Frequently Asked Questions

1. What is discrete uniform distribution?

Answer: In probability theory, the discrete uniform distribution is a probability distribution whereby, a finite number of equally spaced values are equally likely to be observed. Every one of n values has equal probability $1/n$.

2. Explain the pmf of discrete uniform distribution.

Answer: A random variable X is said to follow uniform distribution with parameter n if its pmf is given by,

$p(x)= 1/n$, $a < x < b$

$\quad = 0 \quad$ otherwise

Where $a\varepsilon(. \ldots -2, -1, 0, 1, 2 \ldots)$, $b\varepsilon(. \ldots -2, -1, 0, 1, 2 \ldots)$ and $\quad x\varepsilon(a, a+1, a+2 \ldots b-1,b)$

3. Derive the distribution function of uniform distribution.

Answer:
Distribution function is given by,

$$F(x) = \sum_{x=a}^{x} p(x) = \sum_{x=a}^{x} \frac{1}{n} = \frac{|x|-a+1}{n}$$

Hence we can write

$F(x) = 0 , \qquad$ for x<a

$\quad = \dfrac{|x|-a+1}{n} \quad$ for $a \le x \le b$

$\quad = 1 \qquad\qquad$ for x > b

4. Obtain mean of Discrete uniform distribution having pmf $p(x) = 1/n$, x= 1, 2, . . ., n

Answer: $E(X) = \sum x.p(x)$

$\quad = 1.P(X=1) + 2.P(X=2) + \ldots + (n-1).P(X=n-1) + n.P(X=n)$

$\quad = 1.1/n + 2.1/n + \ldots + (n-1).1/n + n.1/n$

$\quad = (1+2+\ldots n-1+n).1/n$

$\quad = \dfrac{n(n+1)}{2} \times \dfrac{1}{n}$, (sum of n natural numbers is $\dfrac{n(n+1)}{2}$)

$\quad = \dfrac{n+1}{2}$

5. Obtain mean of Discrete uniform distribution having pmf p(x) = 1/n+1, x= 0, 1, 2, . . ., n

Answer: $E(X) = \sum x \cdot p(x)$

$= 0.P(X=0) +1.P(X=1) + 2.P(X=2) + . . . + (n-1).P(X=n-1) + n.P(X=n)$

$= 1.1/n+1 + 2.1/n+1 + . . . + (n-1).1/n+1 + n.1/n+1$

$= (1+2+. . . (n-1)+n).1/n+1$

$= \dfrac{n(n+1)}{2} \times \dfrac{1}{n+1}$, (sum of n natural numbers is $\dfrac{n(n+1)}{2}$)

$= \dfrac{n}{2}$

6. Find variance of Discrete uniform distribution having pmf, p(x) = 1/n, x=1,2,...,n

Answer: Variance is given by $V(X) = E(X^2) + (E(X))^2$

First let us find $E(X)^2$

$E(X^2) = \sum x^2 \cdot p(x)$

$= 1^2.P(X=1) + 2^2.P(X=2) + . . . + (n-1)^2.P(X=n-1) + n^2.P(X=n)$

$= 1^2.1/n + 2^2.1/n + . . . + (n-1)^2.1/n + n^2.1/n$

$= (1^2 2^2+ . . . (n-1)^2+n^2).1/n$

$= \dfrac{n(n+1)(2n+1)}{6} \times \dfrac{1}{n}$

$= \dfrac{(n+1)(2n+1)}{6}$

7. Derive mean deviation of discrete uniform distribution about mean (or median)

Answer MD $= \dfrac{1}{n}\sum_{x=1}^{n} |\, x - \dfrac{1}{2}(n+1)\,|$

To do the sum, we consider separately the cases of n is odd and n is even.

If n is odd, that is we can write in general, n= 2m-1 then

MD $= \dfrac{1}{n}\sum_{x=1}^{n} |\, x - m\,|$

$= \dfrac{1}{n}\left[\sum_{x=1}^{m-1}(m-x) + \sum_{x=m+1}^{n}(x-m)\right]$

$$= \frac{2m^2 - 2m(n+1) + n^2 + n}{2n}$$

$$= \frac{n^2 - 1}{4n}$$

If n is even, that is we can write in general, n= 2m then

$$MD = \frac{1}{n} \sum_{x=1}^{n} | x - (m + \frac{1}{2}) |$$

$$= \frac{1}{n} \left[\sum_{x=1}^{m-1} ((m + \frac{1}{2}) - x) + \sum_{x=m+1}^{n} (x - (m + \frac{1}{2})) \right]$$

$$= \frac{n}{2} + \frac{m^2}{n} - m$$

$$= \frac{n}{4}$$

Hence, the complete solution is given by

$$M D = \frac{n^2 - 1}{4n} \quad \text{for n odd}$$

$$= \frac{n}{4} \quad \text{for n even}$$

8. Comment on skewness of discrete uniform distribution.

Answer Skewness

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = 0$$

Hence, the discrete uniform distribution is symmetric.

9. What is median?

Answer: Median of the distribution is the middle most value.

10. Comment on and kurtosis of discrete uniform distribution.

Answer Kurtosis

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{3(3n^2 - 7)}{5(n^2 - 1)}, \text{ which is less than 3}$$

Hence, the discrete uniform distribution has platikurtic curve.

11. Obtain mgf of discrete uniform distribution.

Answer: $M_x(t) = E(e^{tx})$

$$= \sum_{x=1}^{n} e^{tx} p(x)$$

$$= \frac{1}{n} \frac{e^t - e^{t(n+1)}}{1 - e^t}$$

$$= \frac{e^t (1 - e^{tn})}{n(1 - e^t)}$$

12. What is the other name of discrete uniform distribution?

Answer: The discrete uniform distribution is also known as the 'equally likely outcomes' distribution.

13. Write the equation of cumulative distribution function in general case.

Answer: For general case, cumulative distribution function is given by,

$$F(x) = \sum_{x=a}^{x} p(x) = \sum_{x=a}^{x} \frac{1}{n} = \frac{|x| - a + 1}{n}$$

14. Write the equation of general median for discrete uniform distribution.

Answer: In general, median for discrete uniform distribution is given by

$$\text{Median} = \frac{a+b}{2}$$

15. Highlight the application of discrete uniform distribution.

Answer: Discrete uniform distribution is used in solving 'German tank problem" occurred during world war II.