# Frequently Asked Questions

1. Define marginal distribution of discrete bivariate random variable.

**Answer:**

For discrete bivariate random variable, the marginal probability distribution is given by,

$P(X=x)=\Sigma_y P(X=x, Y=y)$

$P(Y=y)=\Sigma_x P(X=x, Y=y)$

2. Define marginal distribution of continuous bivariate random variable.

**Answer:**

For continuous bivariate random variable, the marginal probability distribution is given by,

$$f(x) = \int_y f(x, y)dy$$

$$f(y) = \int_x f(x, y)dx$$

3. Define conditional probability function for discrete bivariate random variable.

**Answer:**

Let (X, Y) be a discrete bivariate random variable. Then, the conditional discrete density function or the conditional probability mass function of X, given Y=y denoted by $p_{X|Y}(x|y)$, is defined as $p_{X|Y}(x \mid y) = \dfrac{P(X = x, Y = y)}{P(Y = y)}$ provided $P(Y=y)\neq 0$

4. Define conditional probability function for continuous bivariate random variable.

**Answer:**

Let (X, Y) be continuous bivariate random variable.Then the conditional density function or the conditional probability function of X, given Y denoted by $f_{X|Y}(x|y)$, is defined as $f_{X|Y}(x \mid y) = \dfrac{f(x, y)}{f(y)}$ provided $f(y)\neq 0$

5. For the joint probability distribution of 2 random variables X and Y given below. Find the marginal distribution of X and Y. Also find the Conditional distribution of X given the value of Y=1 and that of Y given X=2.

| Y<br>X | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | $^4/_{36}$ | $^3/_{36}$ | $^2/_{36}$ | $^1/_{36}$ |
| 2 | $^1/_{36}$ | $^3/_{36}$ | $^3/_{36}$ | $^2/_{36}$ |
| 3 | $^5/_{36}$ | $^1/_{36}$ | $^1/_{36}$ | $^1/_{36}$ |
| 4 | $^1/_{36}$ | $^2/_{36}$ | $^1/_{36}$ | $^5/_{36}$ |

**Answer:**

The marginal distribution of X is defined as

$P(X=x)=\Sigma_y P(X=x, Y=y)$

$\therefore P(X=1)=\Sigma_y P(X=1, Y=y)$

$=P(X=1,Y=1)+ P(X=1,Y=2) + P(X=1,Y=3)+ P(X=1,Y=4)$

$={}^4/_{36}+{}^3/_{36}+{}^2/_{36}+{}^1/_{36}={}^{10}/_{36}$

Similarly $P(X=2)=\Sigma_y P(X=2, Y=y)={}^9/_{36}$

$P(X=3)=\Sigma_y P(X=3, Y=y)={}^8/_{36}$

$P(X=4)=\Sigma_y P(X=4, Y=y)={}^9/_{36}$

Similarly we can obtain the marginal distribution of Y.

Marginal distribution of X

| x | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| P(X=x) | $^{10}/_{36}$ | $^{9}/_{36}$ | $^{8}/_{36}$ | $^{9}/_{36}$ |

Marginal Distribution of Y

| y | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| P(Y=y) | $^{10}/_{36}$ | $^{9}/_{36}$ | $^{8}/_{36}$ | $^{9}/_{36}$ |

6. For the joint probability distribution of 2 random variables X and Y given below. Find the Conditional distribution of X given the value of Y=1 and that of Y given X=2.

| Y<br>X | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | $^{4}/_{36}$ | $^{3}/_{36}$ | $^{2}/_{36}$ | $^{1}/_{36}$ |
| 2 | $^{1}/_{36}$ | $^{3}/_{36}$ | $^{3}/_{36}$ | $^{2}/_{36}$ |
| 3 | $^{5}/_{36}$ | $^{1}/_{36}$ | $^{1}/_{36}$ | $^{1}/_{36}$ |
| 4 | $^{1}/_{36}$ | $^{2}/_{36}$ | $^{1}/_{36}$ | $^{5}/_{36}$ |

