

## Frequently Asked Questions

1. When do you fit a geometric curve to the given data?

**Answer:**

If the data shows a variation in the geometrical pattern, then we fit a parabolic curve to the given data, which can be easily found using scatter plots.

2. Give some instances which show geometric patterns in real life.

**Answer:**

Some of the processes given below show the pattern of geometric curve or power curve.

- Length of a pendulum (in cms) and its period (the time to complete one oscillation) in seconds.
- The distance of different planets from sun and the revolution time needed
- Length and weight of a trout fish, etc.

3. Write the equation of the geometric curve

**Answer:**

The equation of geometric curve is,

$$Y = ax^b$$

4. Write the normal equations used to fit the geometric curve.

**Answer:**

The normal equations used to fit the geometric curve is,

$$\sum U = n A + b \sum V$$

$$\sum UV = A \sum V + b \sum V^2$$

Where  $U = \log Y$ ,  $V = \log x$  and  $A = \log a$ .

5. Derive the normal equations used to estimate the constants of the geometric curve  $Y = ax^b$

**Answer:**

The equation which represent the geometric curve is  $Y = ax^b$

To find the constants  $a$  and  $b$ , first we take the logarithm on both the sides.

$\log Y = \log a + b \log x$ , which is in the form,

$U = A + bV$ , a linear equation in  $V$  and  $U$  where  $U = \log Y$ ,  $A = \log a$  and  $V = \log x$ . the normal equations can be obtained by minimizing the expression,

$$E = \sum (U - A - bV)^2$$

$$\frac{dE}{dA} = 0 = -2\sum (U - A - bV) \Rightarrow \sum U = nA + b\sum V$$

$$\frac{dE}{db} = 0 = -2\sum V(U - A - bV) \Rightarrow \sum UV = A\sum V + b\sum V^2$$

6. Give one of the methods used to fit geometric curve to a given data.

**Answer:**

To fit a geometric curve we can use principle of Least squares method.

7. How do you select the value of  $x$  while fitting the curve?

**Answer:**

We can take the given  $x$  values as it is. Since we are using the logarithm values, we see that the values taken by  $x$  are not zero or negative.

8. Can we fit the geometric curve for any data?

**Answer:**

We cannot fit the geometric curve if any values taken by  $x$  or  $y$  is zero or negative.

9. Fit a curve of the form  $Y = ax^b$  to the following data. Also estimate  $y$  when  $x = 8$ .

$x$	3	4	5	6	7
$Y$	126	42	20	6	3

**Answer:**

Given the curve fitted to the above data is,  $Y = ax^b$ , where  $a$  and  $b$  are constants, to be determined. Hence to determine these constants, we use the following normal equations.

$$\sum U = nA + b\sum V$$

$$\sum UV = A\sum V + b\sum V^2$$

Where  $U = \log Y$ ,  $V = \log x$  and  $A = \log a$ .

To calculate the values of  $a$  and  $b$ , we construct the following table.

$x$	$Y$	$V = \log x$	$U = \log Y$	$V^2$	$UV$
3	126	0.4771	2.1004	0.2276	1.0021
4	42	0.6021	1.6232	0.3625	0.9773

5	20	0.6999	1.301	0.4899	0.9106
6	6	0.7782	0.7782	0.6056	0.6056
7	3	0.8451	0.4771	0.7142	0.4032
Totals		3.4024	6.2799	2.3992	3.8988

Therefore the normal equations can be written as follows.

$$6.2799 = 5A + 3.4024b$$

$$3.8998 = 3.4024A + 2.3998b$$

On solving above two equations, we get,  $A = 4.2626$  and  $b = -4.4185$

$a = \text{antilog}(A) = \text{antilog}(4.2626) = 18306$ .

The equation of required curve is,  $Y = 18306x^{-4.4185}$

To estimate  $y$  when  $x = 8$ ,  $Y = 18306(8)^{-4.4185} = 1.9$

10. Following data regarding the profit of a company for last 6 years in lakhs of rupees. Estimate the profit for the year 2012 by fitting the geometric curve  $y = ax^b$

Year	2006	2007	2008	2009	2010	2011
Profit (lakhs)	2.98	4.26	5.21	6.1	6.8	7.5

### Answer:

Given the curve fitted to the above data is,  $Y = ax^b$ , where  $a$  and  $b$  are constants, to be determined. Hence to determine these constants, we use the following normal equations.

$$\Sigma U = nA + b\Sigma V$$

$$\Sigma UV = A\Sigma V + b\Sigma V^2$$

Where  $U = \log Y$ ,  $V = \log x$  and  $A = \log a$ .

To calculate the values of  $a$  and  $b$ , we construct the following table.

