1. Introduction

Welcome to the series of eLearning modules on XY graphs.

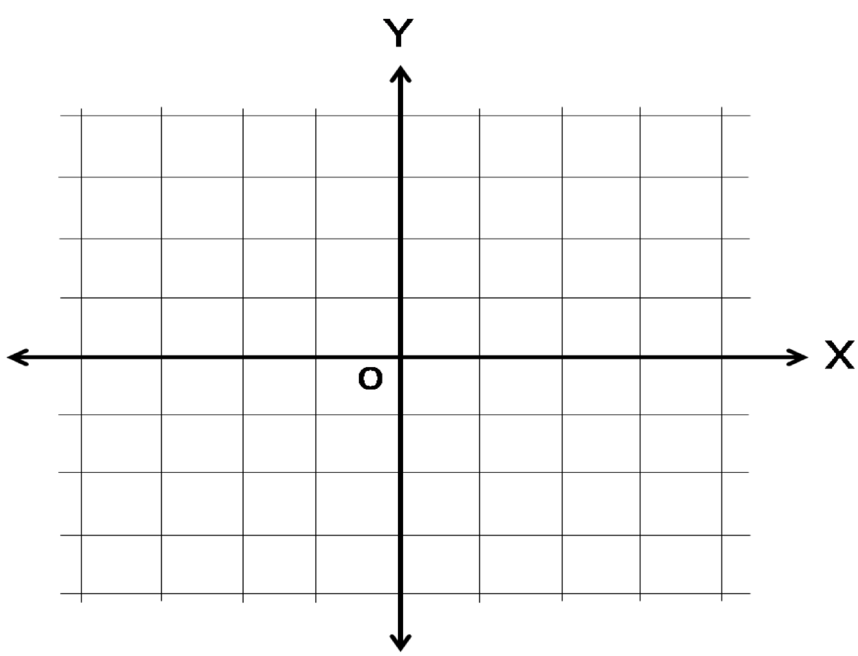
At the end of this session, you will be able to:

- Explain what is a Graph
- Know how to identify points on a graph
- Learn how to plot points on a graph for linear equations
- Learn how to graph equations using intercepts
- Graph equations using slope

XY graphs are based on the Cartesian coordinate system in two dimensions (also known as the rectangular coordinate system) which is defined by an ordered pair of perpendicular lines (axes). A single unit of length is used for both axes.

The lines are commonly called the *x* and *y*-axis, where the *x*-axis is taken to be horizontal axis and the *y*-axis is taken to be vertical axis. The point where the axes meet or intersect is considered the origin, thus turning each individual axis into a number line. The point of the origin is often labelled *O* and if so then the axes are called *Ox* and *Oy*. A plane with *x* and *y*-axes defined is often referred to as the Cartesian plane or *xy* plane.

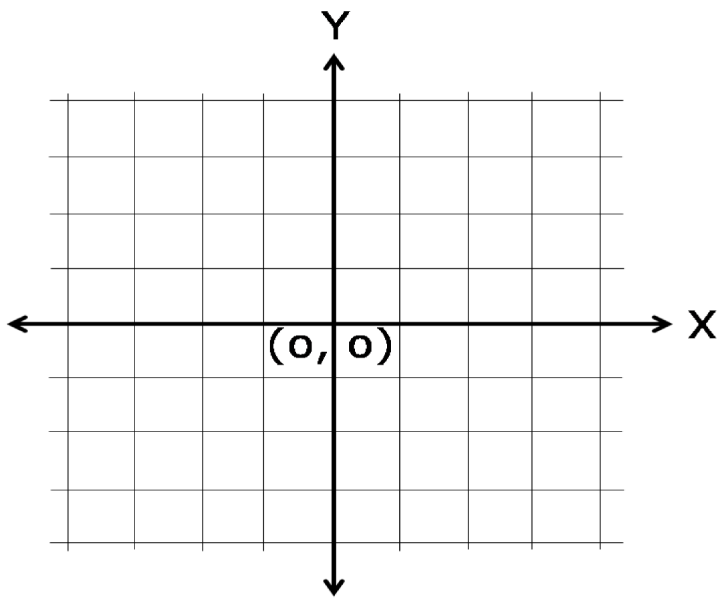
Figure 1



By virtue of having created two number lines with our x & y axes each point on a xy graph will have 2 values which are called co-ordinates. The first number represents the point given by a numerical value on the X axis while the second represents the point or number on the Y axis.

The co-ordinates at the origin are (0, 0).