**Answer:**
Conditional probability function of X given Y is defined as follows.

$$P(X = x \mid Y = y) = \frac{P(X = x, Y = y)}{P(Y = y)}$$

$$P(X = 1 \mid Y = 1) = \frac{P(X = 1, Y = 1)}{P(Y = 1)} = \frac{4/36}{11/36} = \frac{4}{11}$$

$$P(X = 2 \mid Y = 1) = \frac{P(X = 2, Y = 1)}{P(Y = 1)} = \frac{1/36}{11/36} = \frac{1}{11}$$

$$P(X = 3 \mid Y = 1) = \frac{P(X = 3, Y = 1)}{P(Y = 1)} = \frac{5/36}{11/36} = \frac{5}{11}$$

$$P(X = 4 \mid Y = 1) = \frac{P(X = 4, Y = 1)}{P(Y = 1)} = \frac{1/36}{11/36} = \frac{1}{11}$$

Similarly

$$P(Y = 1 \mid X = 2) = \frac{P(X = 2, Y = 1)}{P(X = 2)} = \frac{1/36}{9/36} = \frac{1}{9}$$

$$P(Y = 2 \mid X = 2) = \frac{P(X = 2, Y = 2)}{P(X = 2)} = \frac{3/36}{9/36} = \frac{3}{9}$$

$$P(Y = 3 \mid X = 2) = \frac{P(X = 2, Y = 3)}{P(X = 2)} = \frac{3/36}{9/36} = \frac{3}{9}$$

$$P(Y = 4 \mid X = 2) = \frac{P(X = 2, Y = 4)}{P(X = 2)} = \frac{2/36}{9/36} = \frac{2}{9}$$

7. A two dimensional random variable (X, Y) have a joint probability mass function $p(x,y) = \frac{1}{27}(2x+y)$, where x and y can assume only integer values 0, 1 and 2. Find the conditional distribution of Y for X=x

**Answer:**

The joint probability function is given as,

$$p(x,y) = \frac{1}{27}(2x+y);\ x = 0,1,2;\ y = 0,1,2$$

From p(x,y) we can obtain the following table of joint probability distribution of X and Y

| Y<br>X | 0 | 1 | 2 | Total<br>P(X=x) |
|---|---|---|---|---|
| 0 | 0 | $^1/_{27}$ | $^2/_{27}$ | $^3/_{27}$ |
| 1 | $^2/_{27}$ | $^3/_{27}$ | $^4/_{27}$ | $^9/_{27}$ |
| 2 | $^4/_{27}$ | $^5/_{27}$ | $^6/_{27}$ | $^{15}/_{27}$ |
| Total P((Y=y) | $^6/_{27}$ | $^9/_{27}$ | $^{12}/_{27}$ | 1 |

The row totals give the marginal distribution of X and column totals give the marginal distribution of Y.

The conditional distribution of Y given X =x is given by,

$$P(X = x \mid Y = y) = \frac{P(X = x, Y = y)}{P(Y = y)}$$

Where the denominator P(X=x) is the marginal distribution of X. Here we need to obtain conditional distribution. i.e. for all possible values of x. Hence we calculate for the values of Y=0,1,2 for X=0,1,2 and write it in a tabular form as follows.

| X<br>Y | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 0 | $^1/_3$ | $^2/_3$ |
| 1 | $^2/9$ | $^3/_9$ | $^4/_9$ |
| 2 | $^4/_{15}$ | $^5/_{15}$ | $^6/_{15}$ |

8. Joint distribution of X and Y is given by, $f(x,y) = 4xye^{-(x^2+y^2)};\ x \geq 0, y \geq 0$. Find marginal distribution of X.

**Answer:**

Marginal density of X is given by,

$$f(x) = \int_y f(x,y)dy = 4x\int_0^\infty ye^{-(x^2+y^2)}dy$$

$$= 4xe^{-x^2}\int_0^\infty ye^{-y^2}dy = 4xe^{-x^2}\int_0^\infty e^{-t}\frac{dt}{2}$$

$$= 2xe^{-x^2}\left|-e^{-t}\right|_0^\infty = 2xe^{-x^2},\ x \geq 0$$

9. Joint distribution of X and Y is given by, $f(x,y) = 4xye^{-(x^2+y^2)};\ x \geq 0, y \geq 0$. Find conditional distribution of Y given X.