We select  $x = \text{year} - 2005$

Year	Profit (lakhs) (Y)	x	V=logx	U=log Y	V <sup>2</sup>	UV
2006	2.98	1	0	0.4742	0	0
2007	4.26	2	0.301	0.6294	0.0906	0.1895
2008	5.21	3	0.4771	0.7168	0.2276	0.342
2009	6.1	4	0.6021	0.7853	0.3625	0.4728
2010	6.8	5	0.699	0.8325	0.4886	0.5819
2011	7.5	6	0.7782	0.8751	0.6055	0.6809
Total			2.8573	4.3134	1.7748	2.2671

Therefore the normal equations can be written as follows.

$$5.6983 = 5A + 0B, \text{ implies, } A = 5.6983/5 = 1.1397$$

$$4.7366 = 0A + 10B, \text{ implies, } B = 4.7366/10 = 0.47366$$

$a = \text{antilog}(A) = \text{antilog}(1.1397) = 13.7943$  and  $b = \text{antilog}(B) = \text{antilog}(0.47366) = 2.98$

The equation of required curve is,

$$Y = 13.7943(2.98)^x$$

To estimate  $y$  for the year 2013, we substitute the value of  $x = 2013 - 2010 = 3$  in the above equation.

ie.,  $Y = 13.7943(2.98)^3 = 365.05$  lakhs.

11. Fit a curve of best fit of the form  $y = ax^b$  for the data given below

x	1	10	100	1000
Y	10.95	39.81	50.12	63.8

**Answer:**

Given the curve fitted to the above data is,  $Y=ax^b$ , where a and b are constants, to be determined. Hence to determine these constants, we use the following normal equations.

$$\Sigma U = n A + b \Sigma V$$

$$\Sigma UV = A \Sigma V + b \Sigma V^2$$

Where  $U = \log Y$ ,  $V = \log x$  and  $A = \log a$ .

To calculate the values of a and b, we construct the following table.

X	Y	V	U=log Y	V <sup>2</sup>	UV
1	10.95	0	1.0394	0.0000	0.0000
10	39.81	1	1.6000	1.0000	1.6000
100	50.12	2	1.7000	4.0000	3.4000
1000	63.8	3	1.8048	9.0000	5.4145
		<b>6</b>	<b>6.1442</b>	<b>14.0000</b>	<b>10.4145</b>

Therefore the normal equations can be written as follows.

$$6.1442 = 4A + 6b$$

$$10.4145 = 6A + 14b$$

On solving above equations, we get,  $A = 1.1766$  and  $b = 0.2396$

$$a = \text{antilog}(A) = \text{antilog}(1.1766) = 15.0176$$

The equation of required curve is,

$$Y = 15.0176x^{0.2396}$$

12. Fit a curve  $y = ax^b$  to the following data.

x	1	2	3	4	5
Y	1.1	2.8	5.2	8	11.1

**Answer:** We fit the curve  $Y=ax^b$  to the above data, where a and b are constants, to be determined. Hence to determine these constants, we use the following normal equations.

$$\Sigma U = n A + b \Sigma V$$

$$\Sigma UV = A \Sigma V + b \Sigma V^2$$

Where  $U = \log Y$ ,  $V = \log x$  and  $A = \log a$ .

To calculate the values of a and b, we construct the following table.

X	Y	V	U=log Y	V <sup>2</sup>	UV
1	1.1	0.0000	0.0414	0.0000	0.0000
2	2.8	0.3010	0.4472	0.0906	0.1346

3	5.2	0.4771	0.7160	0.2276	0.3416
4	8	0.6021	0.9031	0.3625	0.5437
5	11.1	0.6990	1.0453	0.4886	0.7306
		<b>2.0792</b>	<b>3.1530</b>	<b>1.1693</b>	<b>1.7506</b>

Therefore the normal equations can be written as follows.

$$3.1530 = 5A + 2.0792b$$

$$1.7506 = 2.0792A + 1.1693b$$

Solving above two equations, we get,  $A = 0.0308$  and  $b = 1.4423$

$$A = \text{antilog}(A) = \text{antilog}(0.0308) = 1.0735$$

The equation of required curve is,

$$Y = 1.0735x^{1.4423}$$

13. For the following data, regarding the process control scheme on the working as one go down into a pit to measure the weir gar. Fit an equation of the form  $Y = ax^b$  to the following data.

x	3	1.4	1.9	4.3	4.8	2.4	2.6	1.6	5.5	6.7
Y	26.5	6.5	11.5	53	64	18	20	8.5	78	110

**Answer:**

We fit the curve  $Y = ax^b$  to the above data, where  $a$  and  $b$  are constants, to be determined. Hence to determine these constants, we use the following normal equations.

$$\sum U = nA + b\sum V$$

$$\sum UV = A\sum V + b\sum V^2$$

Where  $U = \log Y$ ,  $V = \log x$  and  $A = \log a$ .

