Summary

- In probability theory and statistics, the moment generating function of a random variable is an alternative specification of its probability distribution. Thus, it provides the basis of an alternative route to analytical results compared with working directly with probability density functions or cumulative distribution functions

- The moment generating function (mgf) of a random variable X (about origin) having the probability density function f(x) is given by, $M_X(t) = E(e^{tX}) = \int e^{tx} f(x)dx$

- Given the mgf we can also find moments by differentiating it with respect to t r times and then putting t=0

- In general, the moment generating function X about the point X=a is defined as $\mu_r' = E[(X-a)^r]$

- Moment generating function suffers from some drawbacks which have restricted its use in Statistics. Below are the some of the deficiencies of mgf, which we discussed in detail in the first semester.
  - A random variable X may have no moments although its mgf exists.
  - A random variable X can have mgf and some (or all) moments, yet the mgf does not generate the moments
  - A random variable X can have all or some moments but mgf does not exist except perhaps at one point

- Properties of mgf are,
  - $M_{cX}(t) = M_X(ct)$, c being a constant.
  - The moment generating function of the sum of the independent random variables is equal to the product of their respective moment generating functions. Symbolically, if X_1, X_2, X_3,…,X_n are independent random variables, then the moment generating function of their sum $X_1+X_2+X_3+\ldots+X_n$ is given by, $M_{X_1+X_2+\ldots+X_n}(t) = M_{X_1}(t).\ M_{X_2}(t) \ldots\ M_{X_n}(t)$
  - Effect of change of origin and scale on mgf – if U=(X-a)/h then $M_U(t) = e^{-at/h} M_U(t/h)$

- The moment generating function of a distribution, if it exists, uniquely determines the distribution

- Cumulant generating function of the distribution is obtained from moment generating function and is denoted by $K_X(t)$ and is given by, $K_X(t) = \log_e M_X(t)$