Statistics
Computing Marginal and Conditional Probability Distributions

1. Introduction

Welcome to the series of E-learning modules on Computing Marginal and Conditional Probability Distributions.

By the end of this session, you will be able to:

- **Explain marginal probability distribution of discrete bivariate random variables.**
- **Explain conditional probability distribution of discrete bivariate random variables.**
- **Explain marginal probability distribution of continuous bivariate random variables.**
- **Explain conditional probability distribution of continuous bivariate random variables.**

Two discrete random variables X and Y have the joint probability mass function p X Y of (x, y) is equal to lambda power x into e power minus lambda into p power y into 1 minus p whole power m minus y divided by y factorial into x minus y factorial, where y take values zero, 1, 2 up to x and x take values zero, 1, 2 etc., where lambda and p are constants with lambda greater than zero and zero less than p less than 1. Find

- The marginal probability mass functions of X and Y
- The conditional distribution of Y for a given X and of X for a given Y.

The marginal probability mass function of X is given by, p X of x is equal to summation over y is equal to zero to x p of x, y Is equal to summation over y is equal to zero to x lambda power x into e power minus lambda into p power y into 1 minus p whole power x minus y divided by y factorial into x minus y factorial. Taking the terms independent of y outside and then multiplying and dividing by x factorial we get, Lambda power x into e power minus lambda divided by x factorial into summation over y is equal to zero to x, x factorial into p power y into 1 minus p power x minus y divided by y factorial into x minus y factorial.

Writing factorial terms as x c y, we get, Lambda power x into e power minus lambda divided by x factorial into summation over y is equal to zero to x, x c x into p power y into 1 minus p power x minus y Which is equal to Lambda power x into e power minus lambda divided by x factorial into p plus 1 minus p whole power x Is equal to Lambda power x into e power minus lambda divided by x factorial, x take values zero 1, 2, etc., which is the probability function of a Poisson distribution with parameter lambda.

The marginal probability mass function of Y is given by, pY of y is equal to summation over x is equal to y to infinity p of x, y is equal to summation over x is equal to y to infinity lambda power x into e power minus lambda into p power y into 1 minus p power x minus y divided by y factorial into x minus y factorial

Multiplying and dividing by lambda power y, we get, Lambda into p whole power y into e power minus lambda divided by y factorial into summation over x minus y is equal to zero to infinity lambda into 1 minus p whole power x minus y divided by x minus y factorial. Is equal to lambda into p whole power y into e power minus lambda divided by y factorial into summation over t is equal to zero to infinity, lambda into 1 minus p whole power t divided by t factorial, where t is equal to x minus y Is equal to lambda into p whole power y into e power minus lambda divided by y factorial into e power lambda into 1 minus p Is equal to lambda into p whole power y into e power minus lambda into p divided by y factorial, which is the probability mass function of Poisson distribution with parameter lambda into p

Let us find the conditional distribution of Y given X. p of y given x is equal to p of x, y divided by p of x is equal to lambda power x into e power minus lambda into p power y into 1 minus p power x minus y into x factorial divided by (y factorial into x minus y factorial into lambda power x into e power minus lambda) is equal to p power y into 1 minus p power x minus y into x factorial divided by y factorial into x minus y factorial is equal to x c y, p power y into 1 minus p power x minus y, where x is greater than or equal to y, that is y is equal to zero, 1, 2 up to x.

Let us find the conditional distribution of X given Y. p x given y is equal to p of x, y divided by p of y Is equal to lambda power x into e power minus lambda into p power y into 1 minus p power x minus y divided by y factorial into x minus y factorial, into y factorial divided by lambda into p whole power y into e power minus lambda into p On simplification we get, e power lambda into q into lambda into q whole power x minus y divided by x minus y factorial, where q is equal to 1 minus p. And range is given by x is greater than or equal to y, i.e. x is equal to y, y plus 1, y plus 2 etc.,

2. Exercise 2

The joint probability distribution of 2 random variables X and Y is given below. Find the marginal distribution of X and Y. Also find the Conditional distribution of X given the value of Y is equal to 1 and that of Y given X is equal to 2.

