Frequently Asked Questions

1. Simulate a random sample of size 10 from discrete uniform distribution with parameter n=5 using following random numbers. 614, 122, 811, 658, 147, 389, 948, 825, 994, 862. Also compare the sample mean and variance with population measures.

Answer:
Given X ~ discrete uniform (n=5). Therefore pmf is given by,
p(x) = 1/n, x= 0, 1, 2, 3, 4.
= 1/5.
To simulate the random sample, first we complete the following table.

x	P(X=x)	P(X≤x)	Range	Tally bars	f	fx	fx^2
0	0.2	0.2	000-199	II	2	0	0
1	0.2	0.4	200-399	I	1	1	1
2	0.2	0.6	400-599		0	0	0
3	0.2	0.8	600-799	II	2	6	18
4	0.2	1.0	800-999	~~IIII~~	5	20	80
				Total	10	27	99

Random observations are, 3, 0, 4, 3, 0, 1, 4, 4, 4, 4
Now let us find the population mean and variance.
Mean = (n-1)/2=(5-1)/2=2
And variance is, $(n^2-1)/12 = (5^2-1)/12=2$
Sample mean and variance
Mean $=\Sigma fx/N=27/10 = 2.7$ and
Variance $\sigma^2=\Sigma fx^2/N-(\Sigma fx/N)^2 = 99/10-(27/10)^2=2.61$
Observe that sample mean and variance are greater than that of population mean and variance.

2. Simulate a random sample of size 10 from Bernoulli distribution with parameter p=0.4 using following random numbers. 845, 449, 339, 797, 345, 937, 573, 194, 476, 968. Also compare the sample mean and variance with population measures.

Answer:
Given X ~ Bernoulli(p=0.4). Therefore $p(x) =p^x q^{1-x}$,x=0, 1

$$= (0.4)^x(0.6)^{1-x}$$

To simulate the random observations, first we find the following table.

x	P(X=x)	P(X≤x)	Range	Tally bars	f	fx	fx^2
0	0.6	0.6	000-599	~~IIII~~ I	6	0	0
1	0.4	1.0	600-999	IIII	4	4	4
			Total		10	4	4

Random observations are, 1, 0, 0, 1, 0, 1, 0, 0, 0, 1

Now let us find the population mean and variance.

Mean = p=0.4

And variance is, pq = 0.4x0.6 = 0.24

Sample mean and variance:

Mean =Σfx/N=4/10 = 0.4 and

Variance $\sigma^2=\Sigma fx^2/N-(\Sigma fx/N)^2$

$= 4/10-(4/10)^2=0.24$

Observe that sample mean and variance exactly coincide with population mean and variance.

3. Simulate a random sample of size 10 from Bernoulli distribution with parameter p=0.7 using following random numbers. 884, 342, 875, 128, 949, 828, 047, 087, 006, 138

Answer:
Given X ~ Bernoulli(p=0.7). Therefore $p(x) = p^x q^{1-x}$,x=0, 1

$= 0.7^x.0.3^{1-x}$

x	P(X=x)	P(X≤x)	Range
0	0.3	0.3	000-299
1	0.7	1.0	300-999

Random observations are, 1, 1, 1, 0, 1, 1, 0, 0, 0, 0

4. Simulate a random sample of size 15 from binomial distribution with parameters n=5 and p=0.5 using following random numbers. 443, 207, 761, 503, 575, 227, 222, 108, 489, 255, 877, 182, 190, 380, 758. Also compare the sample mean and variance with population measures.

Answer:
Given X ~ B(n=5, p=0.5). Therefore pmf is given by,

$$p(x) = {}^nC_x p^x q^{n-x}, x = 0,1,...,n$$

$$p(x) = {}^5C_x 0.5^x 0.5^{5-x}, x = 0,1,...,5$$

ie.,

To simulate the random observations, first we find the following table.

x	P(X=x)	P(X≤x)	Range	Tally bars	f	fx	fx^2
0	0.031	0.031	000-030		0	0	0
1	0.156	0.187	031-186	II	2	2	2
2	0.313	0.5	187-499	~~IIII~~ III-	8	16	32
3	0.313	0.813	500-812	IIII	4	12	36

4	0.156	0.969	813-968	I	1	4	16
5	0.031	1	969-999		0	0	0
				Total	15	34	86

Random observations are, 2, 2, 3, 3, 3, 2, 2, 1, 2, 2, 4, 1, 2, 2, and 3.