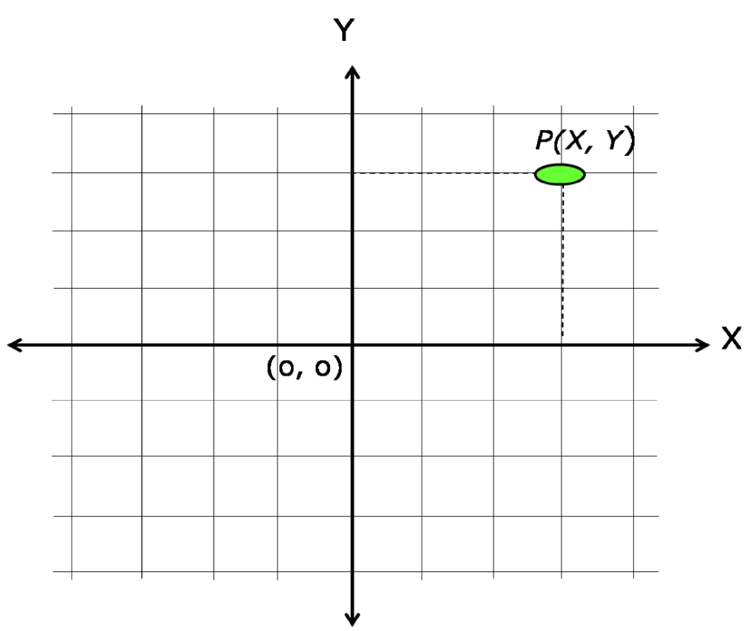
Figure 2



For a given point *P*, if a line is drawn through *P* perpendicular to the *x*-axis to meet it at *X* and second line is drawn through *P* perpendicular to the *y*-axis to meet it at *Y*. The coordinates of *P* are then *X* and *Y* interpreted as numbers *x* and *y* on the corresponding number lines. The coordinates are written as an ordered pair (*x*, *y*).

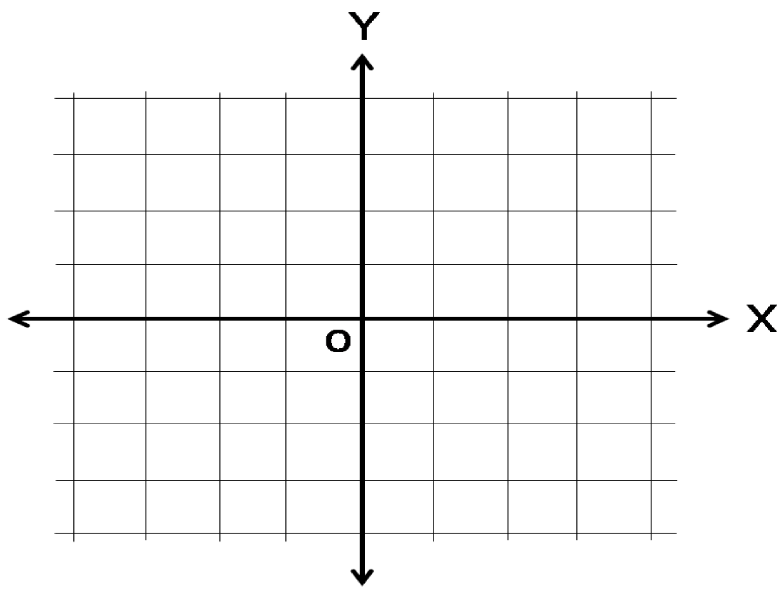
The value of *x* is called the *x*-coordinate or abscissa and the value of *y* is called the *y*-coordinate or ordinate.

Figure 3



A graph is a visual representation of a relationship between two variables. These variables are commonly called x & y. A graph normally consists of 2 axes as explained to you before. The horizontal axis is called the X axis and the vertical axis is called the y axis.

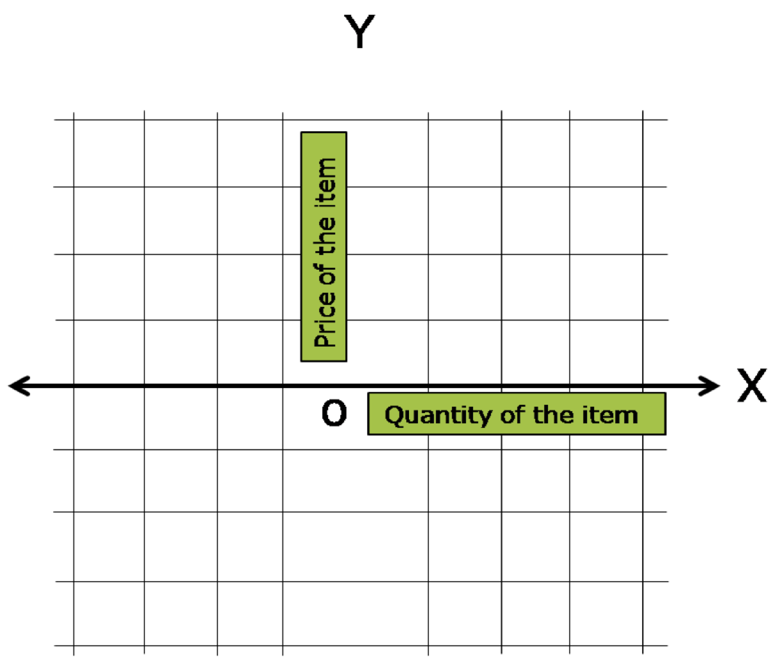
Figure 4



2. Uses of Graphs in Different Type of Data

When graphs are used to represent different types of data these axes are given names or labelled according to the data to be represented by the graph. For example if a graph is to be used in economics to show the law of demand and supply then Price of the item under review maybe represented on the Y axis and quantity of the item represented on the x axis.

Figure 5

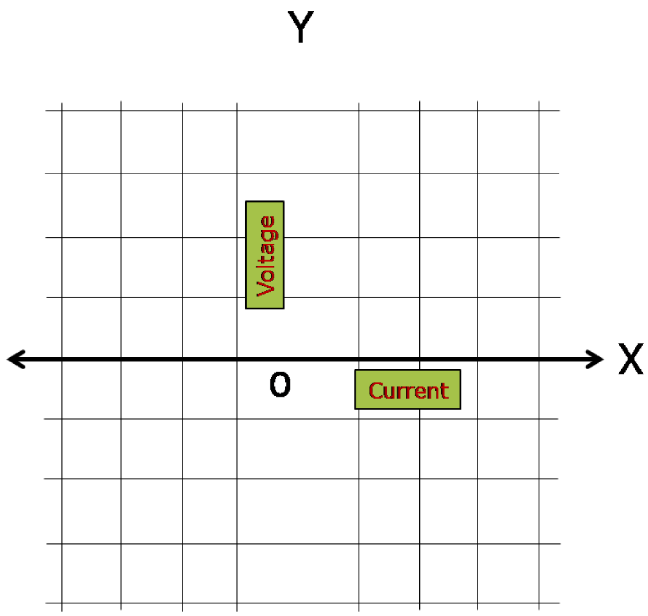


In mathematics, a graph is a geometric representation, a picture or a representation of a relation or function. A relation is a subset of the set of all ordered pairs for a particular set of data (x, y) for which each x is a member of some set X and each y is a member of another set Y. A specific relationship between each x and y determines which ordered pairs are in the subset.

