Frequently Asked Questions

1. Who developed the method of finding the coefficient of rank correlation and when?
   Answer:
   Charles Edward Spearman developed this method in 1904.

2. For which type of data we can use rank correlation coefficient?
   Answer:
   We can use Spearman's rank correlation coefficient for both quantitative and qualitative data.

3. Write any two differences between the Karl Pearson's coefficient of correlation and Spearman coefficient of correlation.
   Answer:

Karl Pearson's Coefficient of correlation	**Spearman's Coefficient of correlation**
It can be used only for quantitative data.	It can be used for both qualitative and quantitative data.
The formula remains same whether we have repeated scores or not.	When the ranks are repeated, then we need to correct Σd^2 and then use in the formula.

4. Write the formula for finding the coefficient of rank correlation when there are ties.
   Answer:

$$\rho = 1 - \frac{6\left[\Sigma d^2 + \frac{1}{12}(m^3 - m) + \frac{1}{12}(m^3 - m)...\right]}{n^3 - n}$$

5. Write the formula for finding the coefficient of rank correlation when there are no ties.
   Answer:

$$\rho = 1 - \frac{6\Sigma d^2}{n^3 - n}$$

6. Write different steps in finding coefficient of rank correlation.
   Answer:
   - Determine the ranks of X and Y
   - Take the difference of two ranks i.e., $(R_1 - R_2)$ and denote them as d
   - Square these differences and total them to get Σd^2
   - Apply required formula

7. How to give ranks when we have quantitative data?

Answer:

- If rank is not given, the highest value is given rank 1, 2nd highest value is given rank 2 and we proceed like this for remaining value
- If there are two or more equal values then we get an obstruction in giving ranks. We have a practice that if two students score equal marks then we give them equal ranks

8. The ranks of students in Hindi and Economics are given as follows.

Hindi	6	1	5	2	4	3	6
Economics	3	1	4	2	5	6	3

Calculate the coefficient of rank correlation.

Answer:

Hindi	**Economics**	**d**	**d^2**
6	3	3	9
1	1	0	0
5	4	1	1
2	2	0	0
4	5	-1	1
3	6	-3	9
			$\Sigma d^2=20$

Since the ranks are not repeated, we use the following formula to find the rank correlation coefficient.

$$\rho = 1 - \frac{6\Sigma d^2}{n^3 - n}$$

By substituting, we get, $\rho = 1 - \frac{6 \times 20}{6^3 - 6} = 0.429$

9. 10 students got the following percentage of marks in Economics and Statistics.

Economics	78	36	98	25	75	82	92	62	65	39
Statistics	84	51	91	60	68	62	86	58	35	49

Calculate rank correlation coefficient.

Answer:

Economics	**Statistics**	**R_1**	**R_2**	**d**	**d^2**
78	84	4	3	1	1
36	51	9	8	1	1
98	91	1	1	0	0
25	60	10	6	4	16
75	68	5	4	1	1
82	62	3	5	-2	4
92	86	2	2	0	0
62	58	7	7	0	0
65	35	6	10	-4	16

39	49	8	9	-1	1
					$\Sigma d^2=40$

$$\rho = 1 - \frac{6\Sigma d^2}{n^3 - n} = 1 - \frac{6 \times 40}{10^3 - 10} = 1 - 0.26 = 0.74$$

It shows a high degree of positive correlation between the percentage of marks in Economics and Statistics.

That is those who score more in Economics are expected to score more in Statistics also.

10. Ten competitors in a beauty contest are ranked by three judges in the following order. Use the rank correlation coefficient to discuss which pair of judges has the nearest approach to common taste in beauty.

First Judge	1	6	5	10	3	2	4	9	7	8
Second Judge	3	5	8	4	7	10	2	1	6	9
Third Judge	6	4	9	8	1	2	3	10	5	7

Answer:
In order to determine which pair of judges has the nearest approach to common taste in beauty we shall have to calculate the rank coefficient of correlation between the rankings of

i. First and Second judge

ii. Second and Third Judge

iii. First and Third Judge

Now let us consider one by one and find the coefficient of rank correlation.

