

## Frequently Asked Questions

1. What do you mean by 3 way classified data?

**Answer:**

In a three way classified data we consider 3 attributes which are divided into different sub groups viz., possession and non-possession of given attribute or possession at different levels.

2. How many classes you get when you have 3 attributes divided into dichotomous classes?

**Answer:**

The total number of classes in which a universe can be divided on the basis of attributes is  $3^n$ .

Similarly in case of three attributes the total number of classes would be  $3^3$  or 27. Out of these 27 classes the number of classes of the ultimate order would be  $2^3$  or 8.

3. Write the order and nature of classes when 3 attributes with dichotomous classes are considered.

**Answer:**

When we have 3 attributes with dichotomous classes, we have 27 classes. The number of classes of the various order in case of three attributes would be as follows

- One zero order class N, which is of positive nature
- 6 classes of first order of which (A), (B), (C), are of positive nature and ( $\alpha$ ), ( $\beta$ ), ( $\gamma$ ), are of negative nature
- 12 classes of 2<sup>nd</sup> order of which (AB), (AC), (BC), are of positive nature. ( $A\beta$ ), ( $A\gamma$ ), ( $B\gamma$ ), ( $\alpha B$ ) ( $\alpha C$ ) ( $\beta C$ ), are pairs of contraries and ( $\alpha\beta$ ) ( $\alpha\gamma$ ) ( $\beta\gamma$ ), are of negative nature
- 8 classes of third order of which (ABC) of positive nature. ( $AB\gamma$ ), ( $A\beta C$ ) ( $\alpha BC$ ), ( $A\beta\gamma$ ) ( $\alpha B\gamma$ ), ( $\alpha\beta C$ ), are pairs of contraries and ( $\alpha\beta\gamma$ ), of negative nature

4. How to express the any class frequency in terms of the frequencies of higher order?

**Answer:**

Here we can express any class frequency in terms of the frequencies of higher order. Thus,

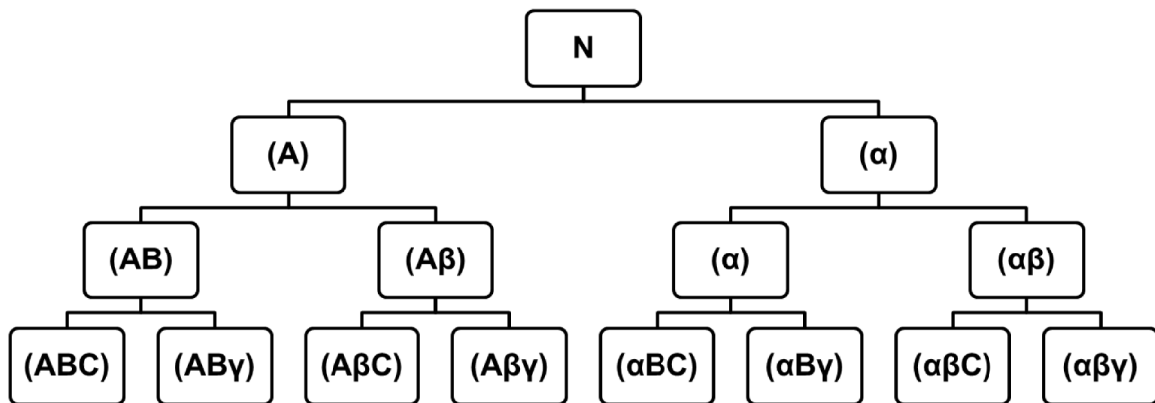
$$(AB) = (ABC) + (AB\gamma)$$

$$(AC) = (ABC) + (A\beta C)$$

$$(BC) = (ABC) + (B\gamma C) \text{ and so on.}$$

5. Show a chart explaining the relationship between various classes when 3 attributes are involved.

**Answer:**



6. If we know number of items possessing the given attribute, then how to find the values of other classes.

**Answer:**

We have general relationship between the known values and unknown values as given below.

$$(\alpha) = N - (A), (\beta) = N - (B), (\gamma) = N - (C)$$

$$(\alpha\beta) = \alpha\beta.N = (1-A)(1-B).N = N - [(A) + (B)] + (AB)$$

$$(\alpha\beta\gamma) = \alpha\beta\gamma.N = (1-A)(1-B)(1-C).N$$

$$= N - [(A) + (B) + (C)] + [(AB) + (AC) + (BC)] - (ABC)$$

$$(AB\gamma) = AB\gamma.N = AB(1-C).N = (AB) - (ABC)$$

$$(A\beta\gamma) = A\beta\gamma.N = A(1-B)(1-C).N = (A) - (AB) - (AC) + (ABC)$$

and so on.