Now let us find the population mean and variance.

Mean = np= 5x0.5=2.5

And variance is, npq = 5x0.5x0.5 = 1.25

Sample mean and variance:

Mean =Σfx/N=34/15 = 2.2667 and

Variance $\sigma^2=\Sigma fx^2/N-(\Sigma fx/N)^2 = 86/15 - (34/15)^2 = 2.0289$

Observe that, Population mean > sample mean and Sample variance > population variance

5. Simulate a random sample of size 15 from binomial distribution with parameters n=6 and p=0.3 using following random numbers. 4462, 6791, 6102, 0215, 7398, 2671, 6175, 0513, 1413, 8771, 1596, 1747, 2005, 2493, 9285.

Answer:
Given X ~ B(n=6, p=0.3). Therefore pmf is given by,

$$p(x)={}^{n}C_{x}p^{x}q^{n-x}, x=0,1,...,n$$

$$p(x)={}^{6}C_{x}(0.5)^{x}(0.5)^{6-x}, x=0,1,...6$$

To simulate the random observations, first we find the following table.

x	P(X=x)	P(X≤x)	Range
0	0.1176	0.1176	0000-1175
1	0.3025	0.4201	1176-4200
2	0.3241	0.7442	4201-7441
3	0.1852	0.9294	7442-9293
4	0.0595	0.9889	9294-9888
5	0.0103	0.9992	9889-9991
6	0.0008	1	9992-9999

Random observations are 2, 2, 3, 3, 3, 2, 2, 1, 2, 2, 4, 1, 2, 2, and 3.

6. Simulate a random sample of size 10 from binomial distribution with parameters n=5 and p=0.7 using following random numbers. 190, 382, 793, 261, 003, 073, 681, 665, 295, 804.

Answer:
Given X ~ B(n=6, p=0.3). Therefore pmf is given by,

$$p(x)={}^{n}C_{x}p^{x}q^{n-x}={}^{5}C_{x}(0.7)^{x}(0.3)^{n-x}, x=0,1,...,n$$

To simulate the random observations, first we find the following table.

x	P(X=x)	P(X≤x)	Range
0	0.002	0.002	000-001
1	0.028	0.030	002-029-
2	0.132	0.162	030-161
3	0.309	0.471	162-470
4	0.360	0.831	471-830
5	0.169	1	831-999

Random observations are 3, 3, 4, 3, 1, 2, 4, 4, 3, 4.

7. Simulate a random sample of size 15 from Poisson distribution with parameter λ=0.25 using following random numbers. 230, 298, 864, 891, 498, 462, 635, 618, 214, 939, 601, 832, 158, 635, 984.

Answer:
Given X ~ P(λ=0.25). Therefore pmf is given by,

$$p(x) = \frac{e^{-\lambda}\lambda^{x}}{x!} = \frac{e^{-0.25}(0.25)^{x}}{x!}, x = 0,1,2,...\infty$$

To simulate the random observations, first we find the following table.

x	P(X=x)	P(X≤x)	Range
0	0.779	0.779	000-778
1	0.195	0.974	779-973
2	0.024	0.998	974-997
≥3	0.002	1	997-999

Random observations are 0, 0, 1, 1, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 2

8. Simulate a random sample of size 25 from Poisson distribution with parameter λ=5 using following random numbers. 633, 570, 128, 966, 219, 045, 054, 260, 001, 042, 986, 685, 763, 564, 982, 593, 652, 003, 080, 861, 812, 679, 627, 430, 896.