**Answer:**

We know that conditional distribution of Y given X is, $f(y \mid x) = \dfrac{f(x,y)}{f(x)}$

Where f(x) is marginal distribution of x.
Marginal density of X is given by,

$$f(x) = \int_y f(x,y)dy = 4x\int_0^\infty ye^{-(x^2+y^2)}dy$$

$$= 4xe^{-x^2}\int_0^\infty ye^{-y^2}dy = 4xe^{-x^2}\int_0^\infty e^{-t}\frac{dt}{2}$$

$$= 2xe^{-x^2}\left|-e^{-t}\right|_0^\infty = 2xe^{-x^2}, x \geq 0$$

$$f(y \mid x) = \frac{f(x,y)}{f(x)} = \frac{4xye^{-(x^2+y^2)}}{2xe^{-x^2}} = 2ye^{-y^2}$$

10. Two discrete random variables X and Y have the joint probability mass function $p_{XY}(x,y) = \dfrac{\lambda^x e^{-\lambda} p^y (1-p)^{x-y}}{y!(x-y)!}\; y = 0,1,2\ldots x;\; x = 0,1,2\ldots$ where λ and p are constants with λ>0 and 0<p<1. Find the marginal probability mass functions of X and Y and identify them.

**Answer:**

The marginal pmf of X is given by,

$$p_X(x) = \sum_{y=0}^{x} p(x,y) = \sum_{y=0}^{x} \frac{\lambda^x e^{-\lambda} p^y (1-p)^{x-y}}{y!(x-y)!}$$

$$= \frac{\lambda^x e^{-\lambda}}{x!}\sum_{y=0}^{x}\frac{x!\,p^y(1-p)^{x-y}}{y!(x-y)!} = \frac{\lambda^x e^{-\lambda}}{x!}\sum_{y=0}^{x}\binom{x}{y}p^y(1-p)^{x-y}$$

$$= \frac{\lambda^x e^{-\lambda}}{x!}[p+(1-p)]^x = \frac{\lambda^x e^{-\lambda}}{x!};\; x = 0,1,2,\ldots$$

which is the probability function of a Poisson distribution with parameter λ.

The marginal pmf of Y is given by,

$$p_y(y) = \sum_{x=y}^{\infty} p(x,y) = \sum_{x=y}^{\infty}\frac{\lambda^x e^{-\lambda} p^y (1-p)^{x-y}}{y!(x-y)!}$$

$$= \frac{(\lambda p)^y e^{-\lambda}}{y!}\sum_{x-y=0}^{\infty}\frac{[\lambda(1-p)]^{x-y}}{(x-y)!} = \frac{(\lambda p)^y e^{-\lambda}}{y!}\sum_{t=0}^{\infty}\frac{[\lambda(1-p)]^t}{(t)!}, (t = x-y)$$

$$= \frac{(\lambda p)^y e^{-\lambda}}{y!}e^{\lambda(1-p)} = \frac{(\lambda p)^y e^{-\lambda p}}{y!};\; y = 0,1,2,\ldots$$

This is the probability mass function of Poisson distribution with parameter λp

11. Two discrete random variables X and Y have the joint probability mass function $p_{XY}(x,y) = \dfrac{\lambda^x e^{-\lambda} p^y (1-p)^{x-y}}{y!(x-y)!}\; y = 0,1,2\ldots x;\; x = 0,1,2\ldots$ where λ and p are constants with λ>0 and 0<p<1. Find the The conditional distribution of Y for a given X

**Answer:**

The marginal pmf of X is given by,

$$p_X(x) = \sum_{y=0}^{x} p(x,y) = \sum_{y=0}^{x} \frac{\lambda^x e^{-\lambda} p^y (1-p)^{x-y}}{y!(x-y)!}$$

$$= \frac{\lambda^x e^{-\lambda}}{x!} \sum_{y=0}^{x} \frac{x!\, p^y (1-p)^{x-y}}{y!(x-y)!} = \frac{\lambda^x e^{-\lambda}}{x!} \sum_{y=0}^{x} \binom{x}{y} p^y (1-p)^{x-y}$$

$$= \frac{\lambda^x e^{-\lambda}}{x!} [p + (1-p)]^x = \frac{\lambda^x e^{-\lambda}}{x!}; x = 0,1,2,\ldots$$

Conditional distribution of Y given X is given by,

$$p(y / x) = \frac{p(x,y)}{p(x)} = \frac{\lambda^x e^{-\lambda} p^y (1-p)^{x-y} x!}{y!(x-y)!\lambda^x e^{-\lambda}}$$

$$= \frac{p^y (1-p)^{x-y} x!}{y!(x-y)!} = \binom{x}{y} p^y (1-p)^{x-y}, x \geq y, i.e. y = 0,1,2,\ldots x$$

12. The joint probability density function of a two-dimensional random variable (X, Y) is given by, f(x,y)=2;0<x<1, 0<y<x,
    Find marginal distribution of X and Y and conditional distribution of X given Y=y.

