Subject	Statistics
Year	1st Year B.Sc
Paper no	05
Paper Name	Probability Distributions-1
Topic no	25
Topic name	Simulation of Random Samples from Standard Univariate Continuous Distributions such as Normal, Exponential and Cauchy Distributions
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E-Learning Module on Simulation of Random Samples from Standard Univariate Continuous Distributions such as Normal, Exponential and Cauchy Distributions

Learning Objectives

By the end of this session, you will be able to:

- Simulate random samples from standard normal and normal distribution
- Simulate samples from exponential distribution
- Simulate samples from standard Cauchy and Cauchy distribution with one and two parameters

Introduction

To simulate random numbers from normal distribution, we use standard normal variate. Using given random numbers, we first convert them into probabilities, which are taken as cumulative probabilities. Hence, from probability we find the corresponding value of standard normal variate Z using standard normal tables and then convert them into the real normal variate using the relation $X=\mu+Z\sigma$.

Normal Tables

Normal probability curve is given by,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} ;-\infty < x < \infty$$

And standard normal probability curve is given by,

$$\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} ;-\infty < z < \infty$$

Where z=(X-μ)/σ~N(0, 1). The table given in the next slide gives the area of the normal curve between zero and z. i.e. P(0<Z<z) for different values of z.

values of z.

TABLE OF AREAS

↓Z→	0	1	2	3	4	5	6	7	8	9
.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0759
.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
.7	.2580	.2611	.2642	.2673	.2703	.2734	.2764	.2794	.2823	.2852
.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3655	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4678	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4959	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998
3.6	.4998	.4998	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.7	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.9	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000

Exponential Distribution

While simulating the random numbers for an exponential distribution, we convert given numbers into probabilities in the beginning, which are taken as cumulative probabilities. Then, by using cumulative distribution function, we solve for the random variable X.

We know that for an exponential distribution with pdf $f(x)=\theta e^{-x\theta}$, $x>0$, cumulative probability function is given by, $F(X)=P(X<x)=1-e^{-x\theta}$.
From this we can solve for x as follows:

$$x = -\frac{\ln[1 - F(x)]}{\theta}$$

Suppose X is an exponential variate with pdf

$$f(x) = \frac{1}{\theta} e^{-x/\theta}, x > 0$$

Then, cumulative distribution function is given by, $F(X)=P(X<x)=1-e^{-x/\theta}$.

From this we can solve for x as follows:
$x=-\theta \ln[1-F(x)]$

Cauchy Distribution

While simulating random samples from Cauchy distribution, we first convert the given random numbers into probabilities. Then, by using cumulative distribution function, we solve for the random variable X.

We know that for Standard Cauchy Distribution with pdf $f(x)=1/[\pi(1+x^2)]$; $-\infty<x<\infty$, the cumulative distribution function is given by,

$$F(x) = P(X < x) = \frac{1}{\pi}\left[\tan^{-1} x + \frac{\pi}{2}\right]$$

From this we get, $x=\tan[\pi\{F(x)-0.5\}]$

Exercise - 1

Simulate a random sample of size 10 from standard normal distribution using following random numbers: 130, 995, 897, 646, 858, 746, 177, 086, 984, 633.

Solution:

We need to generate random sample from standard normal distribution. Cumulative probability $P(X < x) = \int_{-\infty}^{x} f(x)dx$

Random number	P(X<x)	x
130	0.130	-1.11
995	0.995	2.275
897	0.897	1.264
646	0.646	0.375
858	0.858	1.05
746	0.746	0.66
177	0.177	-0.93
086	0.086	-1.37
984	0.984	2.145
633	0.633	0.34

Exercise - 2

Simulate a random sample of size 10 from normal distribution with parameters $\mu=50$ and $\sigma^2=25$ using following random numbers.
Random Numbers – 377, 676, 561, 915, 052, 216, 980, 553, 736, 030

Solution:
We have to simulate a random sample of size n from N(50, 25). Cumulative probability

$$P[X < x] = \int_{-\infty}^{x} f(x)dx$$

Random number	P(X<x)	Z	$X=\mu+Z\sigma$
377	0.377	-0.56	47.2
676	0.676	0.46	52.3
561	0.561	0.15	50.75
915	0.915	1.37	56.85
52	0.052	-1.63	41.85
216	0.216	-0.79	46.05
980	0.980	2.05	60.25
553	0.553	0.13	50.65
736	0.736	0.63	53.15
30	0.030	-1.88	40.6

Exercise - 3

Simulate a random sample of size 10 an exponential distribution with parameter $\theta=0.3$ using following random numbers. 098, 272, 268, 182, 202, 651, 985, 669, 388, 927.

