1. Exercise on Linear Curves or Straight Line (Part -1)

Welcome to the series of E-learning modules on Fitting of curves by Least Square Method.

By the end of this session, you will be able to:

- Explain fitting linear curves or straight line
- Explain fitting quadratic curves
- Explain exponential curves
- Explain power curves

Fit a straight line to the following data.

Figure 1

X	6	7	7	8	8	8	9	9	10
Y	5	5	4	5	4	3	4	3	3

Let the linear equation is given by, Y is equal to a plus b into X.

The normal equations are given by,
Summation Y is equal to n into a plus b into summation X and
Summation X into Y is equal to a into summation X plus b into summation X square.

To find the values to be substituted in the above equations, we obtain the following table.

Figure 2

X	**Y**	**X^2**	**XY**
6	5	36	30
7	5	49	35
7	4	49	28
8	5	64	40
8	4	64	32
8	3	64	24
9	4	81	36
9	3	81	27
10	3	100	30
$\Sigma X=$**72**	$\Sigma Y=$**36**	$\Sigma X^2=$**588**	$\Sigma XY=$**282**

The first two columns are as it is in the question. The 3rd column gives square of values in the first column and the 4th column gives the product of 1st and 2nd columns. The last row, which is written in the bold numbers denote the sum of each columns.

By substituting the different values in the above two equations we get,
36 is equal to 9 into a plus b into 72 and 282 is equal to a into 72 plus b into 588.

We solve the above equations using Cramer’s rule as follows:
Delta is equal to determinant of 9, 72, 72, 588 is equal to 9 into 588 minus 72 into 72 is equal to 108.
Delta 1 is equal to determinant of 36, 72, 282, 588 is equal to 36 into 588 minus 282 into 72 is equal to 864.
Delta 2 is equal to determinant of 9, 36, 72, 282 is equal to 9 into 282 minus 72 into 36 is equal to minus 54.

Hence, a is equal to delta 1 by delta is equal to 864 divided by 108 is equal to 8 and b is equal to delta 2 divided by delta is equal to minus 54 divided by 108 is equal to minus zero point 5.

Therefore, the equation for the straight line is given by,
Y is equal to 8 minus zero point 5 X.

2. Exercise on Linear Curves or Straight Line (Part - 2)

Fit a straight line to the data.

Figure 3

X	1911	1921	1931	1941	1951
Y	15	23	28	32	39

Let the linear equation is given by, Y is equal to a plus b into X.

The normal equations are given by,
Summation Y is equal to n into a plus b into summation X and Summation X into Y is equal to a into summation X plus b into summation X square.

Observe that the values of X are large. Hence, we consider the transformation, x is equal to X minus 1931 divided by 10.

Hence, consider the following table.

Figure 4

X	Y	x	x^2	xY
1911	15	-2	4	-30
1921	23	-1	1	-23
1931	28	0	0	0
1941	32	1	1	32
1951	39	2	4	78
Total	**$\Sigma Y=137$**	**$\Sigma x=0$**	**$\Sigma x^2=10$**	**$\Sigma xY=57$**

The first two columns are as it is in the question.
3rd column that is values of x is obtained by using the above relation.
4th column is obtained by squaring the values in the third column and the last column is obtained by multiplying the values in the 2nd and 3rd columns.
The last row denotes the total of each of the columns.

By substituting the different values in the above two equations we get,

137 is equal to 5a plus b into zero, which implies, a is equal to 27 point 4
57 is equal to a into zero plus b into 10, which implies b is equal to minus 5 point 7.

Therefore, equation for the straight line is given by,
Y is equal to 27 point 4 minus 5 point 7 x.

By substituting, x is equal to X minus 1931divided by 10, we get,
Y is equal to 27 point 4 minus 5 point 7 into X minus -1931 divided by 10
Is equal to 1128.07 minus 0.57X.

The following are the population of a village from 1991to 2008. Fit a straight line and estimate the population for the year 2012.

Figure 5

895	785	784	846	775	816	823	874	750
736	807	734	785	805	750	784	765	715

Let the linear equation is given by, Y is equal to a plus b into X.

The normal equations are given by,
Summation Y is equal to n into a plus b into summation X and
Summation X into Y is equal to a into summation X plus b into summation X square.

Observe that the values of X are large. Here the middle point is 1999 point 5. Hence, we consider the transformation, x is equal to X minus 1999 point 5 into 2.

Now consider the following table.