Answer:
Given X ~ P(λ=5). Therefore pmf is given by,

$$p(x) = \frac{e^{-\lambda}\lambda^{x}}{x!} = \frac{e^{-5}5^{x}}{x!}, x = 0,1,2,...\infty$$

To simulate the random observations, first we find the following table.

x	P(X=x)	P(X≤x)	Range
0	0.007	0.007	000-006
1	0.034	0.041	007-040

2	0.084	0.125	041-124
3	0.140	0.265	125-264
4	0.176	0.441	265-440
5	0.176	0.617	441-616
6	0.146	0.763	617-762
7	0.104	0.867	763-866
8	0.065	0.932	867-931
9	0.036	0.968	932-967
10	0.018	0.986	968-985
11	0.008	0.994	986-993
≥12	0.006	1	994-999

Random observations are, 6, 5, 3, 9, 3, 2, 2, 3, 0, 2, 11, 6, 7, 5, 10, 5, 6, 1, 2, 7, 7, 6, 6, 4, 8.

9. Simulate a random sample of size 15 from geometric distribution with parameter p=0.6 using following random numbers. 271, 478, 866, 828, 436, 250, 005, 939, 048, 031, 759, 124, 601, 105, 715.

Answer:

Given X ~ Geometric(p=0.6)

Therefore pmf is given by,

$p(x) = q^x p$; x=0, 1, 2, …. $0<p\leq 1$

$= (0.4)^x(0.6)$

To simulate the random observations, first we find the following table.

x	P(X=x)	P(X≤x)	Range
0	0.600	0.600	0-599
1	0.240	0.840	600-839
2	0.096	0.936	840-935
3	0.038	0.974	936-973
4	0.015	0.989	974-988
≥5	0.011	1	989-999

Random observations are 0, 0, 2, 1, 0, 0, 0, 3, 0, 0, 1, 0, 1, 0, 1

10. Simulate a random sample of size 15 from geometric distribution with parameter p=0.3 using following random numbers. 521, 000, 009, 209, 818, 833, 180, 161, 714, 426, 804, 510, 734, 891, 912.

Answer:

Given X ~ Geometric(p=0.3). Therefore pmf is given by,

$p(x) = q^x p$; x=0, 1, 2, …. $0<p\leq 1$

$= (0.7)^x(0.3)$

To simulate the random observations, first we find the following table.

x	P(X=x)	P(X ≤ x)	Range

0	0.300	0.300	000-299
1	0.210	0.510	300-509
2	0.147	0.657	510-656
3	0.103	0.760	657-759
4	0.072	0.832	760-831
5	0.050	0.882	832-881
6	0.035	0.917	882-916
7	0.025	0.942	917-941
≥8	0.058	1	942-999

Random observations are, 2, 0, 0, 0, 4, 5, 1, 1, 3, 1, 4, 2, 3, 6, 6.

11. Simulate a random sample of size 10 from negative binomial distribution with parameters r = 7 and p=0.4 based on the following random numbers. 710, 338, 782, 004, 998, 233, 360, 283,155, 025,

Answer:

Given X ~ NB(r=7, p=0.4)

$$\binom{x+r-1}{r-1}p^r q^x = \binom{x+6}{6}(0.4)^7(0.6)^x$$

Therefore pmf is given by,

p(x) =

x=0, 1, 2, …., and 0<p≤1

To simulate the random observations, first we find the following table.

x	P(X=x)	P(X≤x)	Range
0	0.002	0.002	000-001
1	0.007	0.009	002-008
2	0.017	0.025	009-024
3	0.030	0.055	025-054
4	0.045	0.099	055-098
5	0.059	0.158	099-157
6	0.071	0.229	158-228

7	0.079	0.308	229-307
8	0.083	0.390	308-389
9	0.083	0.473	390-472
10	0.079	0.552	473-551
11	0.074	0.626	552-625
12	0.066	0.692	626-691
13	0.058	0.750	692-749
14	0.050	0.800	750-799
15	0.042	0.842	800-841
16	0.034	0.876	842-875
17	0.028	0.904	876-903
18	0.022	0.926	904-925
19	0.018	0.944	925-943
20	0.014	0.958	944-957
21	0.011	0.969	958-968
22	0.008	0.977	969-976
23	0.006	0.983	977-982
24	0.005	0.987	983-986
25	0.003	0.991	987-990
26	0.003	0.993	991-992
27	0.002	0.995	992-994
28	0.001	0.997	995-996
29	0.001	0.998	997-997
30	0.001	0.999	998-998
31 & above	0.001	1	999

Random Observations are, 13, 8, 14, 1, 30, 7, 8, 7, 5, 3.