7. What are the conditions for consistency of a data when we have three attributes?

**Answer:**

The following rules of consistence are applicable in case of three attributes.

1.  $(ABC) \geq 0$ .
2.  $(AB\gamma) \geq 0$ , implies  $(ABC) \leq (AB)$
3.  $(A\beta C) \geq 0$ , implies  $(ABC) \leq (AC)$
4.  $(\alpha BC) \geq 0$ , implies  $(ABC) \leq (BC)$
5.  $(A\beta\alpha) \geq 0$ , implies  $(ABC) \geq (AB) + (AC) - (A)$
6.  $(\alpha B\gamma) \geq 0$ , implies  $(ABC) \geq (AB) + (BC) - (B)$

7.  $(\alpha\beta C) \geq 0$ , implies  $(ABC) \geq 0$ ,  $(AC) + (BC) - (C)$
8.  $(\alpha\beta\gamma) \geq 0$ , implies  $(ABC) \leq (AB) + (BC) + (AC) - (A) - (B) - (C) + N$

8. What is the minimum value taken by  $(ABC)$ ?

**Answer:**

The first rule of consistency says that  $(ABC) \geq 0$ . Hence it gives the minimum value of  $(ABC)$ , which is equal to zero.

9. What is the maximum value taken by  $(ABC)$ ?

**Answer:**

The last rule of consistency of data involving 3 attributes says that  $(ABC) \leq (AB) + (BC) + (AC) - (A) - (B) - (C) + N$  which gives the maximum value of  $(ABC)$ . Hence maximum value of  $(ABC)$  is given by,

$$(ABC) = (AB) + (BC) + (AC) - (A) - (B) - (C) + N$$

10. Reduce the above 8 rules given to test for the consistency of the data into 4 conditions.

**Answer:**

From 1<sup>st</sup> and 8<sup>th</sup> rule given for the consistency of the data we can have,

1.  $(AB) + (BC) + (AC) \geq (A) + (B) + (C) - N$
2. From 2 and 7 we get,  $(AC) + (BC) - (AB) \leq C$
3. From 3 and 6, we get,  $(AB) + (BC) - (AC) \leq (B)$
4. and From 4 and 5, we get,  $(AB) + (AC) - (BC) \leq A$

Hence given 8 conditions of consistency can be reduced to 4 conditions.

11. One more rule of consistence applicable to any number of attribute is that for N attributes  $(A, B, C, D \dots M)$  is  $(ABCD \dots M) \geq [(A) + (B) + (C) + (D) + \dots (M)] - (n-1)N$

**Answer:**

Let us prove this result by induction.

First we consider the two attributes A and B. Then one of the conditions for consistency is,  $(\alpha\beta) \geq 0$  implies  $(AB) \geq (A) + (B) - (N)$

We prove the above condition as follows.

Consider  $(\alpha\beta) = (\alpha) - (\alpha B)$

$$= N - (A) - [(B) - (AB)]$$

$$= N - (A) - (B) + (AB)$$

Implies  $(AB) = (\alpha\beta) + (A) + (B) - N$

From this expression we can observe that, if  $(AB) < (A) + (B) - N$ , then  $(\alpha\beta)$  will be negative.

Hence  $(AB) \geq (A) + (B) - N$

Applying above inequality to the universe of C we get,

$$(ABC) \geq (AC) \text{ plus } (BC) \text{ minus } (C)$$

$$\geq (A) + (C) - N + (B) + (C) - N - (C)$$

$$\geq (A) + (B) + (C) - 2N$$

Applying the above inequality to the universe of D, we get,

$$(ABC) \geq (AD) + (BD) + (CD) - 2D$$

$$\geq (A)+(B)-N+(B)+(D)-N+(C)+(D)-N-2 D$$

$$\geq (A) + (B) + (C) + (D) - 3N$$

Therefore in general we can write for n attributes

$$(ABCD...M) \geq [(A)+(B)+(C)+(D)+...(M)] - (n-1)N$$

12. Show that

1. If all A's are B' and all B's are C's, then all A's are C's
2. If all A's are B's and no B's are C's then no A's are C's.

**Answer:**

In the above result the first one we prove as follows,

We have given all A's are B' implies  $(AB) = (A)$

and all B's are C's implies  $(BC) = (B)$

To prove  $(AC) = (A)$ ,

We have  $(AB) + (BC) - (AC) \leq (B)$  implies  $(A) + (B) - (AC) \leq (B)$

$$\text{Implies } (A) \leq (AC) \text{ or } (AC) \geq (A)$$

But since  $(AC)$  cannot be greater than  $(A)$ , we get  $(AC) = A$ .