A graph, then, is a pictorial representation of the ordered pairs that comprise a relation or function. At the same time, it is a pictorial representation of the relationship between the first and second elements of each of the ordered pairs.

There are many practical applications of graphs.

Figure 6



In the sciences and engineering, sets of numbers represent physical quantities. Graphing a relationship between these physical quantities is a useful tool for understanding the nature of these items. A specific example is the graphing of current versus voltage.

This is used by electrical engineers, to get a picture of the behaviour of various circuit components. The rectangular coordinate system can be used to represent all possible combinations of current and voltage. Nature, however, severely limits the allowed combinations, depending on the particular electrical device through which current is flowing. By plotting the allowed combinations of current and voltage for various devices, engineers are able to "picture" the different behaviours of these devices. They use this information to design circuits with combinations of devices that will behave as predicted.

Chemists and physicists use graphs to discover relationships between quantities. Graphs can be used to predict future values of important figures like population and the national debt.

In the corporate world, Graphs are used to present data in a pictorial manner. The basic tool for many graphical representations is the xy graph. Graphs are used in almost every discipline, so it is important to develop an understanding of how to use them.

How to plot equations on a XY graph
We can only graph points of one variable on a number line; hence, we need a 2-dimensional (2 variable) way of representing points -- the xy-graph:

As stated earlier The horizontal axis, called the x-axis, represents values of *x*, and the vertical axis, called the y-axis, represents values of *y*. The point where the 2 axes intersect is called the origin. The 2 axes divide our graph sheet into 4 quadrants as represented here.

In Quadrant I both x and y are positive, but ...

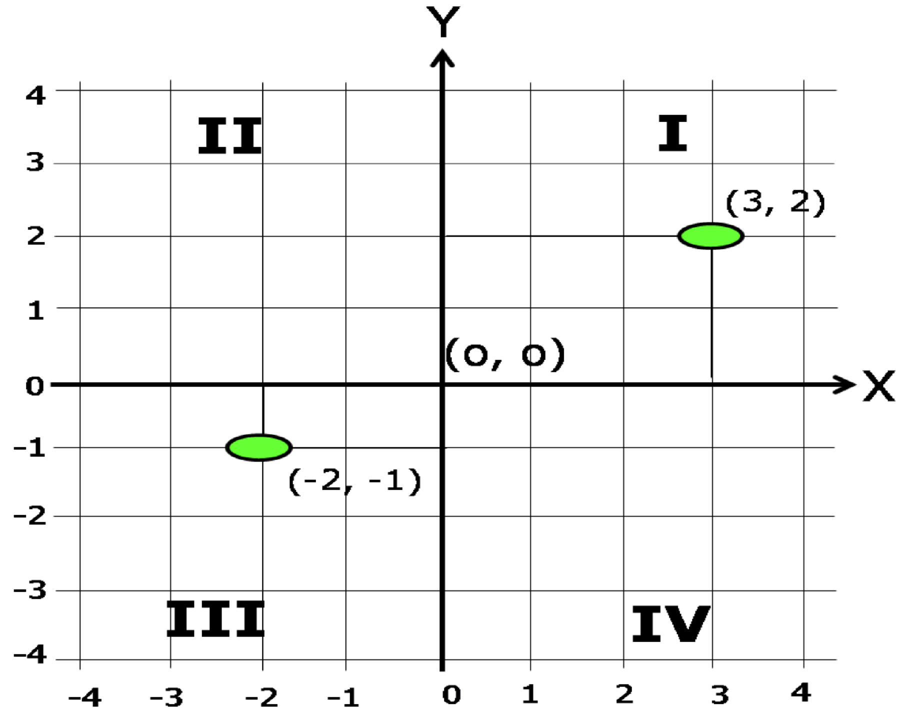
- in Quadrant II x is negative (y is still positive),
- in Quadrant III both x and y are negative, and
- in Quadrant IV x is positive again, while y is negative.

Figure 7

Quadrant	X (Horizontal)	Y (Vertical)	Example
I	Positive	Positive	(3, 2)
II	Negative	Positive	
III	Negative	Negative	(-2,-1)
IV	Positive	Negative	

Like this:The numbers represented at the origin are (0,0) From now on, the word "graph" in our session will refer to the *xy*–graph. To mark a point on the graph, first find the *x* -coordinate on the *x* -axis. Then move up on the graph the number of spaces which is equal to the *y* -coordinate (or move down if the *y* -coordinate is negative). For example, to graph (2, 3), find 2 on the *x* -axis. Then, move up 3 spaces. To graph (- 2, 1), find -2 on the *x* -axis, then move up 1 space.

