Frequently Asked Questions

1. Explain the method of simulating random sample from Normal distribution.

Answer:

To simulate random numbers from normal distribution, we use standard normal variate. Using given random numbers first we convert them into probabilities, which are taken cumulative probabilities. Hence, from probability we find corresponding value of standard normal variate Z using standard normal tables and then convert them into the real normal variate using the relation $X=\mu+Z\sigma$.

2. How will you simulate random sample from exponential distribution with parameter θ?

Answer:

While simulating the random numbers for an exponential distribution, in the beginning we convert given numbers into probabilities, which are taken as cumulative probabilities. Then by using cumulative distribution function, we solve for the random variable X.

We know that for an exponential distribution with pdf $f(x)=\theta e^{-x\theta}$, $x>0$, cumulative probability function is given by, $F(X)=P(X<x)=1-e^{-x\theta}$. From this we solve for x as follows.

$$x = -\frac{\ln[1-F(x)]}{\theta}$$

3. How will you simulate random sample from exponential distribution with parameter 1/θ?

Answer:

Suppose X is an exponential variate with pdf $f(x) = \frac{1}{\theta} e^{-x/\theta}, x > 0$

Then, cumulative distribution function is given by, $F(X)=P(X<x)=1-e^{-x/\theta}$

From this we can solve for x as follows,

$x=-\theta \ln[1-F(x)]$

4. Explain the method of simulating random sample from Cauchy distribution.

Answer:

While simulating random samples from Cauchy distribution, first we convert the given random numbers into probabilities. Then by using cumulative distribution function, we solve for the random variable X. We know that for standard Cauchy Distribution with pdf $f(x)=1/[\pi(1+x^2)]$; $-\infty<x<\infty$, and the cumulative distribution function is given by,

$F(x) = P(X < x) = \frac{1}{\pi}\left[\tan^{-1} x + \frac{\pi}{2}\right]$ From this we get $x=\tan[\pi\{F(x)-0.5\}]$.

5. Explain the method of finding points using standard normal tables while simulating random sample from normal distribution.

Answer:

Outside that is vertically left side horizontally top, we have given the points on the curve z and inside we have given the probabilities. The horizontal numbers are the 2nd digit after point. While generating random numbers, when they are divided by 1 thousand, we consider them as probabilities left to the point z. Then we try to identify the area between zero and z. Because of symmetry any area between minus z to zero is same as zero to z.

6. Write cumulative distribution function of exponential distribution with parameter θ.

Answer:

If X~exp(θ), the cumulative distribution function is given by $F(x)=1-e^{-x\theta}$

7. Write cumulative distribution function of exponential distribution with parameter 1/θ.

Answer:

If X~exp(1/θ), the cumulative distribution function is given by $F(x)=1-e^{-x/\theta}$

8. Write cumulative distribution function of standard Cauchy distribution.

Answer:

If X has standard Cauchy distribution, the cumulative distribution function is given by,

$$F(x) = \frac{1}{\pi}\left[\tan^{-1} x + \frac{\pi}{2}\right]$$

9. Simulate a random sample of size 10 from standard normal distribution using following random numbers. 130, 995, 897, 646, 858, 746, 177, 086, 984, 633.

Answer:

We need to generate random sample from standard normal distribution. Cumulative probability $P(X<x)= \int_{-\infty}^{x} f(x)dx$

Now, let us write the following table.

Random number	**P(X<x)**	**Random sample (x)**
130	0.130	-1.11
995	0.995	2.275
897	0.897	1.264
646	0.646	0.375
858	0.858	1.05
746	0.746	0.66
177	0.177	-0.93
086	0.086	-1.37
984	0.984	2.145
633	0.633	0.34

10. Simulate a random sample of size 10 from normal distribution with parameters μ=50 and σ^2=25 using following random numbers. 377, 676, 561, 915, 052, 216, 980, 553, 736, 030

Answer:

We have to simulate a random sample of size n form N(50, 25). Cumulative probability $P[X<x]= \int_{-\infty}^{x} f(x)dx$

Now, let us write the following table.

Random number	P(X<x)	Z	X=μ+Zσ
377	0.377	0.56	47.2
676	0.676	0.46	52.3
561	0.561	0.15	50.75
915	0.915	1.37	56.85
52	0.052	1.63	41.85
216	0.216	0.79	46.05
980	0.980	2.05	60.25
553	0.553	0.13	50.65
736	0.736	0.63	53.15
30	0.030	1.88	40.6

11. Simulate a random sample of size 10 an exponential distribution with parameter θ=0.3 using following random numbers. 098, 272, 268, 182, 202, 651, 985, 669, 388, 927.