The table shows the joint probability distribution. X take values 1, 2 3 and 4, written vertically and Y take values 1, 2, 3 and 4 written horizontally. The entries inside the table gives the probabilities corresponding to x and y. That the first entry 4 by 36 denotes the probability of X is equal to 1 and Y is equal to 1. The second entry, 3 by 36 denote the probability of X is equal to 1 and Y is equal to 2 and so on.

Solution:
The marginal distribution of X is defined as P(X is equal to x) is equal to summation over y Probability of (X is equal to x and Y is equal to y) Therefore, Probability of (X is equal to 1) is equal to summation over y, Probability of X is equal to 1 and Y is equal to y) Is equal to Probability of (X is equal to 1,Y is equal to 1) plus Probability of (X is equal to 1, Y is equal to 2) plus Probability of (X is equal to 1, Y is equal to 3) plus Probability of (X is equal to 1, Y is equal to 4) Is equal to 4 divided by 36 plus 3 divided by 36 plus 2 divided by 36 plus 1 divided by 36 is equal to 10 divided by 36.

Similarly, Probability of (X is equal to 2) is equal to summation over y, Probability of (X is equal to 2, Y is equal to y) is equal to 9 divided by 36.
Probability of (X is equal to 3) is equal to summation over y, Probability of (X is equal to 3, Y is equal to y) is equal to 8 divided by 36.

Probability of (X is equal to 4) is equal to summation over y Probability of (X is equal to 4, Y is equal to y) is equal to 9 divided by 36.

Similarly, we can obtain the marginal distribution of Y.

Now, we write the marginal distribution in tabular form. First row shows the value taken by x and 2nd row gives the corresponding probability. That is probability of x is equal to 1 is 10 divided by 36. Probability of x is equal to 2 is 9 divided by 36, probability of X is equal to 3 is 8 divided by 36 and Probability of x is equal to 4 is 9 divided by 36. Marginal distribution of Y is as follows.
Probability of y is equal to 1 is 10 divided by 36, probability of Y is equal to 2 is 9 divided by 36, Probability of Y is equal to 3 is 8 divided by 36 and Probability of Y is equal to 4 is 9 divided by 36.
Conditional probability function of X given Y is defined as follows.
Probability of X is equal to x given Y is equal to y is equal to Probability of X is equal to x and Y is equal to y divided by Probability of Y is equal to y.

Therefore, Probability of X is equal to 1 given Y is equal to 1 is equal to Probability of X is equal to 1, Y is equal to 1 divided by Y is equal to 1 Is equal to 4 divided by 36, divided by 11 divided by 36
Is equal to 4 divided by 11.

Probability of X is equal to 2 given Y is equal to 1 is equal to Probability of X is equal to 2, Y is equal to 1 divided by Y is equal to 1 Is equal to 1 divided by 36, divided by 11 divided by 36 Is equal to 1 divided by 11.

Probability of X is equal to 3 given Y is equal to 1 is equal to Probability of X is equal to 3, Y is equal to 1 divided by Y is equal to 1 Is equal to 5 divided by 36, divided by 11 divided by 36 Is equal to 5 divided by 11.

Probability of X is equal to 4 given Y is equal to 1 is equal to Probability of X is equal to 4, Y is equal to 1 divided by Y is equal to 1 Is equal to 1 divided by 36, divided by 11 divided by 36 Is equal to 1 divided by 11.

Now, we will write in tabular form.
The first row denotes the values taken by x and the 2nd row denotes the corresponding conditional probabilities, which are calculated above.

Similarly, Probability of Y is equal to 1 given X is equal to 2 Is equal to Probability of X is equal to 2, Y is equal to 1 divided by X is equal to 2 Is equal to 1 divided by 36 divided by, 9 divided by 36 Is equal to 1 divided by 9.

Probability of Y is equal to 2 given X is equal to 2 Is equal to Probability of X is equal to 2, Y is equal to 2 divided by X is equal to 2 Is equal to 3 divided by 36 divided by, 9 divided by 36 Is equal to 3 divided by 9.

Probability of Y is equal to 3 given X is equal to 2 Is equal to Probability of X is equal to 2, Y is equal to 3 divided by X is equal to 2 Is equal to 3 divided by 36 divided by, 9 divided by 36 Is equal to 3 divided by 9.

Probability of Y is equal to 4 given X is equal to 2 Is equal to Probability of X is equal to 2, Y is equal to 4 divided by X is equal to 2 Is equal to 2 divided by 36 divided by, 9 divided by 36 Is equal to 2 divided by 9.