Figure 8



To graph (1.5, - 1) , find 1.5 on the *x* -axis, then move *down* 1 space:

One of the main uses of a xy-graph is to graph equations. If an equation has both an *x* and *y* variable, then it often has multiple solutions of the form (*x*, *y*). In fact, there are an infinite number of solutions to an equation with two variables.

3. Graphing Equations with two Variables

Now, we move to graphing equations with two variables. For simplicity, our discussion is confined to *linear* equations, i.e. equations of degree 1.

We will first explain how to represent variables as ordered pairs. This is a convenient way of writing corresponding variable values. In this section, we will also learn how to graph ordered pair values (x,y) on a xy-graph. Graphing (x,y) values on a graph is similar to graphing x values on a number line, except that we are working in two dimensions instead of one.

An ordered pair is a pair of numbers in a specific order. For example, (1, 2) and (- 4, 12) are ordered pairs. The order of the two numbers is important: (1, 2) is not equivalent to (2, 1) – $(1, 2)\neq(2, 1)$.

Ordered pairs are often used to represent two variables. When we write (x, y) = (7, - 2), we mean x = 7 and y = - 2. The number which corresponds to the value of x is called the x-coordinate and the number which corresponds to the value of y is called the y-coordinate.

Example 1. If (x, y) = (- 1, 4) , what is the value of $3x + 2y - 4$?
$3x + 2y - 4 = 3(- 1) + 2(4) - 4 = - 3 + 8 - 4 = 1$

Example 2. Which of the following ordered pairs (x, y) are solutions to the equation
$(2x-1)/y- 6 = 1$? {(4, 1),(5, 2),(- 3, 1),(- 3, -1),(1, 4)}
(x, y) = (4, 1) : $(2x-1)/y -6 = (2(4)-1)/1- 6 = 7 - 6 = 1$. Solution.
(x, y) = (5, 2) : $(2x-1)/y- 6 = (2(5)-1)/2-6 = 9/2 -6 = - 3/2\neq1$. Not a solution.

(x, y) = (- 3, 1) : $(2x-1)/y -6 = (2(-3)-1)/1 -6 = - 7 - 6 = - 13\neq1$. Not a solution.
(x, y) = (- 3, - 1) : $(2x-1)/y -6 = (2(-3)-1)/(-1) - 6 = 7 - 6 = 1$. Solution.
(x, y) = (1, 4) : $(2x-1)/y -6 = (2(1)-1)/4 -6 = 1/4 -6 = -23/4\neq1$. Not a solution.
Thus, {(4, 1), (- 3, -1)} are solutions to $(2x-1)/y - 6 = 1$.

The solutions to an equation in two variables can be represented by a curve on a xy-graph; every point on the curve has coordinates which satisfy the equation. In fact, for linear equations, the curve representing the solutions to the equation will actually be a straight line.

Making Data Tables

One way to graph an equation is by use of a data table. A data table is a list of x -values and their corresponding y -values. To make a data table, draw two columns. Label one column x and the other column y. Then list the x -values -2, - 1, 0, 1, 2 in the x column:

Figure 9

X	-2	-1	0	1	2
Y					

Next, plug each value of x into the equation and solve for y. Insert these values of y into the table, under their corresponding x values. For this example, we will use the equation $2x - 4 = 3y$:

$x = -2 : 2(-2) - 4 = 3y . 3y = -8 . y = -2(2/3)$
$x = -1 : 2(-1) - 4 = 3y . 3y = -6 . y = -2$
$x = 0 : 2(0) - 4 = 3y . 3y = -4 . y = -1(1/3)$
$x = 1 : 2(1) - 4 = 3y . 3y = -2 . y = -(2/3)$
$x = 2 : 2(2) - 4 = 3y . 3y = 0 . y = 0$

Thus, the data table looks like:

Figure 10

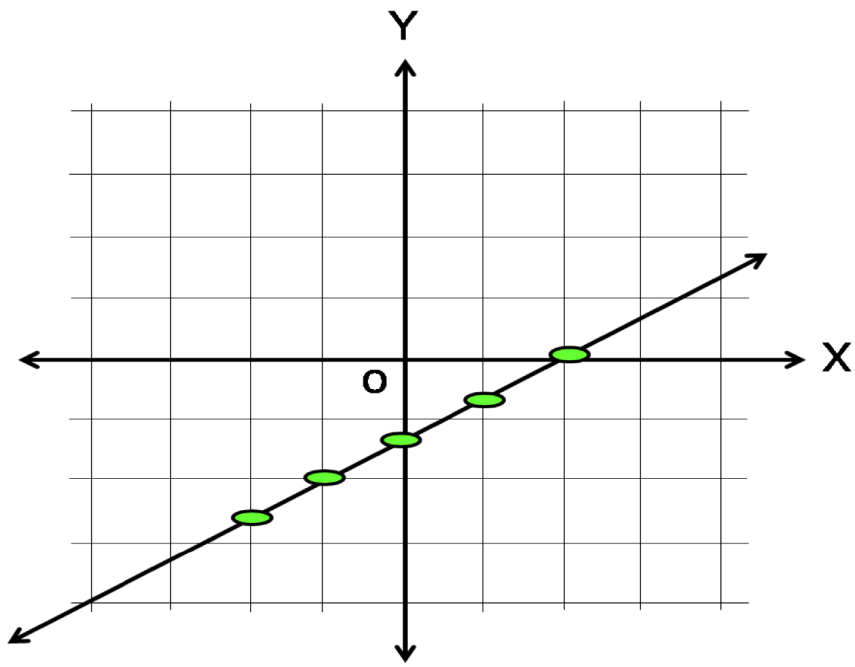
X	-2	-1	0	1	2
Y	-2	-2	-1		0

Making Graphs Using Data Tables

To make a graph using the data table, simply plot all the points and connect them with a straight line. Extend the line on both sides and add arrows. This is to show that the line continues infinitely, even after it can be seen on the graph.

