Frequently Asked Questions

1. When do you fit an exponential curve to the given data?

Answer:
When the data shows the variation in the exponential pattern, then we fit a parabolic curve to the given data, which can be easily found using scatter plot.

2. Give some instances, which show the exponential pattern in real life.

Answer:
Many processes in nature have exponential dependencies.

- The decay with time of the amplitude of a pendulum swinging in air
- The decrease in time of the temperature of an object, i.e. initially warmer than its surroundings
- The growth in time of an initially small bacterial colony

3. What are the different types of exponential curves fitted to the given data?

Answer:
The exponential curves fitted to the above data is given by,
$Y = ab^x$ and ae^{bx}

4. Can we fit exponential curve to any data?

Answer:
Usually we see the pattern to fit an exponential data. Suppose we have given a data and asked to fit an exponential curve, we cannot fit if the values of the dependent variables are negative as we are using logarithms to fit the data and logarithm of negative values is ill defined.

5. How do we obtain normal equations to fit an exponential curve?

Answer:
First, we convert the exponential form of the given curve into the linear form. Once we get the linear form of equation, we find the error term, which is obtained, by finding the squared deviations of the estimated values from the original values. Then the minimum solution of this error term gives the normal equations to find the constants of the curves.

6. Write the normal equations to fit the exponential curve of the type $Y=ab^x$

Answer:
$Y=ab^x$

Taking logarithm on both the sides, we get

$\log Y = \log a + x \log b$

$U = A + B x$, where $U = \log Y$, $A = \log a$ and $B = \log b$

Now let us find the normal equations by the method of least squares. Observe that the function U is a straight line, proceeding as we have done in the case of fitting linear curve or straight line, we minimize the error, $E=\Sigma(U-A-Bx)^2$ ie., by differentiating E with respect to A and B, and then equating to zero and simplifying we can write the normal equations as follows.

$\Sigma U = n A + B\Sigma X$

$\Sigma Ux = A \Sigma x + B \Sigma x^2$

Solving these two simultaneous equations, we can find the values of A and B and consequently we get,

$a = \text{antilog}(A)$ and $b = \text{antilog}(B)$

With these values of a and b, the curve is $Y=ab^x$

7. Write the normal equations to fit the exponential curve of the type $Y=ae^{bx}$.

Answer:
$Y=ae^{bx}$
Taking logarithm on both the sides, we get,
$\log Y= \log a + b x \log e = \log a + (b \log e) x$

Which is in the form, $U = A + B x$, where $U=\log Y$, $A = \log a$ and $B = b \log e$.

Now let us find the normal equations by the method of least squares. Observe that the function U is a straight line, proceeding as we have done in the case a, we get normal equations as follows.

$\Sigma U = n A + B\Sigma X$

$\Sigma Ux = A \Sigma x + B \Sigma x^2$

Solving these two simultaneous equations, we can find the values of A and B and consequently we get, $a = \text{antilog}(A)$ and $b = B/\log e$

With these values of a and b, the curve is $Y=ae^{bx}$

8. Give one of the methods used to fit exponential curve to the given data.

Answer:
To fit a parabolic curve we can use principle of Least squares method.

9. How do you select the value of x while fitting the curve?

Answer:

We can take the given x values as it is. But the data is available for different years, which indicates production, price, sales or population, to make simplifications easy we select the values of x such that sum of x is zero. In addition, when we need to estimated the future values, we convert the original value to the transformed values.

10. Fit an exponential curve of the form $Y=ab^x$ to the following data. Also estimate y when x =10.

X	Y	X	Y
1	1	5	3.6
2	1.2	6	4.7
3	1.8	7	6.6
4	2.5	8	9.1

Answer:
The exponential curve fitted to the above data is,
$Y=ab^x$
The normal equations are given by,
$\Sigma U = n A + B\Sigma X$
$\Sigma Ux = A \Sigma x + B \Sigma x^2$
Where U = log Y, A = log a and B = log b
To calculate the values of a and b, we construct the following table.

x	Y	U=logy	xU	x2
1	1	0	0	1
2	1.2	0.0792	0.1584	4
3	1.8	0.2553	0.7659	9
4	2.5	0.3979	1.5916	16
5	3.6	0.5563	2.7815	25
6	4.7	0.6721	4.0326	36
7	6.6	0.8195	5.7365	49
8	9.1	0.959	7.672	64
36	**30.5**	**3.7393**	**22.7385**	**204**

Therefore, the normal equations can be written as follows.

