Frequently Asked Questions

1. **1.** Write the equation for straight line and the normal equation for finding the constants.

   Answer:

   The equation of straight line is, Y=a+bX

   The normal equations used to find the constants a and b are,

   $\Sigma Y = na + b\Sigma x$ and

   $\Sigma XY = a\Sigma X + b\Sigma X^2$

2. **2.** Write the equation for second degree curve and the normal equation for finding the constants.

   Answer:

   The equation of second degree curve is, $Y = a + b X + cX^2$

   And the normal equations are given by

   $\Sigma Y = na + b\Sigma X+c\Sigma X^2$

   $\Sigma XY = a\Sigma X + b\Sigma X^2+c \Sigma X^3$

   $\Sigma X^2Y = a\Sigma X^2+b\Sigma X^3+c \Sigma X^4$

3. **3.** Write the equation for exponential curve and the normal equation for finding the constants.

   Answer:

   The equation of exponential curve is, $Y=aX^b$

   And the normal equations are,

   $\Sigma U = n A + b\Sigma V$

   $\Sigma UV = A\Sigma V + b \Sigma V^2$

   Where U = log Y

   V= log X

   A = log a

4. **4.** Write the equation for power curve and the normal equation for finding the constants.

Answer:

The equation of power curve is, $Y=ab^x$

The normal equations are given by,

$\Sigma U = nA + B\Sigma X$

$\Sigma UX = A\Sigma X + B\,\Sigma X^2$

Where $U = \log Y$

$A = \log a$

$B = \log b$

5. Fit a straight line to the following data.

X	6	7	7	8	8	8	9	9	10
Y	5	5	4	5	4	3	4	3	3

Answer:

Let the linear equation is given by, $Y = a + bX$

The normal equations are given by,

$\Sigma Y = na + b\Sigma x$ and $\Sigma XY = a\Sigma X + b\Sigma X^2$

X	**Y**	**X^2**	**XY**
6	5	36	30
7	5	49	35
7	4	49	28
8	5	64	40
8	4	64	32
8	3	64	24
9	4	81	36
9	3	81	27
10	3	100	30
72	**36**	**588**	**282**

By substituting the different values in the above two equations we get,

$36=9a+b(72)$ and $282 = a(72)+b(588)$

On solving above two equations simultaneously we get,

$a=8$ and $b=0.5$

Therefore, equation for the straight line is given by,

$Y = 8 – 0.5X$

6. Fit a straight line to the data

X	1911	1921	1931	1941	1951
Y	15	23	28	32	39

Answer:

Let the linear equation is given by, Y = a + bX

The normal equations are given by,

$\Sigma Y = na + b\Sigma x$ and $\Sigma XY = a\Sigma X + b\Sigma X^2$

Observe that the values of X are large. Hence we consider the transformation, x=(X-1931)/10

X	Y	X	X^2	XY
1911	15	-2	4	-30
1921	23	-1	1	-23
1931	28	0	0	0
1941	32	1	1	32
1951	39	2	4	78
Total	**137**	**0**	**10**	**57**

By substituting the different values in the above two equations we get,

137=5a+b(0), implies, a=27.4

57 = a(0)+b(10), implies b=-5.7

Therefore, equation for the straight line is given by,

Y =27.4 – 5.7x.

By substituting x=(X-1931)/10, we get,

Y =27.4 – 5.7((X-1931)/10)

=**1128.07-0.57X**

7. The following are the population of a village from 1991-2008. Estimate the population for the year 2012 by fitting the straight line.