In the second we have given,

All A's are B's implies  $(AB) = (A)$  and no B's are C's implies  $(BC) = 0$

And we want to prove that  $(AC) = 0$ .

We have  $(AB)+(AC)-(BC) \leq A$  Implies  $(A) + (AC) - 0 \leq (A)$ , implies  $(AC) \leq 0$ .

And since  $(AC) \geq 0$ , we must have  $(AC) = 0$ .

13. Show that  $(A)/N = x$ ,  $(B)/N = 2x$  and  $(C)/N = 3x$ , and

$[(AB)/N] = [(BC)/N] = [(CA)/N] = y$ , then the value of neither  $x$  nor  $y$  can exceed 1 by 4.

**Answer:**

Consider the conditions of consistency,

$(AB) \leq (A)$  implies  $Ny \leq Nx$  implies  $y \leq x$  -----(i).

Also  $(BC) \geq (B) + (C) - N \Rightarrow \frac{(BC)}{N} = \frac{(B)}{N} + \frac{(C)}{N} - 1$ .

Implies,  $y \geq 2x + 3x - 1$

Therefore,  $5x - 1 \leq y$ . ----- (ii)

(i) and (ii) give  $5x - 1 \leq x$

Implies,  $4x \leq 1$

Implies,  $x \leq \frac{1}{4}$ . ----- (iii)

Thus, from (i), (ii) and (iii), we have

$\frac{1}{2} \leq x \leq \frac{1}{4}$ , which establishes the result.

14. Given that  $(A) = (\alpha) = (B) = (\beta) = (C) = (\gamma) = \frac{1}{2}N$  and also  $(ABC) = (\alpha\beta\gamma)$ , show that  $2(ABC) = (AB) + (AC) + (BC) - \frac{1}{2}N$ .

**Answer**

From the remark we know that,

$$(\alpha\beta\gamma) = N - [(A) + (B) + (C)] + [(AB) + (AC) + (BC)] - (ABC)$$

Substituting for  $(A)$ ,  $(B)$ ,  $(C)$  and  $(\alpha\beta\gamma)$ , we get,

$$(ABC) = N - [\frac{1}{2}N + \frac{1}{2}N + \frac{1}{2}N] + [(AB) + (AC) + (BC)] - (ABC)$$

$$\text{Implies, } 2(ABC) = N - (3N/2) + (AB) + (AC) + (BC)$$

$$\text{Implies, } 2(ABC) = (AB) + (AC) + (BC) - \frac{1}{2}N$$

15. Among the adult population of a certain town 50 per cent are males, 60 percent are wage earners and 50 percent are 45 years of age or over, 10 percent of the males are not wage earners and 40 percent of the males are under 45. Make a best possible inference about the limits within which the percentage of persons (male or female) of 45 years or over is wage earners.

**Answer:** We have given all the figures in percentages. Hence we take  $N = 100$ . Let us denote male by  $A$ , wage earners by  $B$  and 45 years of age or over by  $C$ .

Hence we have given  $N = 100$ ,  $(A) = 50$ ,  $(B) = 60$  and  $(C) = 50$ .

$$\text{Further } (A\beta) = 50 \times 10\% = 5 \text{ and } (A\gamma) = 50 \times 40\% = 20$$

$$\text{Therefore, } (AB) = (A) - (A\beta) = 45 \text{ and } (AC) = (A) - (A\gamma) = 30.$$

We are required to find the limits for  $(BC)$ .

Using the conditions of consistency,

- i.  $(AB)+(BC)+(AC) \geq (A)+(B)+(C)-N$  Implies  $(BC) \geq (A)+(B)+(C)-N-(AB)-(AC)$
- ii. Implies  $(BC) \geq 50 + 60 + 50 - 100 - 45 - 30 = -15$
- iii.  $(AB) + (AC) - (BC) \leq A$  Implies  $(BC) \geq (AB) + (AC) - (A) = 45 + 30 - 50 = 25$
- iv.  $(AB) + (BC) - (AC) \leq (B)$  Implies  $(BC) \leq (B) + (AC) - (AB) = 60 + 30 - 45 = 45$
- v.  $(AC) + (BC) - (AB) \leq C$  Implies  $(BC) \leq (C) + (AB) - (AC) = 50 + 45 - 30 = 65$

Therefore, from (i) to (iv), we have,  $25 \leq (BC) \leq 45$ .

Hence, the percentage of wage earning population of 45 years or over must lie between 25 and 45.