Frequently Asked Questions

1. When do you fit a parabolic curve to the given data?

Answer:
When the data shows the variation in the parabolic pattern, in that case we fit a parabolic curve to the given data.

2. Write the equation used to fit a parabolic curve.

Answer:
The parabola fitted to the above data is given by,
y = a + bx + cx2.

.

3. How to obtain normal equations to fit a parabolic curve.

Answer:
We can obtain normal equation by differentiating the sum of squared deviations between the given values of y and their estimates, i.e. E=Σ(yi -a-bxi -cxi2)2 with respect to a, b and c and equating each of them equal to zero and solving them.

4. Derive normal equations to fit the quadratic or parabolic curve.

Answer:
The normal equations can be found by differentiating E=Σ(yi -a-bxi -cxi2)2 with respect to a, b and c. i.e.:

$$\frac{dE}{da} = 0 = -2\Sigma(y_i - a - bx_i - cx_i^2) \Rightarrow \Sigma y_i = na + b\Sigma x_i + c\Sigma x_i^2$$

$$\frac{dE}{db} = 0 = -2\Sigma x_i(y_i - a - bx_i - cx_i^2) \Rightarrow \Sigma x_i y_i = a\Sigma x_i + b\Sigma x_i^2 + c\Sigma x_i^3$$

$$\frac{dE}{dc} = 0 = -2\Sigma x_i^2(y_i - a - bx_i - cx_i^2) \Rightarrow \Sigma x_i^2 y_i = a\Sigma x_i^2 + b\Sigma x_i^3 + c\Sigma x_i^4$$

5. Write the normal equations used to find the constants of the parabolic curve.

Answer:
The normal equations are given by,

$\Sigma y = na + b\Sigma x + c\Sigma x^2$

$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\,\Sigma x^3$

$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\,\Sigma x^4$

6. Give one of the methods used to fit a parabolic curve to the given data.

Answer:
To fit a parabolic curve we can use principle of Least squares method.

7. How do you select the value of x while fitting the curve?

Answer:
Usually we select x such that sum of x values becomes zero in turn, the sum of x cube also becomes zero.

8. Is it necessary to make sum of x values zero?

Answer:
Sum of x values are made zero only to simplify the procedure of solving for the constants a, b and c. Otherwise we can take the values of x as it is.

9. What is the use of fitting the parabolic curve to the given data?

Answer:
Suppose if the given data shows the parabolic trend, we can fit the parabolic curve to the given data. It helps in estimating the future value of the phenomenon.

10. Compute the estimated values by fitting the second degree parabola to the given data. Also estimate the import for the year 2013

Year	**Imports ('000 tons)**	**Year**	**Imports ('000 tons)**
2001	98	2006	45
2002	87	2007	57
2003	62	2008	96
2004	47	2009	100
2005	54		

Answer:
The parabola fitted to the above data is given by, Y = a + bx + cx2
And the normal equations are given by,

$\Sigma y = na + b\Sigma x + c\Sigma x^2$

$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\,\Sigma x^3$

$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\,\Sigma x^4$

To solve for a, b and c, we construct the following table.

Year	**Imports ('000 tons)**	x=Year - 2005	x^2	x^3	x^4	**xy**	x^2y
2001	98	-4	16	-64	256	-392	1568
2002	87	-3	9	-	81	-261	783

				27			
2003	62	-2	4	-8	16	-124	248
2004	47	-1	1	1	1	-47	47
2005	54	0	0	0	0	0	0
2006	45	1	1	1	1	45	45
2007	57	2	4	8	16	114	228
2008	96	3	9	27	81	288	864
2009	100	4	16	64	256	400	1600
Total	**646**	**0**	**60**	**0**	**708**	**23**	**5383**

The first normal equation is,

646 = 9a + 0b + 60c

Implies 646=9a+60c---------(1)

23 = 0a + 60b + 0c

23 = 60b--------------------(2)

Implies, b=23/60 = **0.38**

5383=60a+0b+1468c

Implies, 5383=60a+708c---------(3)

To find a and c we solve the equations 1 and 3 as follows.

$$\Delta = \begin{vmatrix} 9 & 60 \\ 60 & 708 \end{vmatrix} = 9 \times 708 - 60 \times 60 = 2772 \qquad \Delta_1 = \begin{vmatrix} 646 & 60 \\ 5383 & 708 \end{vmatrix} = 646 \times 708 - 5383 \times 60 = 134388$$

$$\Delta_2 = \begin{vmatrix} 9 & 646 \\ 60 & 5383 \end{vmatrix} = 9 \times 5383 - 60 \times 646 = 9687$$

$$a = \frac{\Delta_1}{\Delta} = \frac{134388}{2772} = 48.48 \;\; and, c = \frac{\Delta_2}{\Delta} = \frac{9687}{2772} = 3.49$$

Therefore, the parabolic equation is given by,

$Y=48.48+0.38x+3.49x^2$

To find the estimated imports in thousand tons corresponding to different years we substitute the corresponding values of x as indicated in the third column.