895	785	784	846	775	816	823	874	750
736	807	734	785	805	750	784	765	715

Answer:
Let the linear equation is given by, Y = a + bX

The normal equations are given by,

$\Sigma Y = na + b\Sigma x$ and $\Sigma XY = a\Sigma X + b\Sigma X^2$

Observe that the values of X are large. Hence we consider the transformation, x=(X-1999.5).2

Year	Population(Y)	x	x^2	xY
1991	895	-17	289	-15215
1992	785	-15	225	-11775
1993	784	-13	169	-10192
1994	846	-11	121	-9306
1995	775	-9	81	-6975
1996	816	-7	49	-5712
1997	823	-5	25	-4115
1998	874	-3	9	-2622
1999	750	-1	1	-750
2000	736	1	1	736
2001	807	3	9	2421
2002	734	5	25	3670
2003	785	7	49	5495
2004	805	9	81	7245
2005	750	11	121	8250
2006	784	13	169	10192
2007	765	15	225	11475
2008	715	17	289	12155
Total	**14229**	**0**	**1938**	**-5023**

By substituting the different values in the above two equations we get,

14229=18a+b(0), implies, a=790.5

-5023 = a(0)+b(1938), implies b=-2.59

Therefore, equation for the straight line is given by,

Y =790.5 – 2.59x

By substituting x=(X-1999.5).2, we get,

Y =790.5 – 2.59((X-1999.5).2)

=11147.91-5.18X

Estimated population for the year 2012 is given by, substituting X=2012 in the above equation

i.e., Y= 11147.91-5.18(2012)=725.75≈726

8. Fit a curve of the type $y=ax^b$ for the data

X	1	10	100	1000
Y	10.95	39.81	50.12	63.8

Answer:

Given the curve fitted to the above data is, $Y=aX^b$,

where a and b are constants, to be determined. Hence to determine these constants, we use the following normal equations.

$\Sigma U = nA + b\Sigma V$

$\Sigma UV = A\Sigma V + b\Sigma V^2$

Where U = log Y

V= log X
A = log a

To calculate the values of a and b, we construct the following table.

X	Y	U=logY	V=logX	V^2	UV
1	10.95	1.039414	0	0	0
10	39.81	1.599992	1	1	1.599992
100	50.12	1.700011	2	4	3.400022
1000	63.8	1.804821	3	9	5.414462
Total		6.144238	6	14	10.41448

Therefore, the normal equations can be written as follows:

6.1442=4A+6b

10.4145=6A+14b

On solving above two simultaneous equations, we get
A=1.1766 implies a=antilog (A)=antilog (1.1766)=15.0176
The equation of required curve is,
$Y=15.0176X^{0.2396}$

9. The weights of a calf taken at weekly intervals are given below. Fit a straight line using the method of least squares and estimate the weight when the age is 11 years.

Age in years	1	2	3	4	5	6	7	8	9	10
Weight in Kgs.	52.5	58.7	65.0	70.2	75.4	81.1	87.2	95.3	102.2	108.4

Answer:
Let the linear equation is given by,

Y = a + bX

The normal equations are given by,

ΣY = na + bΣx and

$\Sigma XY = a\Sigma X + b\Sigma X^2$

Now let us consider the following table in which we calculate all the necessary totals.

Age in years	Wight in Kgs.	X2	XY
1	52.5	1	52.5
2	58.7	4	117.4
3	65	9	195
4	70.2	16	280.8
5	75.4	25	377
6	81.1	36	486.6
7	87.2	49	610.4
8	95.3	64	762.4
9	102.2	81	919.8
10	108.4	100	1084
55	796	385	4885.9

Therefore, the normal equations are,

$796 = 10a + b(55)$ and

$4\,885.9 = 55a + 385b$

On solving the above two simultaneous equations we get,

$a=45.74$ and $b=6.1564$

Therefore, $Y = 45.74 + 6.1564X$

To estimate the weight at 11 years, we substitute X=11 in the above equation.

i.e., $Y=45.74 + 6.1564(11)=113.4604$ Kg.