3.7393=8A+36B
22.7385=36A+204B

The above equations can be solved by using the Cramer's rule. Hence, consider the following determinants.

$$\Delta = \begin{vmatrix} 8 & 36 \\ 36 & 204 \end{vmatrix} = 8\times 204 - 36\times 36 = 336$$

$$\Delta_1 = \begin{vmatrix} 3.7393 & 36 \\ 22.7385 & 204 \end{vmatrix} = 3.7393\times 204 - 22.7385\times 36 = -55.7688$$

.

$$\Delta_2 = \begin{vmatrix} 8 & 3.7393 \\ 36 & 22.7385 \end{vmatrix} = 8\times22.7385 - 36\times3.7393 = 47.2932$$

$$and, b = \frac{\Delta_2}{\Delta} = \frac{47.2932}{336} = 0.14075 \; a = \frac{\Delta_1}{\Delta} = \frac{-55.7688}{336} = -0.166$$

a=antilog (A)=antilog (-0.166)=0.6823 and b=antilog (B) = antilog(0.14075)=1.38

The equation of required curve is, $Y=0.6821(1.38)^x$

To estimate y when x = 10, we substitute the value of x in the above equation.

i.e. $Y=0.6821(1.38)^x$ $=0.6821(1.38)^{10}$ =17.09

11. Following table gives the profits of an enterprise for 5 consecutive years. Assuming that the data shows the exponential pattern of type $Y=ab^x$ estimate the profit for the year 2013

Year	2008	2009	2010	2011	2012
Profit (lakhs)	1.6	4.5	13.8	40.2	125

Answer:
The exponential curve fitted to the above data is,

$Y=ab^x$

The normal equations are given by,

$\Sigma U = n A + B\Sigma X$

$\Sigma Ux = A \Sigma x + B \Sigma x^2$

Where U = log Y, A = log a and B = log b

To calculate the values of a and b, we construct the following table. We do not consider the year as x. To simplify the calculations, we choose x as the sum of x is equal to zero. That is x=year – 2010.

Year	Profit (Y)	x	U=logy	xU	x2
2008	1.6	-2	0.2041	-0.4082	4
2009	4.5	-1	0.6532	-0.6532	1
2010	13.8	0	1.1399	0	0
2011	40.2	1	1.6042	1.6042	1
2012	125	2	2.0969	4.1938	4
Total	**185.1**	**0**	**5.6983**	**4.7366**	**10**

Therefore, the normal equations can be written as follows.
5.6983=5A+0.B, implies, A=5.6983/5=1.1397
4.7366=0.A+10.B, implies, B=4.7366/10=0.47366
a=antilog (A)=antilog (1.1397)=13.7943 and b=antilog (B) = antilog(0.47366) =2.98
The equation of required curve is,
$Y=13.7943(2.98)^x$
To estimate y for the year 2013, we substitute the value of x = 2013-2010=3 in the above equation.
I.e. $Y=13.7943(2.98)^3$ = 365.05 lakhs.

12. Fit an equation of the form $Y=ab^x$ to the following data

x	Y	x	Y
0.25	93	1.50	38
0.50	71	1.75	29
0.75	63	2.00	26
1.00	54	2.25	22
1.25	48		

Answer:
The exponential curve fitted to the above data is,

$Y=ab^x$

The normal equations are given by,

ΣU = n A + BΣX

$\Sigma Ux = A\ \Sigma x + B\ \Sigma x^2$

Where U = log Y, A = log a and B = log b

To calculate the values of a and b, we construct the following table. We do not consider the x values as it is. We write x = (x-1.25)/0.25 so that we get sum of x is zero.

x	y	x	U	Ux	x2
0.25	93	-4	1.9685	-7.8739	16
0.5	71	-3	1.8513	-5.5538	9
0.75	63	-2	1.7993	-3.5987	4
1	54	-1	1.7324	-1.7324	1
1.25	48	0	1.6812	0	0
1.5	38	1	1.5798	1.5798	1
1.75	29	2	1.4624	2.9248	4
2	26	3	1.415	4.2449	9
2.25	22	4	1.3424	5.3697	16
Total		0	14.8323	-4.6396	60

Therefore, the normal equations can be written as follows.