Here is our sample data table as a graph:

Figure 11



Graph of 2x-4=3y

Note that the large dots on the line are unnecessary -- they are merely there to show the data points we plotted.
To check, pick a data point that is on the line but *not* in the chart -- it should satisfy the equation.

Notice also that it is not necessary to make a huge data table to graph a linear equation effectively. There is only one line through any two points, so already if one plots three points from a data table the redundancy of the third point acts as a check of the calculations. Of course, for more general equations whose graph does not consist of a straight line, more points are necessary to get an idea of the appearance of the graph.

4. Graphing equations using Intercepts

What is an intercept- It is the point at which one of the axes is zero, hence for an X intercept y=0 and for a Y intercept x=0. Thus the x-intercept is the point at which a line crosses the x-axis; i.e. the point at which $y = 0$. The y-intercept is the point at which a line crosses the y-axis; that is, the point at which $x = 0$. These concepts depend upon writing a linear equation using variables x and y, which is both standard and implicit in our identification of such an equation with the straight line that is its graph.

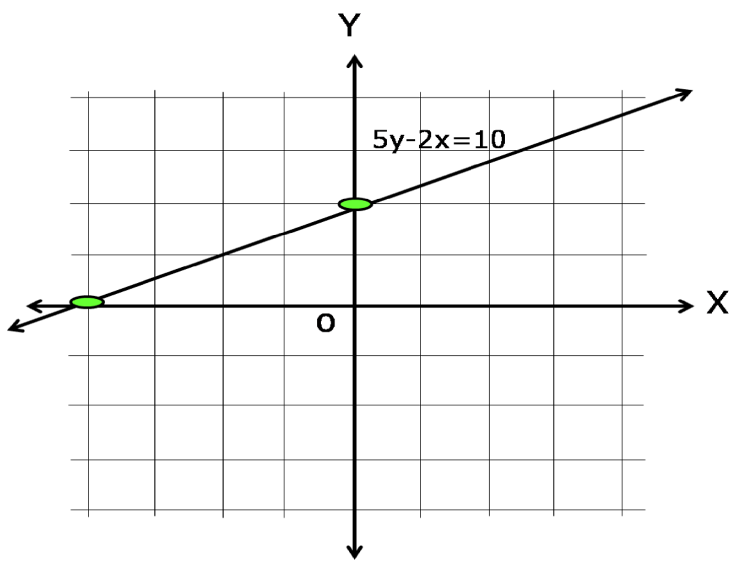
To find the x-intercept, set $y = 0$and solve the equation. For example, to find the x-intercept of$5y - 2x = 10$:
$5(0) - 2x = 10$
$-2x = 10$
$x = -5$
Thus, the x-intercept, or the point at which the line crosses the horizontal axis, is (- 5, 0).

To find the y-intercept, set $x = 0$ and solve the equation. For example to find the y-intercept of $5y - 2x = 10$:
$5y - 2(0) = 10$
$5y = 10$
$y = 2$
Thus, the y-intercept, or the point at which the line crosses the vertical axis, is (0, 2).

Hence, to find the intercept of either variable, set the other variable equal to 0 and solve for the original variable.

As observed in our previous example, we only really need two points to graph a line. Usually, the two easiest points to find are the x-intercept and the y-intercept. Once these have been found, we can plot them, draw a straight line connecting them, and extend the line at either end. Here is a graph of the equation $5y - 2x = 10$, drawn using intercepts:

Figure 12



Graph of 5y-2x=10

Of course, it is useful to test a point on the line to make sure it satisfies the equation; since we are using only two points, there is more room for error.

It is important to point out that, no matter what technique we use to graph an equation, the graph of the equation is always the same -- all techniques will yield the exact same graph.

Graphing Equations using slope

To find the slope of a line, pick *any* two points on the line. Then subtract their x-coordinates and subtract their y-coordinates in the same order. Divide the difference of the *y* -coordinates by the difference of the *x* – coordinates.

Given two points (x_1, y_1) and (x_2, y_2) on a line, the slope of the line is equal to:

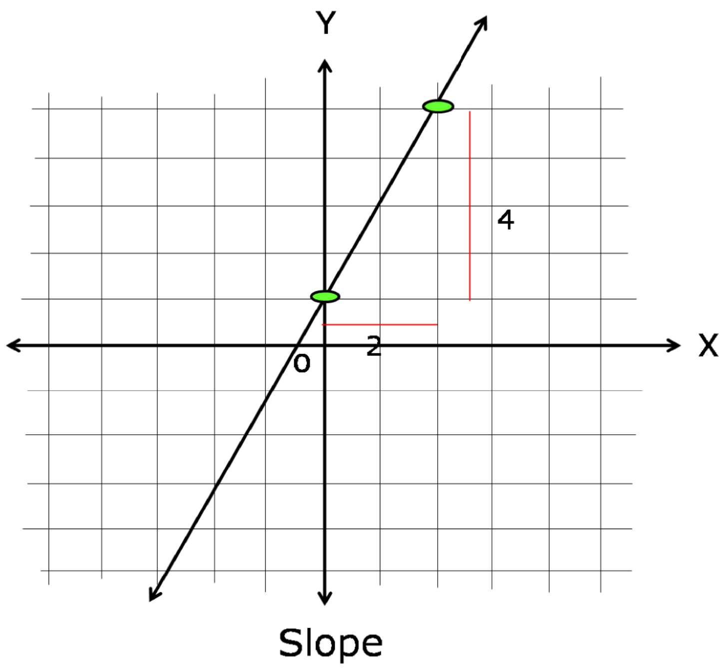
$$m = \frac{\text{Change in y}}{\text{Change in x}} = \frac{y1 - y2}{x1 - x2}$$

Example 1. Find the slope of the line which passes through the points(2, 5) and (0, 1) :

m = (5-1)/(3-1) = 4/2 = 2.