To calculate the values of  $a$  and  $b$ , we construct the following table.

x	Y	V	U=log Y	V <sup>2</sup>	UV
3.0	26.5	0.4771	1.4232	0.2276	0.6791
1.4	6.5	0.1461	0.8129	0.0214	0.1188
1.9	11.5	0.2788	1.0607	0.0777	0.2957
4.3	53	0.6335	1.7243	0.4013	1.0923
4.8	64	0.6812	1.8062	0.4641	1.2304
2.4	18	0.3802	1.2553	0.1446	0.4773
2.6	20	0.4150	1.3010	0.1722	0.5399
1.6	8.5	0.2041	0.9294	0.0417	0.1897
5.5	78	0.7404	1.8921	0.5481	1.4008
6.7	110	0.8261	2.0414	0.6824	1.6863
		<b>4.7825</b>	<b>14.2465</b>	<b>2.7810</b>	<b>7.7103</b>

The normal equations are,  
 $14.2465 = 10A + 4.7825b$ , and  $7.7103 = 4.7825A + 2.7810b$ .  
 On simplifying above two equations we get  
 $A = 0.5559$  and  $b = 1.8165$   
 $a = \text{antilog}(A) = -0.255$  .  
 Therefore the equation is given by,  $Y = (-0.255) x^{1.8165}$

14. Fit an equation of the type  $Y = ax^b$  to the following data regarding the length and weight of trout fish.

<b>Weight (x kg)</b>	0.22	0.85	1.12	0.79	1.86	2.49	3.63	4.4	7.09
<b>Length (y cm)</b>	27	42	46	15	54.5	60	68	72.5	85

**Answer:**

We fit the curve  $Y = ax^b$  to the above data, where a and b are constants, to be determined. Hence to determine these constants, we use the following normal equations.

$$\Sigma U = n A + b \Sigma V$$

$$\Sigma UV = A \Sigma V + b \Sigma V^2$$

Where  $U = \log Y$ ,  $V = \log x$  and  $A = \log a$ .

To calculate the values of a and b, we construct the following table.

<b>Wight</b>	<b>length</b>	<b>V</b>	<b>U=log Y</b>	<b>V<sup>2</sup></b>	<b>UV</b>
0.22	27	-0.6576	1.4314	0.4324	-0.9412
0.85	42	-0.0706	1.6232	0.0050	-0.1146
1.12	46	0.0492	1.6628	0.0024	0.0818
0.79	15	-0.1024	1.1761	0.0105	-0.1204
1.86	54.5	0.2695	1.7364	0.0726	0.4680
2.49	60	0.3962	1.7782	0.1570	0.7045
3.63	68	0.5599	1.8325	0.3135	1.0260
4.4	72.5	0.6435	1.8603	0.4140	1.1970
7.09	85	0.8506	1.9294	0.7236	1.6413
		1.9384	15.0303	2.1310	3.9424

The normal equations are,  
 $15.0303 = 9A + 1.9384b$ , and  $3.9424 = 1.9384A + 2.131b$ .  
 On simplifying above two equations we get

$$A = 1.5813 \text{ and } b = .412$$

$$a = \text{antilog}(A) = 38.1329$$

Therefore the equation is given by,  $Y = 38.1329x^{0.412}$

15. Fit an equation of the type  $Y = ax^b$  to the following data regarding the length (in cms) of a pendulum and its period (in seconds) to complete one oscillation.

Length	6.5	11	13.2	15	18.1	23	24.4	26.5	30.6	34.3	37.5	41.5
Period (s)	0.51	0.67	0.73	0.79	0.89	0.98	1.01	1.08	1.13	1.25	1.28	1.33

**Answer:**

We fit the curve  $Y = ax^b$  to the above data, where  $a$  and  $b$  are constants, to be determined. Hence to determine these constants, we use the following normal equations.

$$\Sigma U = n A + b \Sigma V \text{ and } \Sigma UV = A \Sigma V + b \Sigma V^2$$

Where  $U = \log Y$ ,  $V = \log x$  and  $A = \log a$ .

To calculate the values of  $a$  and  $b$ , we construct the following table.

length	period	V	U=log Y	V <sup>2</sup>	UV
6.5	0.51	0.8129	-0.2924	0.6608	-0.2377
11	0.67	1.0414	-0.1739	1.0845	-0.1811
13.2	0.73	1.1206	-0.1367	1.2557	-0.1532
15	0.79	1.1761	-0.1024	1.3832	-0.1204
18.1	0.89	1.2577	-0.0506	1.5818	-0.0637
23	0.98	1.3617	-0.0088	1.8543	-0.0119
24.4	1.01	1.3874	0.0043	1.9249	0.0060
26.5	1.08	1.4232	0.0334	2.0256	0.0476
30.6	1.13	1.4857	0.0531	2.2074	0.0789
34.3	1.25	1.5353	0.0969	2.3571	0.1488
37.5	1.28	1.5740	0.1072	2.4776	0.1688
41.5	1.33	1.6180	0.1239	2.6181	0.2004
		15.7941	-0.3460	21.4309	-0.1176

The normal equations are,

$$-0.346 = 12A + 15.7941b, \text{ and } -0.1176 = 15.7941A + 21.4309b.$$

On simplifying above two equations we get

$$A = -0.7202 \text{ and } b = 0.5253$$

$$a = \text{antilog}(A) = 0.1905$$

Therefore the equation is given by,  $Y = 0.1905x^{0.5253}$