# 1. Introduction

Welcome to the series of e-learning modules on fitting of geometric curves or power curves. In this section we try to fit the geometric curve to the given data and based on this, we try to estimate the future values of the particular phenomenon.

By the end of this session, you will be able to:

- Fit geometric or power curves using the method of least squares
- Predict the future value assuming that the same pattern continues

Some of the processes given below show the pattern of geometric curve or power curve.
Say, the length of a pendulum and its period in seconds.
The distance of different planets from sum and the revolution time needed. Or say,
Length and weight of a trout fish, etc.

Depending on the pattern suggested by the scatter plot, there are many non-linear equations from which we can select a particular equation to fit a given data.
The equation which represents the geometric curve is given as:
Y is equal to ‘a’ into ‘x power b’.

Suppose the given data suggests the geometric pattern, we fit geometric curve as follows.
The equation which represents the geometric curve is:
Y is equal to ‘a’ into ‘x power b’.
To find the constants ‘a’ and ‘b’, first we take the logarithm to the base 10, denoted as log, on both the sides.
That is ‘log Y’ is equal to ‘log a’ plus ‘b into log x’, which is in the form, U is equal to ‘A’ plus ‘b into V’, where ‘U’ is equal to ‘log Y’, ‘A’ is equal to ‘log a’ and ‘V’ is equal to ‘log x’.

Now, let us find the normal equations using the method of least squares.
Observe that the function ‘U’ is a straight line. Proceeding as we have done in the case of fitting linear curve or straight line,  we minimize the error, E is equal to summation ‘U’ minus ‘A’ minus ‘b into V’ whole square.
That is by differentiating E with respect o A and b and then equating it to zero.

These expressions are simplified to get the normal equations as follows:

That is, differentiation with respect to A, ‘d E’ by ‘d A’ is equal to ‘zero’ is equal to ‘minus 2’ into summation of ‘U’ minus ‘A’ minus ‘b into V’.
Implies, Summation U is equal to ‘n into A; plus ‘b into summation V’.

Differentiation with respect to b is equal to ‘zero’ is equal to ‘minus 2’ into ‘summation V’ into ‘U’ minus ‘A’ minus ‘b into V’/Implies, ‘Summation U into V’ is equal to ‘A into summation V’ plus ‘b into summation V square’.

Solving these two simultaneous equations, we can find the values of A and b and consequently, a is equal to antilog of A.

With these values of a and b, the geometric curve Y is equal to a into x power b is a best fit to the given data set of n points.

Now let us solve some problems of fitting the geometric curve.

# 2. Illustration 1

Fit a curve of the form ‘Y’ is equal to ‘a’ into ‘x power b’ to the following data. Also estimate ‘Y’ when ‘x is equal to 8’.
The values of ‘x’ are given as 3,4,5,6 and 7 and the corresponding values for Y are given as: 126, 42, 20, 6 and 3.

**Figure 1**

| x | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|
| Y | 126 | 42 | 20 | 6 | 3 |

Given the curve fitted to the above data is, ‘Y’ is equal to ‘a’ into ‘x power b’, where ‘a’ and ‘b’ are constants, to be determined.
Hence, to determine these constants, we use the following normal equations.
‘Summation U’ is equal to ‘n into A’ plus ‘b into summation V’ and
‘Summation U into V’ is equal to ‘A into summation V’ plus ‘b into summation V square’.
Where U is equal to ‘log Y’, V is equal to ‘log x’ and A is equal to ‘log a’.

To find the values of different unknowns in the above normal equations, we construct the following table.