Answer:

Pdf of an exponential distribution is given by $f(x)=\theta e^{-x\theta}$, $=0.3e^{-0.3x}$,=x>0

Its distribution function is given by, $F(x)=1-e^{-x\theta}=1-e^{-0.3x}$

$\Rightarrow x=-\ln[1-F(x)]/0.3$

Now, let us write the following table.

Random Number	P(X<x)	X=-ln[1-F(x)]/0.3
98	0.098	0.344
272	0.272	1.058
268	0.268	1.040
182	0.182	0.670
202	0.202	0.752
651	0.651	3.509
985	0.985	13.999
669	0.669	3.685
388	0.388	1.637
927	0.927	8.724

12. Simulate a random sample of size 10 form a distribution having pdf $f(x)=\frac{1}{5}e^{-x/5}, x>0$ using following random numbers. 342, 884, 337, 398, 175, 027, 636, 787, 119, 744.

Answer:

We have given pdf $f(x)=\frac{1}{5}e^{-x/5}, x>0$

Its cumulative distribution function is given by,

$F(x)=1-e^{-x/5} \Rightarrow x=-5\ln[1-F(x)]$

Now let us obtain the following table.

Random Number	F(x)=P(X<x)	X=-5ln[1-F(x)]
342	0.342	2.093
884	0.884	10.771
337	0.337	2.055
398	0.398	2.537
175	0.175	0.962
27	0.027	0.137
636	0.636	5.053
787	0.787	7.732
119	0.119	0.633
744	0.744	6.813

13. Simulate a random sample of size 10 from standard Cauchy distribution using following random numbers. 568, 064, 477, 978, 494, 383, 819, 842, 386, 884

Answer:

X has Standard Cauchy distribution. Hence $f(x)=\frac{1}{\pi(1+x^2)};-\infty<x<\infty$

Cumulative distribution function $F(x) = P(X < x) = \frac{1}{\pi}\left[\tan^{-1} x + \frac{\pi}{2}\right]$

From this we get, x=tan[π{F(x)-0.5}].
Now, let us obtain the following table.

Random Number	**P(X<x)**	**x=tan[π{F(x)-0.5}]**
568	0.568	0.217
64	0.064	-4.920
477	0.477	-0.072
978	0.978	14.573
494	0.494	-0.019
383	0.383	-0.385
819	0.819	1.566
842	0.842	1.848
386	0.386	-0.374
884	0.884	2.625

14. Simulate a random sample of size 10 from Cauchy distribution with parameter 35 using following random numbers. 757, 415, 555, 781, 965, 205, 727, 995, 893, 094.

Answer:

Y has Cauchy distribution with parameter 35(μ). Hence,

$$f(y) = \frac{1}{\pi(1+(y-\mu)^2)} = \frac{1}{\pi(1+(y-35)^2)};-\infty < y < \infty$$

Let us use the transformation, x=y-μ, then X has standard Cauchy distribution and hence cdf is given by, $F(x) = P(X < x) = \frac{1}{\pi}\left[\tan^{-1} x + \frac{\pi}{2}\right]$

From this we get, x=tan[π{F(x)-0.5}] and y=x+μ=x+35. Now, let us obtain the following table.

Random Number	**P(X<x)**	**x=tan[π{F(x)-0.5}]**	**y=x+35**
757	0.757	1.046	36.046
415	0.415	-0.274	34.726
555	0.555	0.175	35.175
781	0.781	1.217	36.217
965	0.965	9.107	44.107
205	0.205	-1.333	33.667
727	0.727	0.866	35.866
995	0.995	66.299	101.299
893	0.893	2.867	37.867
94	0.094	-3.293	31.707

15. Generate a random sample of size 10 from Cauchy (50, 20) using following random numbers. 130, 924, 009, 227, 420, 673, 484, 966, 903, 779.

Answer:

Y has Cauchy distribution with parameters μ=50 and λ=20. Hence, $f(y) = \frac{\lambda}{\pi(\lambda^2 + (y-\mu)^2)} = \frac{20}{\pi(20^2 + (y-50)^2)}; -\infty < y < \infty$

Let us use the transformation, x=(y-μ)/λ, then X has standard Cauchy distribution and hence the cdf is given by, $F(x) = P(X < x) = \frac{1}{\pi}\left[\tan^{-1} x + \frac{\pi}{2}\right]$

From this we get, x=tan[π{F(x)-0.5}] and y=λx+μ=20x+50.
Now, let us obtain the following table.

Random Number	**P(X<x)**	**x=tan[π{F(x)-0.5}]**	**y=20x+50**
130	0.130	-2.314	-11.277
924	0.924	4.118	117.360
009	0.009	-36.153	-688.052
227	0.227	-1.157	11.863
420	0.420	-0.257	29.863
673	0.673	0.604	47.090
484	0.484	-0.050	33.993
966	0.966	9.379	222.571
903	0.903	3.185	98.700
779	0.779	1.202	59.039