete random variable Y give X=xi.

Answer:

Conditional expectation of discrete random variable Y given $X=x_i$ is given by,

$$E[Y \mid X = x_i] = \sum_{j=1}^{\infty} y_j P(Y = y_j \mid X = x_i)$$

5. Write an expression for conditional variance of bivariate distribution.

Answer:

$$V[X \mid Y = y_j] = E[\{X - E(X \mid Y = y_j)^2\} \mid Y = y_j]$$

$$V[Y \mid X = x_i] = E[\{Y - E(Y / X = x_i)^2\} \mid X = x_i]$$

6. Two random variables X and Y have the following joint probability density function. $f(x,y)=k(4-x-y);0\le x\le 2,\ 0\le y\le 2$. Find constant k, E(X), E(Y).

Answer:

Since f(x,y) is a pdf, ∬f(x,y)dx dy=1

$$k\int_0^2\int_0^2 (4 - x - y)dxdy = 1 \Rightarrow k\int_0^2 (6 - 2y)dy = 1 \Rightarrow k(8) = 1 \Rightarrow k = \frac{1}{8}$$

Before we find Expectation and variance, we find marginal distribution of X and Y.

Marginal distribution of X is given by,

$$f(x) = \frac{1}{8}\int_0^2 (4 - x - y)dy = \frac{1}{8}(6 - 2x) = \frac{1}{4}(3 - x);0 \le x \le 2$$

Marginal distribution of Y is given by,

$$f(y) = \frac{1}{8}\int_0^2 (4 - x - y)dx = \frac{1}{8}(6 - 2y) = \frac{1}{4}(3 - y);0 \le y \le 2$$

Now, let us obtain Expectation.

$$E(x) = \int_0^2 xf(x)dx = \frac{1}{4}\int_0^2 x(3 - x)dx = \frac{5}{6}$$

$$E(y) = \int_0^2 yf(y)dx = \frac{1}{4}\int_0^2 y(3 - y)dy = \frac{5}{6}$$

7. Two random variables X and Y have the following joint probability density function. $f(x,y)=(4-x-y)/8; 0 \le x \le 2,\ 0 \le y \le 2$. Find V(X), V(Y) and Cov(X, Y).

Answer:

Before we find variance, let us find marginal distribution of X and Y.

Marginal distribution of X is given by,

$$f(x) = \frac{1}{8}\int_0^2 (4 - x - y)dy = \frac{1}{8}(6 - 2x) = \frac{1}{4}(3 - x); 0 \le x \le 2$$

Marginal distribution of Y is given by,

$$f(y) = \frac{1}{8}\int_0^2 (4 - x - y)dx = \frac{1}{8}(6 - 2y) = \frac{1}{4}(3 - y); 0 \le y \le 2$$

To find variance, first we find expectation of X and its square.

$$E(x) = \int_0^2 xf(x)dx = \frac{1}{4}\int_0^2 x(3 - x)dx = \frac{5}{6}$$

$$E(y) = \int_0^2 yf(y)dx = \frac{1}{4}\int_0^2 y(3 - y)dy = \frac{5}{6}$$

$E(X^2)$ and $E(Y^2)$

$$E(X^2) = \int_0^2 x^2 f(x)dx = \frac{1}{4}\int_0^2 x^2(3 - x)dx = 1$$

$$E(Y^2) = \int_0^2 y^2 f(y)dy = \frac{1}{4}\int_0^2 y^2(3 - y)dy = 1$$

Variances are given as follows.