Solution:

Pdf of an exponential distribution is given by $f(x)=\theta e^{-x\theta}=0.3e^{-0.3x}=x>0$

Its distribution function is given by, $F(x)=1-e^{-x\theta}=1-e^{-0.3x}$

$\Rightarrow x=-\ln[1-F(x)]/0.3$

Random Number	P(X<x)	X=-ln[1-F(x)]/0.3
98	0.098	0.344
272	0.272	1.058
268	0.268	1.040
182	0.182	0.670
202	0.202	0.752
651	0.651	3.509
985	0.985	13.999
669	0.669	3.685
388	0.388	1.637
927	0.927	8.724

Exercise - 4

Simulate a random sample of size 10 from a distribution having pdf $f(x) = \frac{1}{5}e^{-x/5}, x > 0$
using following random numbers. 342, 884, 337, 398, 175, 027, 636, 787, 119, 744

Solution:

We have given pdf $f(x) = \frac{1}{5}e^{-x/5}, x > 0$

Its cumulative distribution function is given by,

$F(x)=1-e^{-x/5}$

$\Rightarrow x=-5\ln[1-F(x)]$

Random Number	F(x)=P(X<x)	X=-5ln[1-F(x)]
342	0.342	2.093
884	0.884	10.771
337	0.337	2.055
398	0.398	2.537
175	0.175	0.962
27	0.027	0.137
636	0.636	5.053
787	0.787	7.732
119	0.119	0.633
744	0.744	6.813

Exercise - 5

Simulate a random sample of size 10 from standard Cauchy distribution using following random numbers. 568, 064, 477, 978, 494, 383, 819, 842, 386, 884.

Solution:

X has Standard Cauchy distribution. Hence,

$$f(x) = \frac{1}{\pi(1+x^2)};-\infty < x < \infty$$

Cumulative distribution function

$$F(x) = P(X < x) = \frac{1}{\pi}\left[\tan^{-1} x + \frac{\pi}{2}\right]$$

From this we get, x=tan[π{F(x)-0.5}].

Random Number	$P(X<x)$	$x=\tan[\pi\{F(x)-0.5\}]$
568	0.568	0.217
64	0.064	-4.920
477	0.477	-0.072
978	0.978	14.573
494	0.494	-0.019
383	0.383	-0.385
819	0.819	1.566
842	0.842	1.848
386	0.386	-0.374
884	0.884	2.625

Exercise - 6

Simulate a random sample of size 10 from Cauchy distribution with parameter 35 using following random numbers. 757, 415, 555, 781, 965, 205, 727, 995, 893, 094.

Solution:

Y has Cauchy distribution with parameter 35(μ). Hence

$$f(y) = \frac{1}{\pi(1+(y-\mu)^2)} = \frac{1}{\pi(1+(y-35)^2)}; -\infty < y < \infty$$

Let us use the transformation, x=y-μ, then X has standard Cauchy distribution and hence cdf is given by,

$$F(x) = P(X < x) = \frac{1}{\pi}\left[\tan^{-1} x + \frac{\pi}{2}\right]$$

From this we get, x=tan[π{F(x)-0.5}] and y=x+μ=x+35.

Random Number	$P(X<x)$	$x=\tan[\pi\{F(x)-0.5\}]$	$y=x+35$
757	0.757	1.046	36.046
415	0.415	-0.274	34.726
555	0.555	0.175	35.175
781	0.781	1.217	36.217
965	0.965	9.107	44.107
205	0.205	-1.333	33.667
727	0.727	0.866	35.866
995	0.995	66.299	101.299
893	0.893	2.867	37.867
94	0.094	-3.293	31.707

Exercise - 7

Generate a random sample of size 10 from Cauchy (50, 20) using following random numbers. 130, 924, 009, 227, 420, 673, 484, 966, 903, 779.

Solution:

Y has Cauchy distribution with parameters $\mu=50$ and $\lambda=20$. Hence,

$$f(y) = \frac{\lambda}{\pi(\lambda^2 + (y-\mu)^2)} = \frac{20}{\pi(20^2 + (y-50)^2)}; -\infty < y < \infty$$

Let us use the transformation, $x=(y-\mu)/\lambda$, then X has standard Cauchy distribution and hence the cdf is given by,

$$F(x) = P(X < x) = \frac{1}{\pi}\left[\tan^{-1} x + \frac{\pi}{2}\right]$$

From this we get,

$x=\tan[\pi\{F(x)-0.5\}]$ and

$y=\lambda x+\mu=20x+50$

Random Number	P(X<x)	x=tan[π{F(x)-0.5}]	y=20x+50
130	0.130	-2.314	-11.277
924	0.924	4.118	117.360
009	0.009	-36.153	-688.052
227	0.227	-1.157	11.863
420	0.420	-0.257	29.863
673	0.673	0.604	47.090
484	0.484	-0.050	33.993
966	0.966	9.379	222.571
903	0.903	3.185	98.700
779	0.779	1.202	59.039