For the year, 2001, x=-4. Hence estimated import is,

$Y = 48.48+(0.38)(-4)+(3.49)(-4^2) = 102.8$

For the year 2002, x=-3. Hence,

$Y= 48.48+(0.38)(-3)+(3.49)(-3^2) = 78.75$

For the year 2003, x=-2. Hence,

$Y= 48.48+(0.38)(-2)+(3.49)(-2^2) = 61.68$

Proceeding like this we can obtain estimated values for all the years.

For 2004, y=51.59

For 2005, y=48.48

For 2006, y=52.35

For 2007, y=63.20

For 2008, y=81.03

For 2009, y=105.84

To estimate the import for the year 2013, we consider x=2013-2005=8

$Y = 48.48+(0.38)(8)+(3.49)(8^2)$

= 274.88 thousand tons.

11. Fit a second degree parabola to the following data using the method of least squares. Estimate Y when X = 4.0

X	Y
0.5	72
1.0	110
1.5	158
2.0	214
2.5	290
3.0	380

Answer:

The parabola fitted to the above data is given by, Y = a + bx + cx2
And the normal equations are given by,
$\Sigma y = na + b\Sigma x + c\Sigma x^2$
$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\,\Sigma x^3$
$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\,\Sigma x^4$
To solve for a, b and c, we construct the following table.

X	Y	x=(X-1.75).4	x^2	x^3	x^4	xy	x^2y
0.5	72	-5	25	-125	625	-360	1800
1.0	110	-3	9	-27	81	-330	990
1.5	158	-1	1	-1	1	--158	158
2.0	214	1	1	1	1	214	214
2.5	290	3	9	27	81	870	2610
3.0	380	5	25	125	625	1900	9500
	1224	**0**	**70**	**0**	**1414**	**2136**	**15272**

The first normal equation is,
1224 = 6a + 0 + 70c
Implies 1224=6a+70c---------(1)

2136 = 0 + 70b + 0
2136 = 70b--------------------(2)
Implies, b=2136/70 = **30.51**

15272=70a+0b+1414c
Implies, 15272=70a+1414c--------(3)

To find a and c we solve the equations 1 and 3 as follows.

$$\Delta = \begin{vmatrix} 6 & 70 \\ 70 & 1414 \end{vmatrix} = 6\times1414 - 70\times70 = 3584$$

$$\Delta_1 = \begin{vmatrix} 1224 & 70 \\ 15272 & 1414 \end{vmatrix} = 1224\times1414 - 15272\times70 = 661696$$

$$\Delta_2 = \begin{vmatrix} 6 & 1224 \\ 70 & 15272 \end{vmatrix} = 6\times15272 - 70\times1224 = 5952$$

$$a = \frac{\Delta_1}{\Delta} = \frac{661696}{3584} = 184.625 \ and, c = \frac{\Delta_2}{\Delta} = \frac{5952}{3584} = 1.66$$

Therefore the parabolic equation is given by,
$Y=184.625 + 30.51x + 1.66x^2$

The estimated value of y when X = 4.0, that is x=(4.0 - 1.75).4 = 9 is given by,
$Y = 184.625 + (30.51)(9)+(1.66)(9^2) = 593.7$

12. Following table gives the population of India in crores over the years. Fit a parabolic curve to the data and estimate the population for the year 2021.

Year	**Population (crores)**	**Year**	**Population (Crores)**
1901	238.4	1961	439.23
1911	252.09	1971	84.16
1921	251.32	1981	683.33
1931	278.98	1991	864.42

1941	318.16	2001	1028074
1951	316.09	2011	1210.19

Answer:

The parabola fitted to the above data is given by, Y = a + bx + cx2

And the normal equations are given by,

$\Sigma y = na + b\Sigma x + c\Sigma x^2$

$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\ \Sigma x^3$

$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\ \Sigma x^4$

To solve for a, b and c, we construct the following table.

Year	**Population (crores)**	**x= (Year – 1956)/ 5**	**x^2**	**x^3**	**x^4**	**xy**	**x^2y**
1901	2.38	-11	121	-1331	14641	-26.18	287.98
1911	2.52	-9	81	-729	6561	-22.68	204.12
1921	2.51	-7	49	-343	2401	-17.57	122.99
1931	2.79	-5	25	-125	625	-13.95	69.75
1941	3.18	-3	9	-27	81	-9.54	28.62
1951	3.16	-1	1	-1	1	-3.16	3.16
1961	4.39	1	1	1	1	4.39	4.39
1971	5.84	3	9	27	81	17.52	52.56
1981	6.83	5	25	125	625	34.15	170.75
1991	8.64	7	49	343	2401	60.48	423.36
2001	10.29	9	81	729	6561	92.61	833.49
2011	12.1	11	121	1331	14641	133.1	1464.1
Total	**64.63**	**0**	**572**	**0**	**48620**	**249.17**	**3665.27**

The first normal equation is,

64.63 = 12a + 572c--------------(1)

2nd equation is given by,

249.17= 572b--------------------(2)

Implies, b=249.17/572 = **0.43**

3665.27=572a+48620c---------(3)