$V(X)= E(X^2)-\{E(X)\}^2 = 1-(^5/_6)^2 = {}^{11}/_{36}$

$V(Y)= E(Y^2)-\{E(Y)\}^2 = 1-(^5/_6)^2 = {}^{11}/_{36}$

We know that Cov(X,Y)=E(XY)-E(X)E(Y)

We have already found E(X) and E(Y). Now let us obtain E(XY).

$$E(XY) = \int_0^2\int_0^2 xyf(,y)dxdy = \frac{1}{8}\int_0^2 y\left[\int_0^2 x(4 - x - y)dx\right]dy = \frac{1}{8}\int_0^2 y\left(\frac{16}{3} - 2y\right)dy = \frac{2}{3}$$

Hence $Cov(X,Y)= {}^2/_3 - (^5/_6)(^5/_6) = -{}^1/_{36}$

8. X and Y have a bivariate distribution given by $P(X = x \cap Y = y) = \dfrac{x + 3y}{24}; (x, y) = (1,1), (1,2), (2,1), (2,2)$. Find E(X), E(Y).

Answer:

First let us obtain marginal distribution of X and Y.

Marginal distribution of X is given by

$$P(X = x) = \sum_{y=1}^{2} \frac{x + 3y}{24} = \frac{x + 3.1}{24} + \frac{x + 3.2}{24} = \frac{2x + 9}{24}; x = 1,2$$

Marginal distribution of Y is given by,

$$P(Y = y) = \sum_{x=1}^{2} \frac{x + 3y}{24} = \frac{1 + 3y}{24} + \frac{2 + 3y}{24} = \frac{3 + 6y}{24}; y = 1,2$$

Now let us find the expectations.

$$E(X) = \sum_{x=1}^{2} xP(X = x) = \sum_{x=1}^{2} x\frac{2x + 9}{24}$$

$$= 1\times\frac{2.1 + 9}{24} + 2\times\frac{2.2 + 9}{24} = 1\times\frac{11}{24} + 2\times\frac{13}{24} = \frac{37}{24}$$

$$E(Y)=\sum_{y=1}^{2} y(P(Y=y)=\sum_{y=1}^{2} y\,\frac{3+6y}{24}$$

$$=1\times\frac{3+6.1}{24}+2\times\frac{3+6.2}{24}=1\times\frac{9}{24}+2\times\frac{15}{24}=\frac{39}{24}=\frac{13}{8}$$

9. X and Y have a bivariate distribution given by $P(X=x\cap Y=y)=\frac{x+3y}{24}\,;(x,y)=(1,1),(1,2),(2,1),(2,2)$. Find E(X), E(Y), V(X), V(Y) and Cov(X,Y).

Answer:

First let us obtain marginal distribution of X and Y.

Marginal distribution of X is given by

$$P(X=x)=\sum_{y=1}^{2}\frac{x+3y}{24}=\frac{x+3.1}{24}+\frac{x+3.2}{24}=\frac{2x+9}{24}\,;x=1,2$$

Marginal distribution of Y is given by,

$$P(Y=y)=\sum_{x=1}^{2}\frac{x+3y}{24}=\frac{1+3y}{24}+\frac{2+3y}{24}=\frac{3+6y}{24}\,;y=1,2$$

Now let us find the expectations.

$$E(X)=\sum_{x=1}^{2} xP(X=x)=\sum_{x=1}^{2} x\,\frac{2x+9}{24}$$

$$=1\times\frac{2.1+9}{24}+2\times\frac{2.2+9}{24}=1\times\frac{11}{24}+2\times\frac{13}{24}=\frac{37}{24}$$

$$E(Y)=\sum_{y=1}^{2} y(P(Y=y)=\sum_{y=1}^{2} y\,\frac{3+6y}{24}$$

$$=1\times\frac{3+6.1}{24}+2\times\frac{3+6.2}{24}=1\times\frac{9}{24}+2\times\frac{15}{24}=\frac{39}{24}=\frac{13}{8}$$

Before finding the variance, let us find $E(X^2)$ and $E(Y^2)$

$$E(X^2)=\sum_{x=1}^{2} x^2P(X=x)=\sum_{x=1}^{2} x^2\,\frac{2x+9}{24}$$

$$=1^2\times\frac{2.1+9}{24}+2^2\times\frac{2.2+9}{24}=1\times\frac{11}{24}+4\times\frac{13}{24}=\frac{63}{24}=\frac{21}{8}$$

$$E(Y^2)=\sum_{y=1}^{2} y^2P(Y=y)=\sum_{y=1}^{2} y^2\,\frac{3+6y}{24}$$

$$=1^2\times\frac{3+6.1}{24}+2^2\times\frac{3+6.2}{24}=1\times\frac{9}{24}+4\times\frac{15}{24}=\frac{69}{24}=\frac{23}{8}$$