First and Second judges

First Judge	**Second Judge**	**d**	**d^2**
1	3	-2	4
6	5	1	1
5	8	-3	9
10	4	6	36
3	7	-4	16
2	10	-8	64
4	2	2	4
9	1	8	64
7	6	1	1
8	9	-1	1
			$\Sigma d^2=200$

$\rho = 1 - \frac{6\Sigma d^2}{n^3 - n}$ Where n=10 and Σd^2 =200.

By substituting the values in the above formula we get,

$$\rho = 1 - \frac{6 \times 200}{10^3 - 10} = 1 - 1.21 = -0.21$$

Second and Third judges

Second Judge	Third Judge	d	d^2
3	6	-3	9
5	4	1	1
8	9	-1	1
4	8	-4	16
7	1	6	36
10	2	8	64
2	3	-1	1
1	10	-9	81
6	5	1	1
9	7	2	4
			$\Sigma d^2=214$

Hence, $\rho = 1 - \frac{6 \times 214}{10^3 - 10} = 1 - 1.297 = -0.297$

First and Third judges

First Judge	Third Judge	d	d^2
1	6	-5	25
6	4	2	4
5	9	-4	16
10	8	2	4
3	1	2	4
2	2	0	0
4	3	1	1
9	10	-1	1
7	5	2	4
8	7	1	1
			$\Sigma d^2=60$

Hence $\rho = 1 - \frac{6 \times 60}{10^3 - 10} = 1 - 0.36 = 0.64$

Hence first and third judge pair has the nearest approach to common taste in beauty.

11. The rank correlation coefficient between marks obtained by some students in Statistics and Accountancy is 0.8. If the total of squares of rank differences is 33, find the number of students.

Answer:

We know that, $\rho = 1 - \frac{6\Sigma d^2}{n^3 - n}$

By substituting the given information in the above, we get,

$$0.8 = 1 - \frac{6(33)}{n^3 - n} = 1 - \frac{198}{n(n^2-1)} \Rightarrow \frac{198}{n(n^2-1)} = 1 - 0.8 \Rightarrow \frac{198}{n(n^2-1)} = 0.2$$

$$\Rightarrow 0.2n(n^2-1) = 198$$

Or $n(n^2 - 1)=198/0.2=990$

By observation, we find that,

$10(10^2 - 1)=990$

Hence the number of students is 10

12. The coefficient of rank correlation of the marks obtained by 10 students I English and Economics was found to be 0.5. It was later discovered that the difference in ranks in the two subjects obtained by one of the students was wrongly taken as 3 instead of 7. Find the correct coefficient of rank correlation.

Answer:

We know that, $\rho = 1 - \frac{6\Sigma d^2}{n^3 - n}$

By substituting the given values in the above, we get,

$$0.5 = 1 - \frac{6\Sigma d^2}{10^3 - 10} = 1 - \frac{6\Sigma d^2}{990}$$

$\frac{6\Sigma d^2}{990} = 1 - 0.5 = 0.5$

$6\Sigma d^2 = 0.5(990)$

$\Sigma d^2 = 495/6 = 82.5$

But this is based on the incorrect d. Hence after correction,

$\Sigma d^2 = 82.5 - 3^2 + 7^2 = 122.5$ and the corrected rank correlation is,

$$\rho = 1 - \frac{6\Sigma d^2}{n^3 - n} = 1 - \frac{6 \times 122.5}{10^3 - 10} = 1 - 0.74 = 0.26$$

13. Obtain the rank correlation coefficient for the following data.

x	68	64	75	50	64	80	75	40	55	64
y	62	58	68	45	81	60	68	48	50	70

Answer:

x	**y**	**R_1**	**R_2**	**d**	**d^2**
68	62	4	5	-1	1
64	58	6	7	-1	1
75	68	2.5	3.5	-1	1
50	45	9	10	-1	1
64	81	6	1	5	25
80	60	1	6	-5	25