Figure 6

Year (X)	Population(Y)	x	x^2	xY
1991	895	-17	289	-15215
1992	785	-15	225	-11775
1993	784	-13	169	-10192
1994	846	-11	121	-9306
1995	775	-9	81	-6975
1996	816	-7	49	-5712
1997	823	-5	25	-4115
1998	874	-3	9	-2622
1999	750	-1	1	-750
2000	736	1	1	736
2001	807	3	9	2421
2002	734	5	25	3670
2003	785	7	49	5495
2004	805	9	81	7245
2005	750	11	121	8250
2006	784	13	169	10192
2007	765	15	225	11475
2008	715	17	289	12155
Total	**$\Sigma Y=14229$**	**$\Sigma x=0$**	**$\Sigma x^2=1938$**	**$\Sigma xY=-5023$**

The first column denotes the year and the second column denote the population of the village for those years.
Third column represent the values taken by x when we consider the above transformation.
The fourth column is obtained by squaring the numbers in the third column and 5^{th} column is obtained by multiplying the numbers in the 2^{nd} and 3^{rd} columns.
The last row denotes the totals of each column.

By substituting the different values in the above two equations we get,
14 thousand 229 is equal to 18 into a plus b into zero, which implies, a is equal to 790 point 5.
Minus 5023 is equal to a into zero plus b into 1 thousand 938, which implies b is equal to minus 2 point 5, 9.

Therefore, equation for the straight line is given by,
Y is equal to 790 point 5 minus 2 point 5, 9 into x .

By substituting, x is equal to X minus 1999 point 5 into 2, we get,
Y is equal to 790 point 5 minus 2 point 59 into X minus 1999 point 5 into 2
is equal to 11 thousand 147 point 9,1 minus 5 point 18 into X.

Estimated population for the year 2012 is given by, substituting X is equal to 2012 in the above equation
That is, Y is equal to 11 thousand 147 point 9, 1 minus 5 point 1, 8 into (2012) is equal to 725 point 7, 5 approximately equal to 726.

3. Exercise on Power Curves

Fit a curve of the type Y is equal to a into X power b for the data.

Figure 7

X	1	10	100	1000
Y	10.95	39.81	50.12	63.8

Given the curve fitted to the above data is, Y is equal to a into X power b, where a and b are constants, to be determined.

Hence, to determine these constants, we use the following normal equations.
Summation U is equal to n into A plus b into summation V.
Summation U into V is equal to A into summation V plus b into summation V square.
Where U is equal to log Y, V is equal to log X and A is equal to log a.

To calculate the values of a and b, we construct the following table.

Figure 8

X	Y	V=logX	U=log Y	V^2	UV
1	10.95	0	1.0394	0	0.0000
10	39.81	1	1.6000	1	1.6000
100	50.12	2	1.7000	4	3.4000
1000	63.8	3	1.8048	9	5.4145
Totals		**6**	**6.1442**	**14**	**10.4145**

In the table, the first two columns are written, as it is in the question. The third column is obtained by taking the logarithm of the numbers in the first column. The fourth column is obtained by taking logarithm of the numbers in the second column. The 5th column is obtained by squaring the numbers in the 3rd column. The last column is obtained by multiplying the numbers in the 3rd and 4th column. The last row gives the totals of all the columns which are necessary to calculate the constants of the fitted curve.

Therefore, the normal equations can be written as follows.
6 point 1,4,4,2 is equal to 4 into A plus 6 into b.
10 point 4,1,4,5 is equal to 6 into A plus 14 into b.

The above equations can be solved by using the Cramer's rule.

Hence, consider the following determinants.

Delta is equal to determinant of 4, 6, 6, 14 is equal to 4 into 14 minus 6 into 6 is equal to 20.
Delta 1 is equal to determinant of (6 point 1,4,4,2), 6, (10 point 4,1,4,5), 14 is equal to 6 point 1,4,4,2 into 14 minus 10 point 4,1,4,5 into 6 is equal to 23 point 5,3,1,8.
Delta 2 is equal to determinant of 4, (6 point 1,4,4,2), 6, (10 point 4,1,4,5) is equal to 4 into 10 point 4,1,4,5 minus 6 into 6 point 1,4,4,2 is equal to 4 point 7,9,2,8.

Hence, A is equal to delta 1 divided by delta is equal to 23 point 5,3,1,8 divided by 20 is equal to 1 point 1,7,6,6 and
B is equal to delta 2 divided by delta is equal to 4 point 7,9,2,8 divided by 20 is equal to zero point 2,3,9,6.

a is equal to antilog of A is equal to antilog of (1 point 1, 7, 6, 6) is equal to 15 point zero 1, 7, 6

The equation of the required curve is,
Y is equal to 15 point zero 1, 7, 6 into X power zero point 2, 3, 9, 6.

Fit an equation of type Y is equal to a into b power x to the following data.

Figure 9

Age in years	1	2	3	4	5	6	7	8	9	10
Weight in Kgs.	52.5	58.7	65.0	70.2	75.4	81.1	87.2	95.3	1022	108.4

The power curve fitted to the above data is,
Y is equal to a into b power x.