**Figure 2**

| x | Y | V=log x | U=log Y | V2 | UV |
|---|---|---|---|---|---|
| 3 | 126 | 0.4771 | 2.1004 | 0.2276 | 1.0021 |
| 4 | 42 | 0.6021 | 1.6232 | 0.3625 | 0.9773 |
| 5 | 20 | 0.6999 | 1.3010 | 0.4899 | 0.9106 |
| 6 | 6 | 0.7782 | 0.7782 | 0.6056 | 0.6056 |
| 7 | 3 | 0.8451 | 0.4771 | 0.7142 | 0.4032 |
| Totals | | 3.4024 | 6.2799 | 2.3992 | 3.8988 |

The first two columns are written as it is in the question. The third column is obtained by taking logarithm of the values in the first column. That is log 3 is equal to 0.4711, log 4 is equal to 0.6021 etc.
The fourth column is obtained by finding the logarithm of numbers in the $2^{nd}$ column, that is, log 126 is equal to 2.1004, log 42 is equal to 1.6232 etc. The fifth column is obtained by finding square of the numbers in $3^{rd}$ column. That is 0.4771 square is equal to 0.2276, 0.6021 square is equal to 0.3625 etc. Finally the last column is obtained by multiplying the number in $3^{rd}$ and $4^{th}$ columns. That is 0.4771 into 2.1004 is equal to 1.0021, 0.6021 into 1.6232 is equal

to 0.9773 etc.
Once we find all these values, we find the total of those columns which are required for the calculations.

Therefore the normal equations can be written as follows:
6.2799 is equal to '5A' plus '3.4024 b', and
3.8998i s equal to '3.4024 A' plus '2.3998 b'.
The above equations can be solved by using the Cramer's rule.

The above equations can be solved by using the Cramer's rule. Hence consider the following determinants.
Delta is equal to determinant of 5, 3.4024, , 3.4024, 2.3998
Is equal to 5 into 2.3998 minus 3.4024 into 3.4024, is equal to 0.4227.
Delta 1 is equal to determinant of 6.2799, 3.4024, 3.8998, 2.3998 is equal to 6.2799 into 2.3998 eight minus 3.8998 into 3.4024,
is equal to 1.8018.
Delta 2 is equal to determinant of 5, 6.2799, 3.4024, 3.8998 is equal to 5 into 3.8998 minus 3.4024 into 6.27993.
Is equal to minus 1.8677.

Therefore A is equal to delta 1 divided by delta is equal to 1.8018 divided by 0.4227 is equal to 4.2626
And b is equal to delta 2 divided by delta is equal to,
Minus 1.8677 divided by 0.4227 is equal to minus 4.4185
Hence 'a' is equal to 'antilog of A' is equal to 'antilog of 4.2626' is equal to 18,306.
The equation of the required curve is:
'Y' is equal to 18,306 into 8 to the power 'minus 4.4185' is equal to 1.9

# 3. Illustration 2

Given below in the table is the data regarding the profit of a company for last 6 years in lakhs of rupees.

**Figure 3**

| Year | 2006 | 2007 | 2008 | 2009 | 2010 | 2011 |
|---|---|---|---|---|---|---|
| Profit (lakhs) | 2.98 | 4.26 | 5.21 | 6.10 | 6.80 | 7.5 |

Estimate the profit for the year 2012 by fitting the geometric curve y is equal to 'a' into x power b.
For the years 2006, 2007, 2008, 2009, 2010 and 2011 the profit in lakhs is given as 2.98, 4.26, 5.21, 6.10, 6.80 and 7.5.

Given the curve fitted to the above data is, 'Y' is equal to 'a' into 'x power b', where a and b are constants, to be determined. Hence to determine these constants, we use the following normal equations.
'Summation U' is equal to 'n into A' plus 'b into summation V' and
'Summation U into V' is equal to 'A into summation V' plus 'b into summation V square'.
Where U is equal to 'log Y', V is equal to 'log x' and A is equal to 'log a'.

To find the values of different unknowns in the above normal equations, we construct the following table.

**Figure 4**

| Year | Profit (lakhs) (Y) | x | V=logx | U=log Y | V2 | UV |
|---|---|---|---|---|---|---|
| 2006 | 2.98 | 1 | 0.0000 | 0.4742 | 0.0000 | 0.0000 |
| 2007 | 4.26 | 2 | 0.3010 | 0.6294 | 0.0906 | 0.1895 |
| 2008 | 5.21 | 3 | 0.4771 | 0.7168 | 0.2276 | 0.3420 |
| 2009 | 6.1 | 4 | 0.6021 | 0.7853 | 0.3625 | 0.4728 |
| 2010 | 6.8 | 5 | 0.6990 | 0.8325 | 0.4886 | 0.5819 |
| 2011 | 7.5 | 6 | 0.7782 | 0.8751 | 0.6055 | 0.6809 |
| Total | | | 2.8573 | 4.3134 | 1.7748 | 2.2671 |