14.8323 =9A+0.B,

implies, A= 14.8323/9=**1.6480**

-4.6396=0.A+60.B,

implies, B=-4.6396/60=**-0.0773**

a=antilog (A)=antilog (1.6480)=44.463 and

b=antilog (B) = antilog(-0.0773)=0.837

Therefore, the equation is given by, **$Y=44.463(0.837)^x$,** where x= (x-1.25)/0.25

13. Fit a curve of the form $y=ae^{bx}$ for the following data regarding the production of a factory in tons.

Year	1990	1995	2000	2005	2010
Production	100	120	144	207.4	248.8

Answer:
The exponential curve fitted to the above data is,

$y=ae^{bx}$

The normal equations to find the constants of the data are,

$\Sigma U = n A + B\Sigma X$

$\Sigma Ux = A \Sigma x + B \Sigma x^2$

Where U=logY, A = log a and B = b loge.

To calculate the values of a and b, we construct the following table. We do not consider the year as x. To simplify the calculations, we choose x as the sum of x is equal to zero. That is x=(year – 2000)/5

Year	Production (Y)	x	U=logY	Ux	x2
1990	100	-2	2	-4	4
1995	120	-1	2.0792	-2.0792	1
2000	144	0	2.1461	0	0
2005	207.4	1	2.3168	2.3168	1
2010	248.8	2	2.3976	4.7952	4
Total		**0**	**10.9397**	**1.0328**	**10**

Therefore, the normal equations can be written as follows.

10.9397=5A+0.B, implies, A=10.9397/5=2.1879

1.0328=0.A+10.B, implies, B=1.0328/10=0.1033

a=antilog (A)=antilog (2.1879)=154.1 and

$b=B/\log e = 0.1033/0.4343=0.2374$

Hence, the required exponential curve is given by, $Y=(154.1)e^{0.2374x}$

14. Fit an equation of the form $Y=ab^x$ for the following data.

x	2	3	4	5	6
Y	8.3	15.3	33.1	62.5	127.4

Answer:

The exponential curve fitted to the above data is, $y=ae^{bx}$

The normal equations to find the constants of the data are,

$\Sigma U = n A + B\Sigma X$

$\Sigma Ux = A \Sigma x + B \Sigma x^2$

Where $U=\log Y$, $A = \log a$ and $B = b \log e$.

To calculate the values of a and b, we construct the following table.

x	Y	U	Ux	X^2
2	8.3	0.9191	1.8382	4
3	15.3	1.1847	3.5541	9
4	33.1	1.5198	6.0793	16
5	62.5	1.7959	8.9794	25
6	127.4	2.1052	12.6310	36
20		**7.5246**	**33.0820**	**90**

The normal equations are,
$7.5246=5A+20B$, and
$33.0820=20A+90.B$.
On simplifying above two equations we get
$A=0.3115$ and $B=0.2984$
$a=$ antilog$(A)=2.0488$ and $b=$antilog$(B)=1.99$
Therefore the equation is given by, $Y=(2.2488)(1.99)^x$

15. Fit an equation of the type $Y=ab^x$ for the following data by the method of least squares.

Year	**Production (in lakhs)**	**Year**	**Production (in lakhs)**
2005	3.1	2008	9.6
2006	5.3	2009	12.9
2007	7.3	2010	17.1

Answer:
The exponential curve fitted to the above data is, $y=ae^{bx}$

The normal equations to find the constants of the data are,

$\Sigma U = n A + B\Sigma X$

$\Sigma Ux = A \Sigma x + B \Sigma x^2$

Where U=logY, A = log a and B = b loge.

To calculate the values of a and b, we construct the following table.

Year	Production (Y)	x	U=log(Y)	Ux	x
2005	3.1	-5	0.4914	-2.4568	25
2006	5.3	-3	0.7243	-2.1728	9
2007	7.3	-1	0.8633	-0.8633	1
2008	9.6	1	0.9823	0.9823	1
2009	12.9	3	1.1106	3.3318	9
2010	17.1	5	1.2330	6.1650	25
total	55.3	0	5.4048	4.9861	70

The normal equations are,
5.4048 = 6A+0, implies A=5.4048/6=0.9008
4.9861=0+70B, implies, B=4.9861/70=0.0712
a=antilog(0.9008)=7.9580 and b=antilog(0.0712)=1.18
Therefore the required exponential curve is given by, $Y=(7.9580)(1.18)^x$