10. Fit an equation of type $Y=ab^x$ to the following data.

X	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25
Y	93	71	63	54	48	38	29	26	22

Answer:

The power curve fitted to the above data is, $Y=ab^x$

The normal equations are given by,

$\Sigma U = nA + B\Sigma X$

$\Sigma UX = A\Sigma X + B\Sigma X^2$

Where $U = \log Y$

$A = \log a$

$B = \log b$

To calculate the values of a and b, we construct the following table.
To make calculations simple, let us consider the transformation $x=(X-1.25)/0.25$

X	Y	U=logY	x	x^2	Ux
0.25	93	1.9685	-4	16	-7.8739
0.5	71	1.8513	-3	9	-5.5538
0.75	63	1.7993	-2	4	-3.5987
1	54	1.7324	-1	1	-1.7324
1.25	48	1.6812	0	0	0.0000
1.5	38	1.5798	1	1	1.5798
1.75	29	1.4624	2	4	2.9248
2	26	1.4150	3	9	4.2449
2.25	22	1.3424	4	16	5.3697

Total	14.8323	0	60	-4.6396

Hence, by substituting in normal equations, we get,

14.8323=9A implies A=14.8323/9=1.648 and

-4.6396=60B implies B=-4.6396/60=-0.07733

We know that, a=antilog(A)=antilog(1.648)=44.4665 and b=antilog(B) = antilog(-0.07733)=0.8369

Hence, the equation to be fitted to the given data can be written as follows.

$Y=(44.4665)(0.8369)^x$ where x=(X-1.25)/0.25.

11. Fit an equation of type Y=a+bx to the following data.

X	0	1	2	3	4	5	6
Y	32	47	65	92	132	190	275

Answer:
The equation fitted to the above data is, Y=a+bx

The normal equations are given by,
$\Sigma Y = na + b\Sigma x$ and
$\Sigma XY = a\Sigma X + b\Sigma X^2$
Now let us consider the following table in which we calculate all the necessary totals.

X	Y	X^2	XY
0	32	0	0
1	47	1	47
2	65	4	130
3	92	9	276
4	132	16	528
5	190	25	950
6	275	36	1650
21	833	91	3581

Hence, the normal equations are,
833=7a+21b
3581=21a+91b
On solving the above two simultaneous equations, we get,
a= 3.07 and b=38.64
Therefore, the equation is Y=3.07+38.64x.

12. Fit a parabola to the following data

X	-1	0	0	1
Y	2	0	1	2

Answer:

The parabola fitted to the above data is given by, $Y = a + b X + cX^2$

And the normal equations are given by

$\Sigma Y = na + b\Sigma X + c\Sigma X^2$
$\Sigma XY = a\Sigma X + b\Sigma X^2 + c\Sigma X^3$
$\Sigma X^2Y = a\Sigma X^2 + b\Sigma X^3 + c\Sigma X^4$

To solve for a, b and c, we construct the following table.

X	Y	X^2	X^3	X^4	XY	X^2Y
-1	2	1	-1	1	-2	2
0	0	0	0	0	0	0
0	1	0	0	0	0	0
1	2	1	1	1	2	2
0	**5**	**2**	**0**	**2**	**0**	**4**

Hence, the normal equations are,

5= 5a + 0.b+2c ------------(1)
0 = a.0 + b.2+c.0 implies b=0
4=2a+0.b+2c --------------(2)

From equations 1 and 2, we get,
a=1/3 and c=5/3.

Therefore, the equation of parabola fitted to the given data is,
$Y= (1/3) + (0) X + (5/3)X^2$
ie., $Y= (1/3) + (5/3)X^2$

13. Fit a curve of the form $y=ax+bx^2$ for the data given below

X	1	2	3	4	5
Y	1.8	5.1	8.9	14.1	19.8

Answer:
The curve fitted to the given data is $y=ax+bx^2$

Hence, the normal equations are,

$\Sigma y=a\Sigma x+ b\Sigma x^2$
$\Sigma xy=a\Sigma x^2 + b\Sigma X^3$

X	Y	X^2	X^3	XY
1	1.8	1	1	1.8
2	5.1	4	8	10.2
3	8.9	9	27	26.7
4	14.1	16	64	56.4
5	19.8	25	125	99
15	**49.7**	**55**	**225**	**194.1**