To simplify the calculations, we do not consider the year as x values, instead we take deviation from the value after looking at the smallest value, that is from 2006.
Therefore x is equal to year minus 2005. We cannot choose the year 2006 because we cannot find logarithm of zero or any negative number.
First two columns of the table, we write as it is given in the problem.
The third column is obtained by taking deviation from the year 2005. Hence we get the

values of x as 1, 2, 3 etc.
The $4^{th}$ column is obtained by taking the logarithm of the numbers in the third column.
The fifth column is obtained by taking the logarithm of the numbers in the $2^{nd}$ column which we consider as Y.
The sixth column is obtained by taking the squares of the numbers in the fourth column.
The last column is obtained by multiplying the numbers in the $4^{th}$ and $5^{th}$ column.
Once we finish all the entries, we find the totals of the required columns for calculation.

Therefore the normal equations can be written as follows.
4.3134 is equal to ‘6 into A’ plus ‘2.8573 into b’ and
2.2671 is equal to ‘2.8573 into A’ plus ‘1.7748 into b’
The above equations can be solved by using the Cramer’s rule.

Delta is equal to determinant of 6, 2.8573, 2.8573, 1.7748 is equal to '6 into 1.7748' minus '2.8573 into 2.8573’ is equal to 2.4846.
Delta One is equal to determinant of 4.3134, 2.8573, 2.2671, 1.7748 is equal to '4.3134 into 1.7748' minus '2.2671 into 2.8573' is equal to 1.1776.
Delta Two is equal to determinant of 6, 4.3134, 2.8573, 2.2671 is equal to '6 into 2.2671' minus '2.8573 into 4.3134' is equal to 1.2779

Hence, A is equal to Delta one by delta, is equal to 1.1776 divided by 2.4846 is equal to 0.4739, and
b is equal to delta two divided by delta is equal to 1.2779 divided by 2.4846 is equal to 0.5143.
a is equal to 'antilog of A' is equal to antilog of 0.4739 is equal to 2.9778.
The equation of the required curve is:
Y is equal to '2.9778 into x to the power 0.5143'.
Putting 'x' is equal to 7 in the above equation.
That is Y is equal to '2.9778 into 7 power 0.5143' is equal to 'Rupees 8.1 lakh’.

# 4. Illustration 3

Fit a curve of best fit of the form y is equal to ‘a’ into x power b for the data given below. The values given for x are 1, 10, 100 and 1000 and the corresponding values of Y are given as 10.95, 39.81, 50.12 and 63.8.

**Figure 5**

| x | 1 | 10 | 100 | 1000 |
|---|---|---|---|---|
| Y | 10.95 | 39.81 | 50.12 | 63.8 |

Given the curve fitted to the above data is, ‘Y’ is equal to ‘a’ into ‘x power b’, where a and b are constants, to be determined. Hence to determine these constants, we use the following normal equations.
‘Summation U’ is equal to ‘n into A’ plus ‘b into summation V’ and
‘Summation U into V’ is equal to ‘A into summation V’ plus ‘b into summation V square’.
Where U is equal to ‘log Y’, V is equal to ‘log x’ and A is equal to ‘log a’.

To find the values of different unknowns in the above normal equations, we construct the following table.

**Figure 6**

| x | Y | V | U=log Y | V2 | UV |
|---|---|---|---|---|---|
| 1 | 10.95 | 0 | 1.0394 | 0 | 0.0000 |
| 10 | 39.81 | 1 | 1.6000 | 1 | 1.6000 |
| 100 | 50.12 | 2 | 1.7000 | 4 | 3.4000 |
| 1000 | 63.8 | 3 | 1.8048 | 9 | 5.4145 |
| Totals | | 6 | 6.1442 | 14 | 10.4145 |

First two columns of the table are written as it is in the question.
Third column is obtained by finding logarithm of the values in the first column.
The fourth column is obtained by finding the logarithm of the numbers in the 2nd column.
Fifth column is obtained by finding the square of the values in the 3rd column.
The last column is obtained by multiplying the numbers in 3rd and 4th columns.
Once we find all the entries in different columns, we find the totals of required columns.