The normal equations are given by,
Summation U is equal to n into A plus B into summation X.
Summation U into X is equal to A into summation X plus B into summation X square.
Where U is equal to log Y, A is equal to log a and B is equal to log b.

To calculate the values of a and b, we construct the following table.

To make calculations simple, let us consider the transformation x is equal to (X minus 1point 2, 5) divided by zero point 2, 5.

First 2 columns are written as given in the problem.

Figure 10

Age in years(X)	Weight in Kgs.(Y)	X^2	XY
1	52.5	1	52.5
2	58.7	4	117.4
3	65	9	195
4	70.2	16	280.8
5	75.4	25	377
6	81.1	36	486.6
7	87.2	49	610.4
8	95.3	64	762.4
9	102.2	81	919.8
10	108.4	100	1084
$\Sigma X=55$	$\Sigma Y=796$	$\Sigma X^2=385$	$\Sigma XY=4885.9$

3rd column is obtained by taking logarithm of the numbers in the 2nd column. The 4th column is obtained by considering the transformation x is equal to X minus 1point 2, 5 divided by zero point 2, 5.
The 5th column is obtained by finding squares of the numbers in the 4th column. The last column is obtained by multiplying the numbers in 3rd and 4th columns.
The last row gives the totals of required columns.

Hence, by substituting in normal equations, we get,
14 point 8, 3, 2, 3 is equal to 9 into A which implies A is equal to (14 point 8, 3, 2, 3) divided by 9 is equal to 1 point 6, 4, 8 and
Minus 4 point 6, 3, 9, 6 is equal to 60 into B which implies B is equal to minus 4 point 6, 3, 9, 6 divided by 60 is equal to zero point zero 7, 7, 3, 3.

We know that, a is equal to antilog of (A) is equal to antilog of (1 point 6, 4, 8) is equal to 44 point 4, 6, 6, 5
And b is equal to antilog of (B) is equal to antilog of (minus zero point zero 7, 7, 3, 3) is equal to zero point 8, 3, 6, 9.

Hence, the equation to be fitted to the given data can be written as follows.
Y is equal to (44 point 4, 6, 6, 5) into (zero point 8, 3, 6, 9) power x where x is equal to (X minus 1 point 2, 5) divided by zero point 2, 5.

4. Exercise on Exponential Curves

The weights of a calf taken at weekly intervals are given below.

Figure 11

X	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25
Y	93	71	63	54	48	38	29	26	22

Fit a straight line using the method of least squares and estimate the weight when the age is 11 years

Let the linear equation is given by, Y is equal to a plus b into X.

The normal equations are given by,
Summation Y is equal to n into a plus b into summation X and summation X into Y is equal to a into summation X plus b into summation X square.

Now let us consider the following table in which we calculate all the necessary totals.

Figure 12

X	Y	U=logY	x	x^2	Ux
0.25	93	1.9685	-4	16	-7.8739
0.5	71	1.8513	-3	9	-5.5538
0.75	63	1.7993	-2	4	-3.5987
1	54	1.7324	-1	1	-1.7324
1.25	48	1.6812	0	0	0.0000
1.5	38	1.5798	1	1	1.5798
1.75	29	1.4624	2	4	2.9248
2	26	1.4150	3	9	4.2449
2.25	22	1.3424	4	16	5.3697
Total		**14.8323**	**0**	**60**	**-4.6396**

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The first two columns of the table are written from the question.
The 3rd column is obtained by squaring the numbers in the first column and the 4th column is obtained by multiplying the numbers in 1st and 2nd columns.
The last row which is written in bold give the totals of all the columns.

Therefore, the normal equations are,
796 is equal to 10 into a plus b into 55 and
4 thousand 885 point 9 is equal to 55 into a plus 385 into b.

On solving the above two simultaneous equations we get,
a is equal to 45 point 7, 4 and b is equal to 6 point 1, 5, 6, 4.

Therefore, Y is equal to 45 point 7, 4 plus 6 point 1, 5, 6, 4 into X

To estimate the weight at 11 years, we substitute X is equal to 11 in the above equation.
That is, Y is equal to 45 point 7, 4 plus 6 point 1, 5, 6, 4 into (11) is equal to 113 point 4, 6, zero 4 Kg.

5. Exercise on Quadratic Curves

Fit a parabola to the following data.

Figure 13

X	-1	0	0	1
Y	2	0	1	2

The parabola fitted to the above data is given by, Y is equal to a plus b into X plus c into X square.

And the normal equations are given by,
Summation Y is equal to n into a plus b into summation X plus c into summation X square
Summation X into Y is equal to a into summation X plus b into summation X square plus c into summation ΣX cube.
Summation X square into Y is equal to a into summation X square plus b into summation X cube plus c into summation X power 4.