75	68	2.5	3.5	-1	1
40	48	10	9	1	1
55	50	8	8	0	0
64	70	6	2	4	16
					$\Sigma d^2=72$

As we have repeated ranks in the above problem, we use the following formula.

$$\rho = 1 - \frac{6\left[\Sigma d^2 + \frac{1}{12}(m^3 - m) + \frac{1}{12}(m^3 - m)...\right]}{n^3 - n}$$

Observe that in the above table, 3 ranks are repeated.

Therefore we find $\frac{1}{12}(m^3 - m)$ for each value repeated. In x series 75 is repeated twice. Hence m=2. Therefore $\frac{1}{12}(m^3 - m) = \frac{1}{12}(2^3 - 2) = \frac{1}{2}$. 64 is repeated thrice. Hence m=3. Therefore, $\frac{1}{12}(m^3 - m) = \frac{1}{12}(3^3 - 3) = 2$. In y series, 63 occur twice. Hence m=2. Therefore $\frac{1}{12}(m^3 - m) = \frac{1}{12}(2^3 - 2) = \frac{1}{2}$.

Therefore coefficient of rank correlation is given by,

$$\rho = 1 - \frac{6\left[72 + \frac{1}{2} + 2 + \frac{1}{2}\right]}{10^3 - 10} = 0.545$$

14. Find rank correlation coefficient between the marks of the subjects Mathematics and Hindi

Mathematics	29	32	53	47	45	32	70	45	70	53
Hindi	56	60	72	48	72	35	67	67	75	31

Answer:

Mathematics	**Hindi**	**R_1**	**R_2**	**d**	**d^2**
29	56	10	7	3	9
32	60	8.5	6	2.5	6.25
53	72	3.5	2.5	1	1
47	48	5	8	-3	9
45	72	6.5	2.5	4	16
32	35	8.5	9	-0.5	0.25
70	67	1.5	4.5	-3	9
45	67	6.5	4.5	2	4
70	75	1.5	1	0.5	0.25
53	31	3.5	10	-6.5	42.25

					$\Sigma d^2=97$

As we have repeated ranks in the above problem, we use the following formula.

$$\rho = 1 - \frac{6\left[\Sigma d^2 + \frac{1}{12}(m^3 - m) + \frac{1}{12}(m^3 - m)...\right]}{n^3 - n}$$

Observe that in the above table, altogether 6 ranks are repeated, 4 in Mathematics and 2 in Hindi. Further each rank is repeated twice.

Hence for each repeated ranks, the value of $\frac{1}{12}(m^3 - m) = \frac{1}{12}(2^3 - 2) = \frac{1}{2}$

Therefore coefficient of rank correlation is given by,

$$\rho = 1 - \frac{6\left[97 + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}\right]}{10^3 - 10} = 1 - \frac{6(100)}{990} = 0.394$$

15. From the following data calculate Spearman's Rank Coefficient of Correlation.

Sl. No.	Rank Differences
1	-2
2	-4
3	-1
4	3
5	2
6	0
7	?
8	3
9	3
10	-2

Answer:

We know that the sum of the rank differences is always equal to zero. So assuming the unknown rank difference as x,

-2-4-1+3+2+0+x+3+3-2=0

Implies, x+2=o or x=-2.

Therefore the unknown rank difference is -2.

Now let us write the complete table and find d square values.

Sl. No.	Rank Differences	d^2
1	-2	4
2	-4	16

3	-1	1
4	3	9
5	2	4
6	0	0
7	-2	4
8	3	9
9	3	9
10	-2	4
		$\Sigma d^2=60$

The formula is given by,

$$\rho = 1 - \frac{6\Sigma d^2}{n^3 - n} = 1 - \frac{6(60)}{10^3 - 10} = 1 - 0.364 = 0.636$$