Hence, the normal equations are,
49.7=15a+55b
194.1=55a+225b

On solving the simultaneous equations we get,
a=1.45, b=0.51

Therefore, the required curve is, $y=1.45x+0.51x^2$

14. The following table gives the results of measurements of train resistance

V	20	40	60	80	100	120
R	5.5	9.1	14.9	22.8	33.3	46.0

Here V is the velocity in Kms/hour and R is the resistance in KG/puintal. Fit a relation of the form $R= a+bV+cV^2$ to the data. Estimate R when V = 90

Answer:

The parabola fitted to the above data is given by,

$R= a+bV+Cv^2$

And the normal equations are given by,

$\Sigma R = na + b\Sigma V+c\Sigma V^2$

$\Sigma VR = a\Sigma V + b\Sigma V^2+c\ \Sigma V^3$

$\Sigma V^2R= a\Sigma V^2+b\Sigma V^3+c\Sigma V^4$

Since values of V are large, to make calculations easier, we use the transformation $X=(V-70)/10$. Hence the normal equations become,

$\Sigma R = n\ a + b\Sigma X+c\Sigma X^2$

$\Sigma XR = a\Sigma X + b\Sigma X^2+c\Sigma X^3$

$\Sigma X^2R= a\Sigma X^2+b\Sigma X^3+c\Sigma X^4$

To solve for a, b and c, we construct the following table.

V	R	X	X^2	X^3	X^4	XR	X^2R
20	5.5	-5	25	-125	625	-27.5	137.5
40	9.1	-3	9	-27	81	-27.3	81.9
60	14.9	-1	1	-1	1	-14.9	14.9
80	22.8	1	1	1	1	22.8	22.8
100	33.3	3	9	27	81	99.9	299.7
120	46	5	25	125	625	230	1150
Total	**131.6**	**0**	**70**	**0**	**1414**	**283**	**1706.8**

Hence, the normal equations become,

131.6=6a+70c-------------(1)

283=70b implies b=4.0429

1706.8=70a+1414c-------(2)

By solving equations 1 and 2, we get,

a=18.5844 and b=0.287

Hence, the relation fitted to the given data is,

$R= (18.5844)+(4.0429)X+(0.287)X^2$

To estimate R when V=90, substitute X=2

Hence, $R= (18.5844)+(4.0429)2+(0.287)2^2=27.84$

15. Fit a curve of the form ax^b to the following data

X	1	2	3	4	5	6
Y	2.98	4.26	5.21	6.10	6.80	7.50

Answer:

Given the curve fitted to the above data is, $Y=aX^b$

where a and b are constants, to be determined. Hence to determine these constants, we use the following normal equations.

$\Sigma U = n A + b\Sigma V$

$\Sigma UV = A \Sigma V + b \Sigma V^2$

Where U = log Y

V= log X

A = log a

To calculate the values of a and b, we construct the following table.

X	Y	U=logY	V=logX	V^2	UV
1	2.98	0.4742	0.0000	0.0000	0.0000
2	4.26	0.6294	0.3010	0.0906	0.1895
3	5.21	0.7168	0.4771	0.2276	0.3420
4	6.10	0.7853	0.6021	0.3625	0.4728
5	6.80	0.8325	0.6990	0.4886	0.5819
6	7.50	0.8751	0.7782	0.6055	0.6809
21	**32.85**	**4.3134**	**2.8573**	**1.7748**	**2.2671**

By substituting the values in the above normal equations we get,

4.3134=6A+2.8573b

2.2671=2.8573A+1.1778B

On solving above two equations, we get, A=1.27, implies a=18.62 and b=-1.16

Hence, the curve is $1.27x^{-1.16}$.