This means that every time *x* increases by 1 (anywhere on the line), *y* increase by 2, and whenever *x* decrease by 1, *y* decreases by 2.

Figure 13



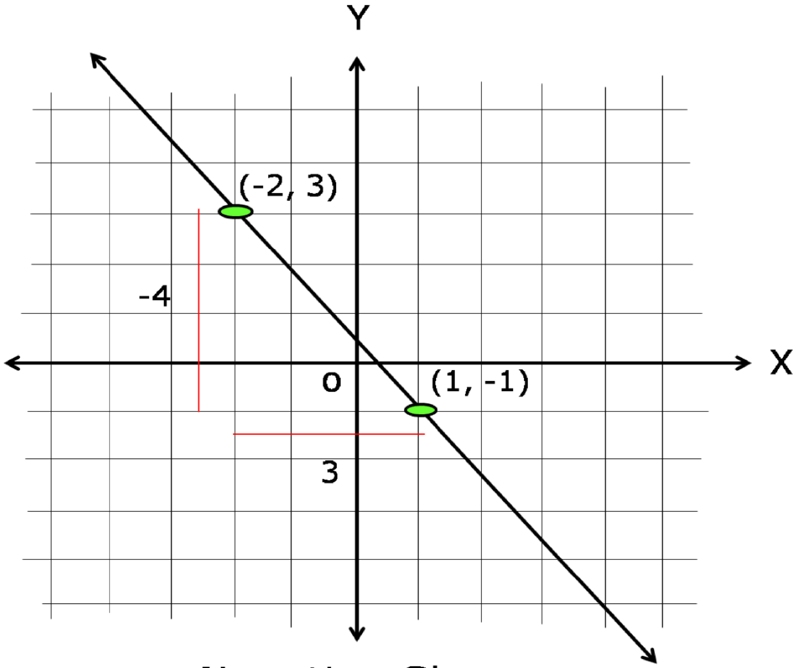
Negative Slope

If a line has a positive slope (i.e. $m > 0$), then *y* always increases when *x* increases and *y* always decreases when *x* decreases. Thus, the graph of the line starts at the bottom left and goes towards the top right.

Often, however, the slope of a line is negative. A negative slope implies that *y* always decreases when *x* increases and *y* always increases when *x* decreases.

Here is an example of a graph with negative slope:
m = (3-(-1))/(-2-1)=4/(-3)= -(4/3)

Figure 14



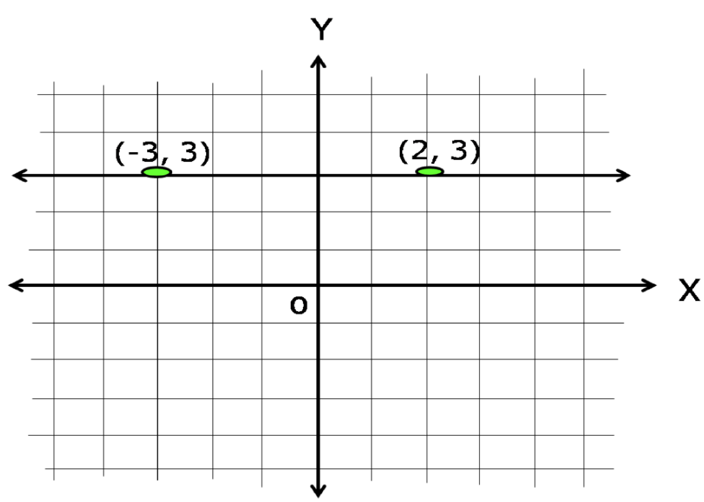
Negative Slope

Thus, as *x* increases by 3, *y* decreases by 4, and as *x* decreases by 3, *y* increases by 4.

Horizontal and Vertical Lines

Sometimes, we will see equations whose graphs are horizontal lines. These are graphs in which *y* remains constant -- that is, in which $y_1 - y_2 = 0$ for any two points on the line:

Figure 15



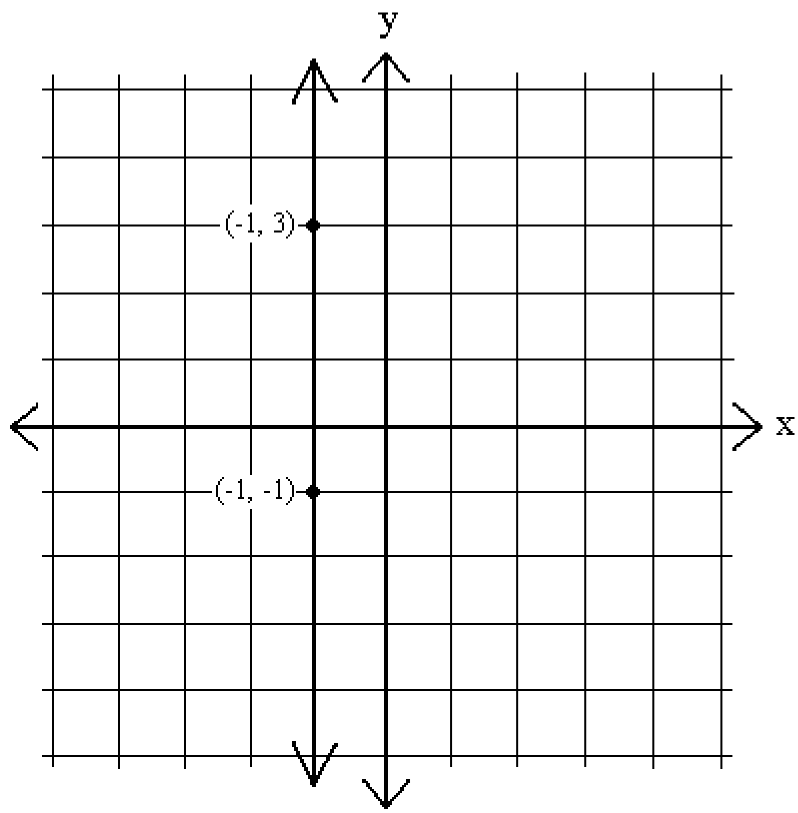
Graph of a Horizontal Line

m = (3-3)/(2-(-3)) =0/5=0

The slope of any horizontal line is 0. In other words, as x increases or decreases, y does not change. x takes every possible value at a specific y value.

We will also see equations whose graphs are vertical lines. These are graphs in which x remains constant -- that is, in which $x_1 - x_2 = 0$ for any two points on the line:

Figure 16



Graph of a Vertical Line

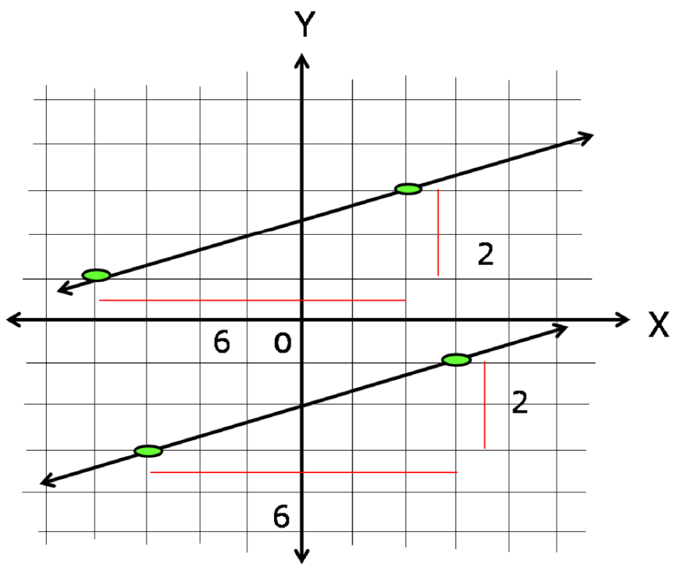
M=(3-(-1))/(-1-(-1))=4/0=undefined. We cannot divide a number by zero. The slope of any vertical line is undefined. x does not increase or decrease; rather, y takes every possible value at a specific x value.

Intuitively, it makes sense that one cannot assign a slope to a vertical line; a line that is "almost" vertical (i.e. is very steeply inclined) could have a very large positive or negative slope. So there is no way to decide even whether or not a vertical line should have positive or negative slope (and it clearly cannot have zero slope).

5. Parallel Lines and Perpendicular Lines

Two lines are parallel if they have the same slope. Parallel lines, when extended, do not intersect at any point.

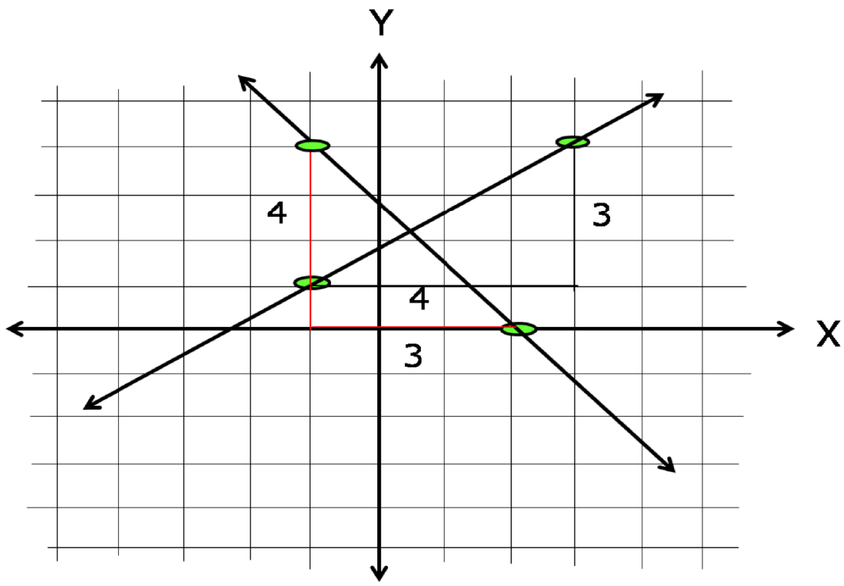
Figure 17



Graph of a Parallel Lines

Two lines are perpendicular if their slopes are opposite reciprocals of each other. For example, if a line has a slope of $\frac{3}{4}$, a perpendicular line has a slope of - $\frac{4}{3}$. Perpendicular lines intersect each other at right angles.

Figure 18



Graph of a Perpendicular Lines

Using Slope to Graph Lines

One can graph a line if we know only its slope and one point on it.
First, plot the point.
Next, if the slope is a fraction, move to the right the number of spaces equal to the denominator, and move up (or down, if the slope is negative) the number of spaces equal to the numerator.