$V(X)=E(X^2)-\{E(X)\}^2={}^{21}/_{8}-({}^{37}/_{24})^2=0.2483$

$V(Y)=E(Y^2)-\{E(Y)\}^2={}^{23}/_{8}-({}^{13}/_{8})^2=0.2344$

To find cov(X, Y), let us find E(XY)

$$E(XY)=\sum_{x=1}^{2}\sum_{y=1}^{2} xyP(X=x\cap Y=y)=\sum_{x=1}^{2}\sum_{y=1}^{2} xy\,\frac{x+3y}{24}$$

$$=\sum_{x=1}^{2} x\left(\sum_{y=1}^{2} y\,\frac{x+3y}{24}\right)=\sum_{x=1}^{2} x\left(1.\frac{x+3.1}{24}+2\,\frac{x+3.2}{24}\right)$$

$$=\sum_{x=1}^{2} x\left(\frac{3x+15}{24}\right)=1.\frac{3.1+15}{24}+2\,\frac{3.2+15}{24}=\frac{60}{24}=\frac{5}{2}$$

Hence covariance
Cov(X, Y)=E(XY)-E(X)E(Y)

$= {}^{5}/_{2} - [{}^{37}/_{24} \,.\, {}^{13}/_{8}]$

$=-0.0052$

10. Let f(x, y)=8xy, 0<x<y<1; and zero elsewhere. Find E(Y|X=x).

Answer:

First let us find the marginal pdf of given distribution of X

$$f_X(x) = \int_x^1 f(x,y)dy = 8x\int_x^1 ydy = 4x(1-x), 0 < x < 1$$

Conditional pdf of Y given X is given by,

$$f_{Y|X}(y \mid x) = \frac{f(x,y)}{f_X(x)} = \frac{2y}{1-x^2}; 0 < x < y < 1$$

$$E(Y \mid X = x) = \int_x^1 y\left[\frac{2y}{1-x^2}\right]dy = \frac{2}{3}\left[\frac{1-x^3}{1-x^2}\right] = \frac{2}{3}\left[\frac{1+x+x^2}{1+x}\right]$$

11. Let f(x, y)=8xy, 0<x<y<1; and zero elsewhere. Find E(XY|X=x).

Answer:

First let us find the marginal pdf of given distribution of X

$$f_X(x) = \int_x^1 f(x,y)dy = 8x\int_x^1 ydy = 4x(1-x), 0 < x < 1$$

Conditional pdf of Y given X is given by,

$$f_{Y|X}(y \mid x) = \frac{f(x,y)}{f_X(x)} = \frac{2y}{1-x^2}; 0 < x < y < 1$$

$$E(Y \mid X = x) = \int_x^1 y\left[\frac{2y}{1-x^2}\right]dy = \frac{2}{3}\left[\frac{1-x^3}{1-x^2}\right] = \frac{2}{3}\left[\frac{1+x+x^2}{1+x}\right]$$

$$E(XY \mid X = x) = xE(Y \mid X = x) = \frac{2}{3}\frac{x(1+x+x^2)}{1+x}$$

12. Let f(x, y)=8xy, 0<x<y<1 and zero elsewhere. Find V(Y|X=x).

Answer:

First let us find the marginal pdf of given distribution of X

$$f_X(x) = \int_x^1 f(x,y)dy = 8x\int_x^1 ydy = 4x(1-x), 0 < x < 1$$

Conditional pdf of Y given X is given by,

$$f_{Y|X}(y \mid x) = \frac{f(x,y)}{f_X(x)} = \frac{2y}{1-x^2}; 0 < x < y < 1$$

$$E(Y \mid X = x) = \int_x^1 y\left[\frac{2y}{1-x^2}\right]dy = \frac{2}{3}\left[\frac{1-x^3}{1-x^2}\right] = \frac{2}{3}\left[\frac{1+x+x^2}{1+x}\right]$$

