1. Introduction

Welcome to the series of E-learning modules on Practicals on Partial and Multiple Correlation and Regression. In this module we discuss about the computation of partial and multiple correlation coefficients, regression line of one variable on the remaining two variables and predicting the future value based on the fitted regression equation.

By the end of this session, you will be able to

- Find partial and multiple correlation
- Understand regression line of one variable on the other two variables and predict the future value
- Understand Partial regression and partial correlation coefficients

Partial correlation and multiple correlations are used when we have more than two variables.
Here in particular we discuss the partial and multiple correlation between three variables.
Hence either we can find the correlation between two variables by eliminating the effect of third variable or simple correlation between one variable and the joint effect of other two variables.

Partial correlation:
Sometimes the correlation between two variables X one and X two may be partly due to the correlation of a third variable, X three with both X one and X two. In such a situation, one may want to know what the correlation between X one and X two would be if the effect of X three on each of X one and X two were eliminated.

The correlation between x one and x two when the effect of x three on each of x one and x two is eliminated is called partial correlation. Correlation coefficient between x one and x two when the linear effect of x three on each of them has been eliminated is called the partial correlation coefficient

The formula for the partial correlation coefficient is given by,
R one two point three is equal to r one two minus r one, three into r two three divided by square root of one minus r one three square into one minus r two three square.
Similarly we can obtain r one three point two and r two three point one.

Multiple correlation :
In trivariate distribution in which each of the variables x one, x two and x three has N observations, the multiple correlation coefficient between x one on x two and x three is, usually denoted by R one point two three which is the simple correlation coefficient between x one and the joint effect of x two and x three on x one.

In other words R one point two three is the correlation coefficient between X one and its estimated value as given by the plane of regression of x ox two on X_{two} and x three.

The formula for finding the multiple correlation coefficient is given by,
R one point two three square is equal to r one two square plus r one three square minus two into r one two into r one three into r two three divided by one minus r two three square.

Similarly we can obtain the formula for R two point one three square and R three point one two square.
Square root of the expression gives the multiple correlation coefficient.

2. Exercise (Part 1)

Exercise 1
From the data relating to the yield of dry bark (x one), height (x two) and girth (x three) for eighteen cinchona plants, the following correlation coefficients are obtained.
R one two is equal to zero point seven, seven, r one three is equal to zero point seven two, r two three is equal to zero point five two
Find the partial correlation coefficient r one two point three and multiple correlation coefficient R one point two three.

We know that partial correlation coefficient
R one two point three is equal to r one two minus r one three into r two three divided by square root of one minus r one three square into one minus r two three square.
Is equal to zero point seven, seven minus zero point seven two into zero point five two divided by square root of one minus zero point seven two square into one minus zero point five two square.
On simplification we get,
R one two point three is equal to zero point six two.

To find multiple correlation coefficient, we have,
R one point two three square is equal to r one two square plus r one three square minus two into r one two into r one three into r two three divided by one minus r two three square.
Is equal to zero point seven, seven square plus zero point seven two square minus two into zero point seven, seven into zero point seven two into zero point five two divided by one minus zero point five two square.
On simplification, we get, R one point two three is equal to zero point seven three, three four.
Multiple correlation coefficient is obtained by taking the square root of the above value.
Therefore R one point two three is equal to plus zero point eight five six four (since multiple correlation coefficient is non-negative)

3. Exercise (Part 2)

Exercise 2
Is it possible to get the following set of experimental data?
R two three is equal to zero point eight, r three one is equal to minus zero point five, r one two is equal to zero point six.

Solution:
R one two point three is equal to r one two minus r one three into r two three divided by square root of one minus r one three square into one minus r two three square.
Is equal to zero point six minus, minus zero point five into zero point eight divided by square root of one minus, minus zero point five square into one minus zero point eight square is equal to one point nine, two four, six which is greater than one.
But r one, two point three cannot be greater than one.
Hence the given set of experimental data is wrong.

In a tri-variate distribution, σ one is equal to two, σ two is equal to σ three is equal to three, r one, two is equal to zero point seven, r two, three is equal to r three, one is equal to zero point five
Find
a) R two, three point one
b) R one point two, three
c) B one, two point three and b one, three point two
d) Sigma one point two, three

We know that partial correlation coefficient
r one, two point three is equal to r one, two minus r one, three into r two, three divided by square root of one minus r one, three square into one minus r two, three square. Is equal to zero point five minus zero point seven into zero point five divided by square root of one minus zero point seven square into one minus zero point five square
On simplification we get, zero point two, four two, five.

