Frequently Asked Questions

1. Give the normal curve fitted to the given data.

Answer:

While fitting normal distribution, the curve fitted to the given data is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} ; -\infty < x < \infty$$

2. Write the exponential curve with parameter θ fitted to the given data.

Answer:

The exponential curve with parameter θ fitted to the given curve is, $f(x)=\theta e^{-x\theta}$, x>0.

3. Give an exponential curve with mean θ fitted to the given data.

Answer:

We know that parameter is reciprocal of mean in exponential distribution. Hence, parameter of the distribution is, 1/θ.

Therefore, the exponential curve with mean θ fitted to the given curve is, $f(x)=(1/\theta)e^{-x/\theta}$, x>0.

4. Mention the cumulative distribution function of an exponential distribution with parameter θ.

Answer:

The cumulative distribution function of an exponential distribution with parameter θ is given by, $F(x)= P(X<x)=1-e^{-x\theta}$.

5. Mention the cumulative distribution function of an exponential distribution with mean θ.

Answer:

The cumulative distribution function of an exponential distribution with mean θ is given by, $F(x)= P(X<x)=1-e^{-x/\theta}$.

6. How to obtain expected frequencies while fitting a data to the given frequency distribution?

Answer:

While fitting a distribution we find the probabilities of corresponding class intervals. When these probabilities are multiplied by total frequency we get expected frequencies.

7. What should be total of expected frequencies?

Answer:

While fitting any distribution to a given data, finally the total of expected frequencies should be equal to the total of original frequencies.

8. Explain the method of fitting Normal distribution.

Answer:

In order to fit normal distribution to the given data we first calculate mean μ and standard deviation σ from the given data. Then the normal curve fitted to the given data is given by,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} ; -\infty < x < \infty$$

To calculate the expected normal frequencies we first find the standard normal variates corresponding to the lower limits of each of the class intervals, i.e., we compute z=(x-μ)/σ, where x is the upper limit of the class interval. Then the areas under the normal curve to the left of the ordinate z, say Φ(z)=P(Z≤z) are computed from the tables. Finally, the areas for the successive class intervals are obtained by subtraction $\Phi(z)$- $\Phi(z-1)$ and on multiplying these areas by N, the total frequency, we get the expected normal frequencies.

9. How will you compute $\Phi(z)=P(Z\leq z)$ from the tables?

Answer:

While finding the probabilities we have given standard normal tables, which give probabilities between zero and z and $\Phi(z)=P(Z\leq z)$. Hence, for any value of Z which is negative, we find the table value and subtract from 0.5 and for any positive value of z, table value is added to 0.5.

10. How will you fit an exponential distribution with parameter θ?

Answer:

In order to fit an exponential distribution to the given data, we first estimate the parameter of the distribution by its m.l.e. $\hat{\theta} = 1/\overline{x}$

Then the exponential curve fitted to the given data is given by, $f(x)=\theta e^{-x\theta}$, $x>0$.

To calculate expected frequencies, we first write the upper limits of each class interval. Then the probability below the upper limit is calculated using the formula $P(X<x)=1-e^{-x\theta}$.

Finally, the probabilities for the successive class intervals are obtained by subtraction and on multiplying these probabilities by N, the total frequency, we get the expected exponential frequencies.

11. How will you fit an exponential distribution with mean θ?

Answer:

We know that in an exponential distribution, mean is reciprocal of the parameter. Hence parameter will be 1/θ.

In order to fit an exponential distribution to the given data, we first estimate the parameter of the distribution by its m.l.e. $\hat{\theta} = \overline{x}$

Then the exponential curve fitted to the given data is given by, $f(x)=(1/\theta)e^{-x/\theta}$, $x>0$.

To calculate expected frequencies, we first write the upper limits of each class interval. Then the probability below the upper limit is calculated using the formula P(X<x)=1-e-x/θ.

Finally the probabilities for the successive class intervals are obtained by subtraction and on multiplying these probabilities by N, the total frequency, we get the expected exponential frequencies.

12. Fit an exponential distribution with parameter θ for the following data.

CI	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	130	60	34	12	9	3	2	0

Answer:

Given X~Exp(θ). Hence exponential curve fitted to the given data is, $f(x)=\theta e^{-x\theta}$, $-\infty<x<\infty$

Since parameter is not known, we estimate using its m.l.e. $\hat{\theta} = 1/\overline{x}$. Now, consider the following table.