To solve for a, b and c, we construct the following table.

The first 2 columns are written as it is in the question.

Figure 14

X	**Y**	**X^2**	**X^3**	**X^4**	**XY**	**X^2Y**
-1	2	1	-1	1	-2	2
0	0	0	0	0	0	0
0	1	0	0	0	0	0
1	2	1	1	1	2	2
$\Sigma X=0$	$\Sigma Y=5$	$\Sigma X^2=2$	$\Sigma X^3=0$	$\Sigma X^4=2$	$\Sigma XY=0$	$\Sigma X^2Y=4$

The third column is obtained by squaring the numbers in the first column.
The 4th column is obtained by cubing the numbers in the first column.
The fifth column is obtained by finding power 4 of the numbers in the first column.
The 6th column is obtained by multiplying the numbers in 1st and 2nd column and the last

column is obtained by multiplying the numbers in the 2nd and 3rd columns.
The last row which is written in bold gives the corresponding totals of each column.

Hence, the normal equations are,
5 is equal to 5 into a plus zero into b plus 2 into c, name it as 1.
Zero is equal to zero into a plus 2 into b plus zero into c implies b is equal to zero.
4 is equal to 2 into a plus zero into b plus 2 into c, name it as 2.

From equations 1 and 2 we get, a is equal to 1 by 3 and c is equal to 5 by 3.

Therefore, the equation of parabola fitted to the given data is,
Y is equal to 1 by 3 plus zero into X plus 5 by 3 into X square
That is, Y is equal to 1 by 3 plus 5 by 3 into X square.

The following table gives the results of measurements of train resistance.

Figure 15

V	20	40	60	80	100	120
R	5.5	9.1	14.9	22.8	33.3	46.0

Here, V is the velocity in Kilometres per hour and R is the resistance in KG per quintal.

Fit a relation of the form R is equal to a plus b into V plus c into V square to the data.
Estimate R when V is equal to 90.

The parabola fitted to the above data is given by,
R is equal to a plus b into V plus c into V square

And the normal equations are given by,
Summation R is equal to n into a plus b into summation V plus c into summation V square .
Summation V into R is equal to a into summation V plus b into summation V square plus c into summation V cube.
Summation V square into R is equal to a into summation V square plus b into summation V cube plus c into summation V power 4.

Since, values of V are large & to make calculations easier, we use the transformation X is equal to (V minus 70) divided by 10.

Hence, the normal equations become,
Summation R is equal to n into a plus b into summation X plus c into summation X square .
Summation X into R is equal to a into summation X plus b into summation X square plus c into summation X cube.
Summation X square into R is equal to a into summation X square plus b into summation X cube plus c into summation X power 4.

To solve for a, b and c, we construct the following table.

Figure 16

V	R	X	X^2	X^3	X^4	XR	X^2R
20	5.5	-5	25	-125	625	-27.5	137.5
40	9.1	-3	9	-27	81	-27.3	81.9
60	14.9	-1	1	-1	1	-14.9	14.9
80	22.8	1	1	1	1	22.8	22.8
100	33.3	3	9	27	81	99.9	299.7
120	46	5	25	125	625	230	1150
Total	**131.6**	**0**	**70**	**0**	**1414**	**283**	**1706.8**

Here first 2 columns are written as it is in the question.
The third column is found using the above transformation.
4^{th}, 5^{th} and 6^{th} columns are obtaining by squaring, cubing and finding the power 4 of the numbers in the 3^{rd} column respectively.
The 7^{th} column is obtained by multiplying the numbers in 2^{nd} and 3^{rd} column and the last column is obtained by multiplying the numbers in 2^{nd} and 4^{th} columns. The last row which in written in bold denotes the totals of each column.

Hence, the normal equations become,
131 point 6 is equal to 6 into a plus 70 into c. Name it as (1)
283 is equal to70 into b implies b is equal to4 point0,4,2,9
1706 point 8 is equal to70 into a plus1414 into c. Name it as (2)

By solving equations 1 and 2, we get,
A is equal to 18.5,8,4,4 and b is equal to0.2,8,7.

Hence, the relation fitted to the given data is,
R= 18 point 5,8,4,4 plus 4 point 0,4,2,9 into X plus 0 point 2,8,7 into X square

To estimate R when V is equal to 90, substitute X is equal to2
Hence, R is equal to 18 point 5,8,4,4 plus 4 point 0,4,2,9 into 2 plus 0.2,8,7 into 2 square is equal to 27.8,4.

Here's a summary of our learning in this session, where we have understood:

- The fitting of linear curves or straight line
- The fitting quadratic curves
- The exponential curves and
- The power curves