To find multiple correlation coefficient, we have,
R one point two, three square is equal to r one, two square plus r one, three square minus two into r one, two into r one, three into r two, three divided by one minus r two, three square.
Is equal to zero point seven square plus zero point five square minus two into zero point seven into zero point five into zero point five divided by one minus zero point five square
On simplification ,we get, zero point five, two
R one point two, three is equal to plus zero point seven, two, one, one since multiple correlation coefficient is non-negative.

We know that, b one, two point three is equal to minus sigma one into omega one, two divided by sigma two into omega one, one and
b two, one point three is equal to minus sigma two into omega two, one divided by sigma one into omega two, two
Where

Omega is equal to determinant of one, r one, two, r one, three, r two, one, one, r two, three, r three, one, r three, two, one
Is equal to determinant of one, zero point seven, zero point five, zero point seven, one, zero point five, zero point five, zero point five, one.
From the above determinant,
Omega one, one is equal to one minus zero point five into zero point five is equal to zero point seven, five
Omega one, two is equal to minus zero point seven minus zero point five, into zero point five is equal to minus zero point four, five
Omega two, one is equal to minus of (zero point seven minus zero point five into zero point five is equal to minus zero point four, five.
Omega two, two is equal to one minus zero point two, five is equal to zero point seven, five.

Therefore by substituting the values in the above formula, we get,
b one, two point three is equal minus two into minus zero point four, five divided by three into zero point seven, five is equal to zero point four
b two, one point three is equal to minus three into minus zero point four, five divided by two into zero point seven, five is equal to zero point nine.

We know that from partial regression coefficient as follows.
r one, two point three is equal to square root of b one, two point three into b two, one point three is equal to square root of zero point four into zero point nine is equal to zero point six.

We have sigma one point two, three is equal to sigma one into square root of omega divided by omega one, one
Where omega is defined as earlier.
The value of omega is given by,
Omega is equal to one minus r one, two square minus r one, three square minus r two, three square plus two into r one, two into r one, three into r two, three

By substituting and simplifying, we get
Omega is equal to zero point three, six and we have already found that, omega one, one is equal to zero point seven, five. therefore,
sigma one point two, three is equal to two into square root of zero point three, six divided by zero point seven, five
Is equal to two into square root of zero point four, eight which is equal to one point three, eight, five, six.

4. Exercise (Part 3)

Exercise 4
Find the regression equation of X one on X two and X three given the following results

Figure 1

Trait	Mean	S. D	r_{12}	r_{23}	r_{31}
X1	28.02	4.42	+ 0.8	-	-
X2	4.91	1.1	-	- 0.56	-
X3	594	85	-	-	- 0.4

Here we have given the different traits X one, X two and X three with respective means and standard deviations.
Also we have given coefficient of correlations r one, two, r two, three and r three, one.
Where X one is equal to seed per acre
X two is equal to Rainfall in inches
X three is equal to Accumulated temperature above forty two degree Fahrenheit.

Solution:
The regression equation of X one on X two and X three is given by
X one minus X one bar into omega one, one divided by sigma one plus X two minus X two bar into omega one, two divided by sigma two plus X three minus X three bar into omega one, three divided by sigma three is equal to zero
Where Omega is equal to determinant of one, r one two, r one three, r two one, one, r two three, r three one, r three two, one.

Omega one, one is equal to determinant of one, r two three, r three two, one is equal to one minus r two three square is equal to one minus (minus zero point five six) square is equal to zero point six, eight, six.
Omega one, two is equal to r one, three into r two, three minus r two, one is equal to (minus zero point four) into (minus zero point five, six) minus (zero point eight) is equal to minus zero point five, seven, six.

Omega one, three is equal to r two, three into r one, two minus r one, is equal to (minus zero point five six into zero point eight) minus (minus zero point four) is equal to minus zero point zero four eight.

Therefore required equation of plane of regression of X one on X two and X three is given by
Zero point six eight six divided by four point four, two into X one minus twenty eight point zero two plus
(Minus zero point five, seven, six) divided by one point one into X two minus four point nine, one plus
(Minus zero point zero four, three)divided by eighty five into X three minus five hundred and

ninety four
Is equal to zero.