Therefore the normal equations can be written as follows.
6.1442 is equal to ‘4 into A’ plus ‘6 into b’ and 10.4145 is equal to ‘6 into A’ plus ‘14 into b’.
The above equations can be solved by using the Cramer’s rule.

Hence, consider the following determinants;
Delta is equal to determinant of 4, 6, 6, 14 is equal to ‘4 into 14’ minus ‘6 into 6’ is equal to 20.
Delta 1 is equal to determinant of 6.1442, 6, 10.4145, 14 is equal to ‘6.1442 into 14’ minus

10.4145 into 6’ is equal to 23.5318.
Delta 2 is equal to determinant of 4, 6.1442, 6, 10.4145 is equal to ‘4 into 10.4145’ minus ‘6 into 6.1442’ is equal to 4.7928.

Therefore, A is equal to delta one by delta is equal to 23.5318 divided by 20 is equal to 1.1766, and,
b is equal to delta 2 divided by delta is equal to 4.7928 divided by 20 is equal to 0.2396.
Another constant 'a' is equal to 'antilog of A' is equal to 'antilog of 1.1766 is equal to 15.0176.
Therefore the equation of required curve is given by Y is equal to 15.0176 into x power 0.2396.

# 5. Illustration 4

In this illustration, we need to fit a curve y is equal to a into x power b to the data given below, where the values of x are given as 1,2,3,4 and 5 and the respective values of Y are given as 1.1, 2.8, 5.2, 8.0 and 11.1.

**Figure 7**

| x | 1.0 | 2.0 | 3.0 | 4.0 | 5.0 |
|---|---|---|---|---|---|
| Y | 1.1 | 2.8 | 5.2 | 8.0 | 11.1 |

Given the curve fitted to the above data is, 'Y' is equal to 'a' into 'x power b', where a and b are constants, to be determined. Hence to determine these constants, we use the following normal equations.
'Summation U' is equal to 'n into A' plus 'b into summation V' and
'Summation U into V' is equal to 'A into summation V' plus 'b into summation V square'.
Where U is equal to 'log Y', V is equal to 'log x' and A is equal to 'log a'.

To calculate the values of a and b, we construct the following table.

**Figure 8**

| x | Y | V | U=log Y | V2 | UV |
|---|---|---|---|---|---|
| 1.0 | 1.1 | 0.0000 | 0.0414 | 0.0000 | 0.0000 |
| 2.0 | 2.8 | 0.3010 | 0.4472 | 0.0906 | 0.1346 |
| 3.0 | 5.2 | 0.4771 | 0.7160 | 0.2276 | 0.3416 |
| 4.0 | 8.0 | 0.6021 | 0.9031 | 0.3625 | 0.5437 |
| 5.0 | 11.1 | 0.6990 | 1.0453 | 0.4886 | 0.7306 |
| Totals | | 2.0792 | 3.1530 | 1.1693 | 1.7506 |

First and $2^{nd}$ column in the table we write as it is in the given table. $3^{rd}$ column is obtained by taking logarithm of values in the first column. The $4^{th}$ column is obtained by taking logarithm of the values in the $2^{nd}$ column. Fifth column is obtained by squaring the values in the $3^{rd}$ column and finally the last column is obtained by multiplying the numbers in $3^{rd}$ and $4^{th}$ column.

Therefore the normal equations can be written as follows:
3.1530 is equal to '5 into A' plus '2.0792 into b', and,
1.7506 is equal to '2.0792 into A' plus '1.1693 into b'.
Solving the above two equations we get,
A is equal to 0.0308 and b is equal to 1.4423.
Hence, A is equal to 'antilog of A' is equal to 'antilog of 0.0308' is equal to 1.0735.
Therefore, the equation of the required curve is Y is equal to 1.0735 into x to the power

1.4423.

Here’s a summary of our learning from this session:

- Fitting geometric curves using the method of least squares
- Predicting the future value assuming that the same pattern continues