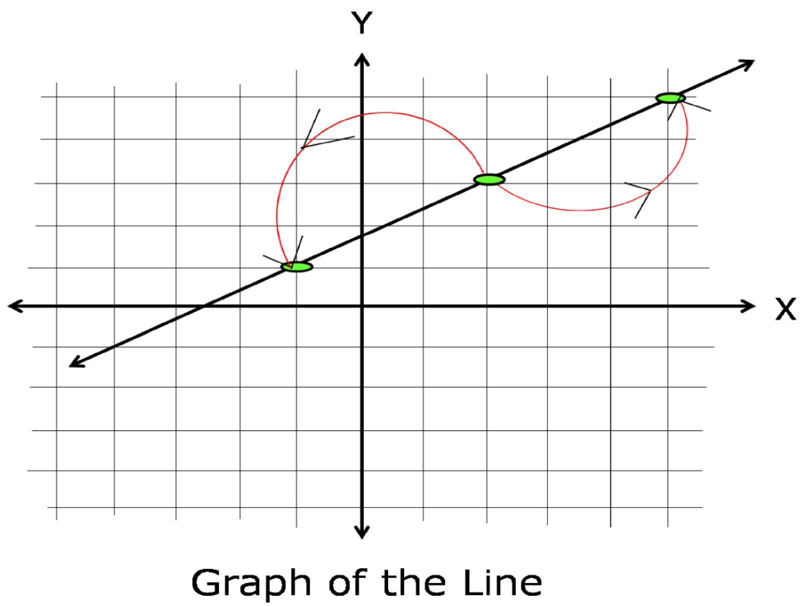
Plot a point at the spot you end up.

If the slope is not a fraction, move 1 space to the right and then move up or down the number of spaces equal to the slope. You can also move *left* the number of spaces in the denominator and *down* (or *up* if negative) the number of spaces in the numerator and plot a point at the spot you end up.

Connecting these points with a straight line and extending on both sides yields a line with the desired slope and containing the given point.

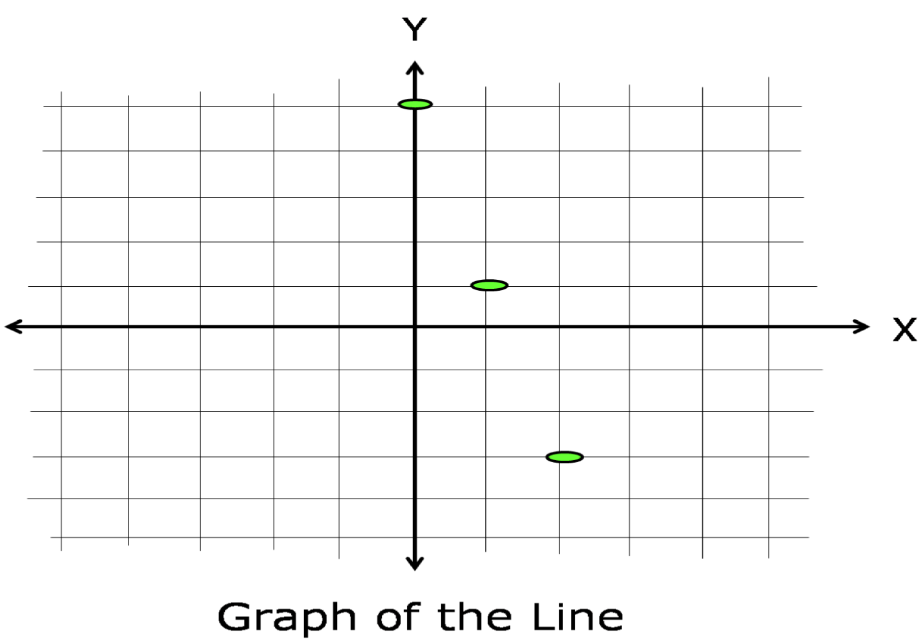
Example 1. Graph the line which passes through (2, 3) and has a slope of 2/ .

Figure 19



Example 2. Graph the line which passes through (1, 1) and has a slope of -4.

Figure 20



Finding Slope from an Equation

We can find the slope of any line represented by an equation. Here are the steps:

1. Using inverse operations get the two variables on opposite sides.
2. Write a fraction with the coefficient of *x* in the numerator and the coefficient of *y* in the

denominator (this is counterintuitive, since we have been dealing with *y* in the numerator and *x* in the denominator)

3. This fraction is the slope.

Example 3. Find the slope of the line given by the equation 5*y* = - 2*x* + 1

1. 5*y* = - 2*x* + 1 (no change)
2. -2/5 = -(2/5)
3. *m* = - 2/5

Example 4. Find the slope of the line given by the equation 2*y* - *x* + 15 = 0

1. 2y + 15 = x
2.
3. ½
4. m = 1/2

Example 5. Find the slope of the line given by the equation 3 = - *y* - 15*x*

1. y + 3 = - 15x
2. -15/1 = - 15
3. m = - 15

We can see from the steps for finding slope and from these examples that *the constant term does not affect the slope.*

Once we know the slope of an equation, we can find a point that satisfies the equation by plugging in a value for *x* and solving. Then we can plot this point and graph the equation.

Here's a summary of our learning in this session:

- Explain what is a Graph
- Know how to identify points on a graph
- Learn how to plot points on a graph for linear equations
- Learn how to graph equations using intercepts
- Graph equations using slope