Exercise 5
Five hundred students were examined in three subjects I, II and III, each subject carrying hundred marks.
The Coefficient of Correlation between pairs of subjects is given by,
R one two is equal to zero point six, r one three is equal to zero point seven, r two, two three is equal to zero point eight

Assuming that the marks are normally distributed
a) Find the correlation between marks in subjects I and II among students who scored equal marks in subject III
b) If r two three was not known, obtain the limits within which it may lie from the value of r one two and r one three (ignoring sampling error)

The correlation between marks in subjects I and II among students who scored equal marks in subject III is r one two point three and is given by,
R one two point three is equal to r one two minus r one three into r two three divided by square root of one minus r one three square into one minus r two three square. Is equal to zero point six minus zero point seven into zero point eight divided by square root of one minus zero point seven square into one minus zero point eight square. Is equal to zero point zero nine three four.

We have
R one two point three square is equal to r one two minus r one three into r two three whole square divided by one minus r one three square into one minus r two three square is less than or equal to one. Since r two three is not known, we consider it as ‘a’. Therefore zero point six minus zero point seven into ‘a’ whole square divided by one minus zero point four nine into one minus ‘a’ square is less than or equal to one.

Which implies, zero point three six plus zero point four nine into ‘a’ square minus zero point eight four into ‘a’ is less than or equal to zero point five one into one minus ‘a’ square. Implies ‘a’ square minus zero point eight four into ‘a’ minus zero point one five is less than or equal to zero. Thus ‘a’ lies between the roots of the equation ‘a’ square minus zero point eight four into ‘a’ minus zero point one five is equal to zero, which are zero point nine, nine and minus zero point one five. Hence r two three should lie between minus zero point one five and zero point nine, nine.

5. Exercise (Part 4)

Exercise 6
A teacher in mathematics wishes to determine the relationship of marks on final examination to those on two tests given during the semester. Calling X one, X two and X three, the marks of a student in first, second and final examination respectively, he made the following computations from a total of one hundred and twenty students.
X one bar is equal to six point eight, x two bar is equal to seven point zero and x three bar is equal to seventy four.

Sigma one is equal to one, sigma two is equal to zero point eight and sigma three is equal to nine. R one two is equal to zero point six, r one three is equal to zero point seven and r two three is equal to zero point six five. Find the least square regression equation of X three on X one and X two. Estimate the final marks of two students who scored respectively nine and seven, four and eight on two tests.

The least square regression equation of X three on x one and x two is given by,
X three is equal to 'a' three point one two plus 'b' three, one point two into X one plus 'b' three two point one into X two.
The regression equation of x three on x one and x two in the deviation form can be written as x three is equal to b three, one point two into x one plus b three two point one into x two. Where x three is equal to X three minus X three bar, x one is equal to X one minus X one bar and x two is equal to X two minus X two bar.

B three one point two is equal to sigma three by sigma one into r three one minus r three two into r one two. Is equal to nine by one into zero point seven minus zero point six five into zero point six divided by one minus zero point six square. Is equal to four point three six. B three two point one is equal to sigma three divided by sigma two into r three two minus r three one into r two one divided by one minus r two one square. Is equal to nine divided by zero point eight into zero point six five minus zero point seven into zero point six divided by one minus zero point six square. Which is equal to four point zero four.

Thus regression equation of X three on X one and X two is given by,
X three minus seventy four is equal to four point three six into X one minus six point eight plus four point zero four into X two minus seven.
Implies, X three is equal to sixteen point zero seen plus four point three six into X one plus four point zero four into X two.

Final marks of students who scored nine and seven marks, that is, substituting X one is equal to nine and X two is equal to seven, in the above regression equation, we get,
X three is equal to sixteen point zero seven plus four point three six into nine plus four point zero four into seven.
Is equal to eighty three point five nine or eighty four.
Final marks of students who scored four and eight marks, that is substituting X one is equal to four and X two is equal to eight, in the above regression equation, we get,
X three is equal to sixteen point zero seven plus four point three six into four plus four point zero four into eight. This is equal to sixty five point eight or sixty six.

Exercise 7

Find r one, two if r one three point two is equal to zero point six and R one point two three is equal to zero point eight.

Solution:

We know that one minus R one point two three square is equal to (one minus r one two square) into (one minus r one three point two square)

By substitution, we get,

one-(zero point eight square) = (one-r one two square) into (one minus zero point six square)

On simplification, we get,

R one two square is equal to zero point four, three, seven, five or

R one two is equal to plus or minus zero point six, six, one, four

Here's a summary of our learning in this session where we have understood how to:

- Find partial and multiple correlation
- Find regression line of one variable on the other two variables and predicting the future value
- Derive partial regression and partial correlation coefficients.