To find V(Y|X=x), first let us find the $E(Y^2|X=x)$.

$$E(Y^2 \mid X = x) = \int_x^1 y^2\left[\frac{2y}{1-x^2}\right]dy = \frac{1}{2}\left[\frac{1-x^4}{1-x^2}\right] = \frac{1+x^2}{2}$$

$$\therefore V(Y \mid X = x) = E(Y^2 \mid X = x) - [E(Y \mid X = x)]^2$$

$$= \frac{1+x^2}{2} - \frac{4}{9}\frac{(1+x+x^2)^2}{(1+x)^2}$$

13. Let $f(x, y)=21x^2y^3$, $0<x<y<1$ and zero elsewhere be the joint pdf of X and Y. Find the conditional mean of X given Y=y, $0<y<1$.

Answer:

Given $f(x, y)=21x^2y^3$, $0<x<y<1$ and zero elsewhere. Marginal pdf of Y is given by

$$f_Y(y) = \int_0^y f(x,y)dx = 21y^3\int_0^y x^2dx = 7y^6$$

Therefore the conditional pdf of X given Y is given by

$$f(x \mid y) = \frac{f(x,y)}{f_Y(y)} = \frac{21x^2y^3}{7y^6} = 3\frac{x^2}{y^3};0<x<y;0<y<1$$

Conditional mean of X is

$$\text{E(X | Y = y)} = \int_0^y xf(x \mid y)dx = \frac{3}{y^3}\int_0^y x^3dx = \frac{3y}{4};0<y<1$$

14. Let $f(x, y)=21x^2y^3$, $0<x<y<1$ and zero elsewhere be the joint pdf of X and Y. Find the variance of X given Y=y, $0<y<1$.

Answer:

Given $f(x, y)=21x^2y^3$, $0<x<y<1$ and zero elsewhere. Marginal pdf of Y is given by

$$f_Y(y) = \int_0^y f(x,y)dx = 21y^3\int_0^y x^2dx = 7y^6$$

Therefore the conditional pdf of X given Y is given by

$$f(x \mid y) = \frac{f(x,y)}{f_Y(y)} = \frac{21x^2y^3}{7y^6} = 3\frac{x^2}{y^3};0<x<y;0<y<1$$

Conditional mean of X is

$$\text{E(X | Y = y)} = \int_0^y xf(x \mid y)dx = \frac{3}{y^3}\int_0^y x^3dx = \frac{3y}{4};0<y<1$$

To find variance, let us find,

$$\text{E}(\text{X}^2 \mid \text{Y} = \text{y}) = \int_0^y x^2f(x \mid y)dx = \frac{3}{y^3}\int_0^y x^4dx = \frac{3}{y^3}\frac{y^5}{5} = \frac{3}{5}y^2;0<y<1$$

$$\therefore V(X|Y=y)=E(X^2|Y=y)-[E(X|Y=y)]^2 = \frac{3}{5}y^2 - \frac{9}{16}y^2 = \frac{3}{80}y^2;0<y<1$$

15. Let X and Y be two random variable each taking three values -1, 0, 1 and having the following joint probability distribution, find V(Y|X=-1)

Y \ X	-1	0	1	Total
-1	0	0.1	0.1	0.2
0	0.2	0.2	0.2	0.6
1	0	0.1	0.1	0.2
Total	0.2	0.4	0.4	1.0

Answer:

We know that $V(Y|X=-1)=E(Y^2|X=-1) -[E(Y|X=-1)]^2$

$E(Y|X=-1)=\sum_y yP(Y=y,X=-1)=(-1)0+0(0.2)+1(0)=0$

$E(Y^2|X=-1)=\sum_y y^2P(Y=y,X=-1)=(-1^2)0+0^2(0.2)+1^2(0)=0$

$\therefore V(Y|X=-1)=0$