**Answer:**

The marginal distribution of X is given by,

$$f(x) = \int_{-\infty}^{\infty} f(x,y)dy = \int_0^x 2dy = 2x, 0 < x < 1$$

To find marginal distribution, range for Y is given as follows. We have 0<x<1, 0<y<x. Combining both we get, 0<y<x<1

$$f(y) = \int_{-\infty}^{\infty} f(x,y)dx = \int_y^1 2dx = 2(1-y), 0 < y < 1$$

Conditional distribution of X given Y is equal to y is given by,

$$f(x \mid y) = \frac{f(x,y)}{f(y)} = \frac{2}{2(1-y)} = \frac{1}{(1-y)}, y < x < 1$$

13. The joint probability density function of a two-dimensional random variable (X, Y) is given by, f(x,y)=2;0<x<1, 0<y<x. Test whether X and Y are independent.

**Answer:**

X and Y are independent if f(x,y)=f(x).f(y)

Hence first let us find the marginal distribution of X and Y.

The marginal distribution of X is given by,

$$f(x) = \int_{-\infty}^{\infty} f(x,y)dy = \int_0^x 2dy = 2x, 0 < x < 1$$

To find marginal distribution, range for Y is given as follows. We have 0<x<1, 0<y<x. Combining both we get, 0<y<x<1

$$f(y) = \int_{-\infty}^{\infty} f(x,y)dx = \int_y^1 2dx = 2(1-y), 0 < y < 1$$

Consider f(x).f(y)=2x.[1/(1-y)]≠f(x, y)

Therefore X and Y are not independent.

14. Joint distribution of X and Y is given by, $f(x,y) = 4xye^{-(x^2+y^2)}; x \geq 0, y \geq 0$. Verify whether X and Y are independent.

**Answer:**

If X and Y are independent, we can write f(x,y)=f(x).f(y). Hence first we find marginal distributions.

Marginal density of X is given by,

$$f(x) = \int_y f(x,y)dy = 4x\int_0^{\infty} ye^{-(x^2+y^2)}dy$$

$$= 4xe^{-x^2}\int_0^\infty ye^{-y^2}dy = 4xe^{-x^2}\int_0^\infty e^{-t}\frac{dt}{2}$$

$$= 2xe^{-x^2}\left|-e^{-t}\right|_0^\infty = 2xe^{-x^2}, x \geq 0$$

Marginal density of Y is given by,

$$f(y) = \int_x f(x,y)dx = 4y\int_0^\infty xe^{-(x^2+y^2)}dx$$

$$= 4ye^{-y^2}\int_0^\infty xe^{-x^2}dx = 4ye^{-y^2}\int_0^\infty e^{-t}\frac{dt}{2}$$

$$= 2ye^{-y^2}\left|-e^{-t}\right|_0^\infty = 2ye^{-y^2}, x \geq 0$$

$$f(x)f(y) = (2xe^{-x^2})2ye^{-y^2} = 4xye^{-(x^2+y^2)} = f(x,y)$$

Hence X and Y are independent.

15. A two dimensional random variable (X,Y) have a bivariate distribution given by, P(X=x,Y=y)=(x2+y)/32 for x=0, 1, 2, 3 and y=0, 1. Find the marginal distributions of X and Y.

**Answer:**

We have

| X<br>Y | 0 | 1 | 2 | 3 | Marginal distribution of Y |
|---|---|---|---|---|---|
| 0 | 0 | 1/32 | 4/32 | 9/32 | 14/32 |
| 1 | 1/32 | 2/32 | 5/32 | 10/32 | 18/32 |
| Marginal distribution of X | 1/32 | 3/32 | 9/32 | 10/32 | 1 |

The marginal probability distribution of X is given by,

$P(X=x)=\Sigma_y P(X=x,Y=y)$ and is tabulated in last row of above table.

Marginal probability distribution of Y is given by,

$P(Y=y)=\Sigma_x P(X=x,Y=y)$ and is tabulated in the last column of the table.