To find a and c we solve the equations 1 and 3 as follows.

$$a = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} 64.63 & 572 \\ 3665.27 & 48620 \end{vmatrix}}{\begin{vmatrix} 12 & 572 \\ 572 & 48620 \end{vmatrix}} = \frac{1046776.16}{256256} = 4.08$$

$$c = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} 12 & 64.63 \\ 572 & 3665.27 \end{vmatrix}}{\begin{vmatrix} 12 & 572 \\ 572 & 48620 \end{vmatrix}} = \frac{7014.88}{256256} = 0.027$$

Therefore, the parabolic equation is given by,

$Y=4.08 + 0.43x + 0.027x^2$

To estimate the population for the year 2021, we consider x=(2021-1956)/5=13

$Y = 4.08 + 0.43(13) + 0.027(13^2)$

= 4.08 + 5.59 + 4.563

= 14.233 crores

13. The following figures are the production data of a certain factory manufacturing air-conditioners. Fit a second degree parabolic curve to the data.

Answer:

The parabola fitted to the above data is given by, $Y = a + bx + cx^2$

And the normal equations are given by

$\Sigma y = na + b\Sigma x+c\Sigma x^2$

$\Sigma xy = a\Sigma x + b\Sigma x^2+c\ \Sigma x^3$

$\Sigma x^2y = a\Sigma x^2+b\Sigma x^3+c\ \Sigma x^4$

To solve for a, b and c, we construct the following table.

Year	**Production ('000 units)**	**x=year-1995**	**x^2**	**x^3**	**x^4**	**xy**	**x^2y**
1990	17	-5	25	-125	625	-85	425
1991	20	-4	16	-64	256	-80	320
1992	19	-3	9	-27	81	-57	171
1993	26	-2	4	-8	16	-52	104
1994	24	-1	1	-1	1	-24	24
1995	40	0	0	0	0	0	0
1996	35	1	1	1	1	35	35
1997	55	2	4	8	16	110	220

1998	51	3	9	27	81	153	459
1999	74	4	16	64	256	296	1184
2000	79	5	25	125	625	395	1975
Total	**440**	**0**	**110**	**0**	**1958**	**691**	**4917**

440=11a+110c
691=110b, that is b=691/110=6.28
4917=110a+1958c

On simplification, a=34 and c=0.60

Therefore, $Y = 34 + 6.28x + 0.60x^2$

14. Fit a parabolic curve to the given data.

Year	Production ('000 units)
1991	4
1992	8
1993	9
1994	12
1995	11
1996	14
1997	16
1998	17
1999	26

Answer:

The parabola fitted to the above data is given by, $Y = a + bx + cx^2$

And the normal equations are given by

$\Sigma y = na + b\Sigma x + c\Sigma x^2$

$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\,\Sigma x^3$

$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\,\Sigma x^4$

To solve for a, b and c, we construct the following table.

Year	**Production ('000 units)**	**x=year-1995**	**x^2**	**x^3**	**x^4**	**xy**	**x^2y**
1991	4	-4	16	-64	256	-16	64
1992	8	-3	9	-27	81	-24	72
1993	9	-2	4	-8	16	-18	36
1994	12	-1	1	-1	1	-12	12
1995	11	0	0	0	0	0	0
1996	14	1	1	1	1	14	14
1997	16	2	4	8	16	32	64

1998	17	3	9	27	81	51	153
1999	26	4	16	64	256	104	416
total	**117**	**0**	**60**	**0**	**708**	**131**	**831**

117=9a+6c
131=60b, implies b=131/60=2.183
831=60a+708c
On simplification, we get, a=11.89 and c=0.116
Therefore, the equation for fitting parabola is given byy=11.89+2.183x+0.116x^2.

15. The prices of a commodity during 2005-2010 are given below. Fit a parabola to these data. Estimated the price of the commodity for the year 2011.

Year	**Price**	**Year**	**Price**
2005	100	2008	140
2006	107	2009	181
2007	128	2010	192

Answer:

The parabola fitted to the above data is given by, $Y = a + bx + cx^2$

And the normal equations are given by

$\Sigma y = na + b\Sigma x + c\Sigma x^2$

$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\,\Sigma x^3$

$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\,\Sigma x^4$

To solve for a, b and c, we construct the following table.

Year	**price**	**x**	**x^2**	**x^3**	**x^4**	**xy**	**x^2y**
2005	100	-5	25	-125	625	-500	2500
2006	107	-3	9	-27	81	-321	963
2007	128	-1	1	-1	1	-128	128
2008	140	1	1	1	1	140	140
2009	181	3	9	27	81	543	1629
2010	192	5	25	125	625	960	4800
Total	**848**	**0**	**70**	**0**	**1414**	**694**	**10160**

848=6a+70c
694=70b, implies b=9.91
10160=70a+1414c

On simplification we get, a=136.125 and c=0.45.

Therefore, the equation for the parabola is given by,
Y=136.125+9.91x+0.45x^2
Estimated price for the year 2011 is given by, 227.54.