CI	Frequency(f)	mid-point(x)	fx
0-10	130	5	650
10-20	60	15	900
20-30	34	25	850
30-40	12	35	420
40-50	9	45	405
50-60	3	55	165
60-70	2	65	130

70-80	0	75	0
	250		3520

The mean of the distribution is given by,

$$\overline{X} = \frac{\sum fx}{N} = \frac{3520}{250} = 14.08$$

$$\therefore \hat{\theta} = \frac{1}{\overline{x}} = \frac{1}{14.08} = 0.071$$

Hence equation fitted to given data is $f(x)=0.071e^{-0.071x}$

Let us obtain the expected frequencies for the given data.

CI	upper limit	$F(x)=1-e^{-0.071x}$	ΔF(x)	Expected Frequencies
0-10	10	0.508	0.508	127
10-20	20	0.758	0.250	62
20-30	30	0.881	0.123	31
30-40	40	0.942	0.060	15
40-50	50	0.971	0.030	7
50-60	60	0.986	0.015	4
60-70	70	0.993	0.007	2
70-80	80	0.997	0.004	1
80 and above	∞	1.000	0.003	1
				250

13. Fit an exponential distribution with mean 17 for the following data.

CI	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	22	13	5	5	3	2

Answer:

Here we have to fit an exponential distribution with mean 17 i.e. parameter 1/17. Hence an exponential curve fitted to the given data is,

$$f(x) = \frac{1}{17} e^{-x/17}; x > 0$$

CI	upper limit	$F(x)=1-e^{-x/17}$	ΔF(x)	Expected Frequencies
0-10	10	0.445	0.445	22
10-20	20	0.692	0.247	12
20-30	30	0.829	0.137	7
30-40	40	0.905	0.076	4
40-50	50	0.947	0.042	2
50-60	60	0.971	0.023	1
60 and above	∞	1.000	0.029	2
				50

14. Obtain equation of a normal curve that may be fitted to the following data

CI	60-65	65-70	70-75	75-80	80-85	85-90	90-95	95-100
Frequency	3	21	150	335	326	135	26	4

Also obtain the expected normal frequencies.

Answer:

For the given data let us obtain mean and standard deviation, which are taken as mean and standard deviation of the normal distribution, which are given by,

$$\mu = \frac{\Sigma fx}{N} \;\&\; \sigma = \sqrt{\frac{\Sigma fx^2}{N} - \left(\frac{\Sigma fx}{N}\right)^2}$$

Let us obtain the following table.

CI	Frequency	x	fx	fx^2
60-65	3	62.5	187.5	11718.75
65-70	21	67.5	1417.5	95681.25
70-75	150	72.5	10875	788437.5
75-80	335	77.5	25962.5	2012094
80-85	326	82.5	26895	2218838
85-90	135	87.5	11812.5	1033594
90-95	26	92.5	2405	222462.5
95-100	4	97.5	390	38025
Total	**1000**	**-**	**79945**	**6420850**

$$\mu = \frac{\Sigma fx}{N} = \frac{79945}{1000} = 79.945$$

$$\sigma = \sqrt{\frac{\Sigma fx^2}{N} - \left(\frac{\Sigma fx}{N}\right)^2} = \sqrt{\frac{6420850}{1000} - \left(\frac{79945}{1000}\right)^2} = 5.545$$

Hence, the equation if the normal curve fitted to the given data is,

$$f(x) = \frac{1}{5.545\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-79.945\mu}{5.545}\right)^2}$$

Now, consider the following table, where we calculate expected frequencies.

CI	Upper limit	Z=(x-79.945)/5.545	Φ(z)	ΔΦ(z)	Expected frequency
below 60	60	-3.60	0.0002	0.0002	0
60-65	65	-2.70	0.0030	0.0028	3
65-70	70	-1.79	0.0341	0.0311	31
70-75	75	-0.89	0.1819	0.1478	148
75-80	80	0.01	0.5040	0.3221	322
80-85	85	0.91	0.8233	0.3193	319
85-90	90	1.81	0.9674	0.1441	144
90-95	95	2.72	0.9971	0.0297	30
95-100	100	3.62	0.9999	0.0028	3
100 and above	∞	∞	1.0000	0.0001	0
Total					**1000**

15. Fit a Standard normal distribution for the following data.

CI	-5-(-3)	-3-(-1)	-1-1	1-3	3-5
Frequency	5	9	19	10	7

Answer:

Here we need to fit standard normal distribution. The standard normal curve fitted to the given data is, $f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$

Consider the following table.

CI	upper limit (z)	Φ(z)	ΔΦ(z)	Expected Frequency
below -5	-5	0.000	0.000	0
-5-(-3)	-3	0.001	0.001	0
-3-(-1)	-1	0.159	0.157	8
-1-1	1	0.813	0.654	33
1-3	3	0.999	0.186	9
3-5	5	1.000	0.001	0
5 and above	∞	1.000	0.000	0
Total				50