Now, we will write in tabular form.
The first row denotes the values taken by y and the 2nd row denotes the corresponding conditional probabilities, which are calculated above.

3. Exercise 3

A two dimensional random variable (X, Y) have a joint probability mass function p of x, y is equal to 1 divided by 27, into 2 into x plus y where x and y can assume only integer values 0, 1 and 2. Find the conditional distribution of Y for X is equal to x.

Solution:
The joint probability function is given as, p of x, y is equal to 1 divided by 27, into 2 into x plus y, where x is equal to 0, 1, 2 and y is equal to zero 1, 2.

From p of x, y, we can obtain the following table of joint probability distribution of X and Y. X take values zero, 1, 2 and Y also take values zero, 1, 2. The entries are obtained as follows. P of x is equal to zero, y is equal to zero, is p of zero, zero is equal to 1 divided by 27 into 2 into zero plus zero is equal to zero. p of 1, zero is equal 1 divided by 27 into 2 into 1 plus zero is equal to 2 divided by 27, p of 2, zero is equal to 1 divided by 27 into 2 into 2 plus zero is equal to 4 divided by 27 and so on.

And these values are written in the table. The row totals give the marginal distribution of X, that is P of X is equal to x and column totals give the marginal distribution of Y, P of Y is equal to y.

The conditional distribution of Y given X is equal to x is given by, Probability of Y is equal to y, given X is equal to x Is equal to Probability of X is equal to x, Y is equal to y divided by Probability of X is equal to x, where the denominator Probability of (X is equal to x) is the marginal distribution of X which is obtained in the joint probability table. Here we need to obtain conditional distribution. That is for all possible values of x. Hence, we calculate for the value of Y is equal to zero, 1, 2 for X is equal to zero 1, 2 and write it in a tabular form as follows.

In the table, the first entry zero is conditional distribution of Y is equal to zero when X is also zero. The second entry in the first row is the conditional distribution of Y is equal to zero when X is equal to 1, the third entry in the first row is the conditional probability of Y is equal to zero when x is equal to 2 and hence like this we have written all combination of X and Y.

4. Exercise 4

Joint distribution of X and Y is given by, f of x, y is equal to 4 into x into y into e power minus of x square plus y square, where x is greater than or equal to zero, y greater than or equal to zero. Find the marginal density of x.

Solution:
Marginal density of X is given by f of x is equal to integral over y, f of x, y dy Is equal to 4 into x into integral from zero to infinity y into e power minus of x square plus y square dy Is equal to 4 into x into e

power minus x square into integral from zero to infinity y into e power minus y square dy Is equal to 4 into x into e power minus x square into integral from zero to infinity e power minus t dt divided by 2, where t is taken as y square and we get y into dy is equal to dt divided by 2.
Is equal to 2 into x into e power minus x square into minus e power minus t ranges from zero to infinity
Is equal to 2 into x into e power minus x square, where x is greater than or equal to zero.

5. Exercise 5

The joint probability density function of a two-dimensional random variable (X, Y) is given by, f of (x, y) is equal to 2; zero less than x less than 1, and zero less than y less than x, Find the marginal distribution of X and Y and conditional distribution of X given Y is equal to y. Also verify whether X and Y are independent.

Marginal distribution of X is given by, f of x is equal to integral from minus infinity to infinity f of x, y dy Is equal to integral from zero to x, 2 into dy Is equal to 2 into x, where zero less than x less than 1

To find marginal distribution, range for Y is given as follows. We have zero less than x less than 1, and zero less than y less than x. Combining both we get, zero less than y less than x less than 1. f of y is equal to integral from minus infinity to infinity, f of x, y d x Is equal to integral from y to 1 2 into dx Is equal to 2 into 1 minus y, where zero less than y less than 1.

Conditional distribution of X given Y is equal to y is given by, f of x given by is equal to f of x, y divided by f of y is equal to 2 divided by 2 into 1 minus y Is equal to f1 divided by 1 minus y, where y less than x less than 1. X and Y are independent if f of (x, y) is equal to f(x) into f(y) Consider f of x into f of y Is equal to 2 x into 1 by 1 minus y, which is not equal to f of x, y. Therefore, X and Y are not independent.