12. Simulate a random sample of size 20 from negative binomial distribution with parameters r=5 and p=0.75 based on the following random numbers. 989, 761, 511, 755, 386, 894, 565, 999, 979, 525, 083, 912, 994, 865, 915, 052, 208, 785, 073, 690

Answer:

Given X ~ NB(r=5, p=0.75)

$$\binom{x+r-1}{r-1} p^r q^x = \binom{x+4}{4} (0.75)^5 (0.25)^x$$

Therefore pmf is given by,

p(x) =

x=0, 1, 2, …., and 0<p≤1

To simulate the random observations, first we find the following table.

x	P(X=x)	P(X≤x)	Range
0	0.237	0.237	000-236
1	0.297	0.534	237-533
2	0.222	0.756	534-755
3	0.130	0.886	756-885
4	0.065	0.951	886-950
5	0.029	0.980	951-979
6	0.012	0.992	980-991
7	0.005	0.997	992-996
8	0.002	0.999	997-998
9	0.001	1.000	999-999

Random observations are 6, 3, 1, 2, 2, 4, 2, 9, 5, 1, 0, 4, 7, 3, 4, 0, 0, 3, 0, 2

13. Simulate a random sample of size 20 from negative binomial distribution with parameters r=6 and p=0.9 based on the following random numbers. 989, 761, 511, 755, 386, 894, 565, 999, 979, 525, 083, 912, 994, 865, 915, 052, 208, 785, 073, 690

Answer:

$$\binom{x+r-1}{r-1} p^r q^x = \binom{x+5}{5} (0.75)^6 (0.25)^x$$

Therefore pmf is given by,

p(x) =

x=0, 1, 2, …., and 0<p≤1

To simulate the random observations, first we find the following table.

x	P(X=x)	P(X≤x)	Range
0	0.531	0.531	000-530
1	0.319	0.850	531-849
2	0.112	0.962	850-961
3	0.030	0.992	962-991
4	0.007	0.998	992-997

5	0.001	1.000	998-999

Random observations are 3, 1, 0, 1, 0, 2, 1, 5, 3, 0, 0, 2, 4, 2, 2, 0, 0, 1, 0, 1.

14. Simulate a random sample of size 10 from Hypergeometric distribution with parameters N=10, M=5 and n=4 based on the following random numbers. 884, 342, 875, 128, 949, 828, 047, 087, 006, 138

Answer:

X ~ Hypergeometric (N=10, M=5, n=4). Hence pmf is given by,

$$p(x) = \binom{M}{x}\binom{N-M}{n-x} \div \binom{N}{n}, x = 0,1,\ldots \min(M, n)$$

$$= \binom{5}{x}\binom{5}{4-x} \div \binom{10}{4}$$

To simulate the random observations, first we find the following table.

x	P(X=x)	P(X≤x)	Range
0	0.024	0.024	000-023
1	0.238	0.262	024-261
2	0.476	0.738	262-737
3	0.238	0.976	738-975
4	0.024	1.000	976-999

Hence random observations are, 3, 2, 3, 1, 3, 3, 1, 1, 0, 1.

15. Simulate a random sample of size 10 from Hypergeometric distribution with parameters N=12, M=10 and n=4 based on the following random numbers. 884, 342, 875, 128, 949, 828, 047, 087, 006, 138

Answer:

X ~ Hypergeometric (N=20, M=10, n=4). Hence pmf is given by,

$$p(x) = \binom{M}{x}\binom{N-M}{n-x} \div \binom{N}{n}, x = 0,1,\ldots \min(M, n)$$

$$= \binom{10}{x}\binom{10}{4-x} \div \binom{20}{4}$$

To simulate the random observations, first we find the following table.

x	P(X=x)	P(X≤x)	Range

0	0.043	0.043	000-042
1	0.248	0.291	043-290
2	0.418	0.709	291-708
3	0.248	0.957	709-956
4	0.043	1.000	957-999

Hence random observations are, 3, 2, 3, 1, 3, 3, 1, 1, 0, 1.