Frequently Asked Questions

1. State Bayes' Theorem (Rule for inverse probability).

Answer:
Let $E_1, E_2, E_3, \ldots, E_n$ be a set of n mutually exclusive events with $P(E_i) \neq 0$ $(i=1,2\ldots.n)$, then for any arbitrary event A which is a subset of $\bigcup_{i=1}^{n} Ei$ such that $P(A) > 0$ we have

$$P(Ei / A) = \frac{P(Ei)P(A / Ei)}{\sum P(Ei)P(A / Ei)}; \quad i = 1,2,\ldots,n$$

2. Prove Bayes' theorem for n mutually exclusive prior events $E_1, E_2, E_3, \ldots,$ E_n with $P(E_i) \neq 0$ ($i=1,2\ldots.n$) and for any arbitrary event A which is a subset of $\bigcup_{i=1}^{n} Ei$ such that $P(A) > 0$

Answer:
Since $A \subset \bigcup_{i=1}^{n} Ei$ $\qquad A = A \cap (\bigcup_{i=1}^{n} Ei)$

$$= \bigcup_{i=1}^{n}(A \cap Ei) \quad \text{(By distributive Law)}$$

Since $(A \cap Ei) \subset E_i$ ($i= 1,2,\ldots\ldots.,n$) are mutually exclusive events we have by addition theorem of Probability or Axiom 3 of Probability

$$P(A) = P(\bigcup_{i=1}^{n}(A \cap Ei)) = \sum_{i=1}^{n} P(A \cap Ei) = \sum_{i=1}^{n} P(Ei) P(A / Ei) \text{ ----------- } (*)$$

Also we have,

$$P(A \cap Ei) = P(A) P(Ei / A)$$

$$P(Ei / A) = \frac{P(A \cap Ei)}{P(A)} = \frac{P(Ei) P(A / Ei)}{\sum_{i=1}^{n} P(Ei) P(A / Ei)} \quad \text{from } (*)$$

3. Distinguish between conditional and posterior probability?

Answer:

Conditional probability asks "What is the probability of the sample or experimental result given the state value"? Whereas posterior probability asks "What is the probability of the event given the sample or experimental result"?

Posterior probabilities are always conditional probabilities, the conditional event being the sample information.

It may be pointed out that the classical theory of conditional probability is mainly empherical since it employs only sample information as the basis for estimation and testing while the posterior probability approach employs any and all available information whether it is personal judgment or empirical evidence.

4. State few applications of Bayes' Theorem

Answer:

- Bayes' Theorem can be used in Internet Marketing to track profile visitors to a Website. With the quantitative methods in the action, Internet marketing, explains how click-stream data can be used for this purpose.

- Bayes' Theorem can be used in decision analysis. The prior probabilities often are subjective estimates provided by a decision maker. Sample information is obtained and posterior probabilities are computed for use in developing a decision strategy

- Bayes' methods are applied to both virtual screening and the chemical biology arena, where applications range from bridging phenotypic and mechanistic space of drug action to the prediction of ligand-target interactions.
- Bayesian statistical analysis, is useful in business and political applications involving actual and opportunity costs where decisions often must be made under uncertain conditions. Bayesian theorem is particularly helpful in financial modelling. pricing problems in foreign exchange trading and to solve social policy-making problems

- Bayesian rule holds a similar promise for researchers of business and management problems.

5. Derive Bayes' theorem for any two priori events A1 and A2?

Answer:

Let A_1 and A_2 be equal to the set of events which are mutually exclusive (the two events cannot occur together) and exhaustive (the combination of two events is the entire experiment) and

B is equal to a simple event which intersects each of A events

Since $B \subset (A1 \cup A2))$ $\quad$ $B = B \cap (A1\ U\ A2)$

$B = (B \cap A1)\ U\ (B \cap A2)$

Hence $P(B) = P(A1 \cap B) + P(A2 \cap B)$ (by axiom 3) --------(1)

The probability of event A_1 given event B is

$$P(A_1 / B) = \frac{P(A1 \cap B)}{P(B)}$$

And similarly the probability of event A_2 given event B is

$$P(A_2 / B) = \frac{P(A2 \cap B)}{P(B)}$$

where

$$P(A_1 \cap B) = P(A_1)\, P(B / A_1) \quad \text{-----------------------(2)}$$

$$P(A_2 \cap B) = P(A_2) \,.\, P(B / A_2) \quad \text{----------------------(3)}$$

Hence

$$P(A1/B) = \frac{P(A1 \cap B)}{P(B)} = \frac{P(A1)P(B/A1)}{P(A1 \cap B) + P(A2 \cap B)} \text{ From equations (1) and (2)}$$

$$P(A_1 / B) = \frac{P(A1)P(B / A1)}{P(A1)P(B/A1)+P(A2)P(B/A2)} \text{ From equations (2) and (3)}$$

$$P(A2/B) = \frac{P(A2 \cap B)}{P(B)} = \frac{P(A2)P(B/A2)}{P(A1 \cap B) + P(A2 \cap B)} \text{ From equations (1) and (3)}$$

$$P(A2 / B) = \frac{P(A2)P(B/A2)}{P(A1)P(B/A1)+P(A2)P(B/A2)} \text{ From equations (2) and (3)}$$

6. Write a note on the tabular approach for the computation of Posterior probabilities using Bayes' rule

Answer:

The Tabular Approach is helpful in conducting the Bayes' theorem calculations simultaneously for all events Ei. The computation involves the following steps

Step 1: Prepare three columns

Column 1: The mutually exclusive events for which posterior probabilities are desired

Column 2: The prior probabilities for the events

Column 3: The conditional probabilities of the new information, given each event

Step 2: In column 4, compute joint probabilities for each event and new information A by using the multiplication law. To get these joint probabilities multiply the prior probabilities in column 2 by corresponding conditional probabilities in column 3.

That is

$$P(E1 \cap A) = P(E1)P(A/E1)$$

Step 3: Sum of the joint probabilities in column 4 to obtain the probability of the new information, P(A).

Step 4: In column 5 compute the posterior probabilities by using the basic relationship of conditional probability.

(1)	(2)	(3)	(4)	(5)
Events E_i	Prior Probabilities $P(E_i)$	Conditional probabilities P(A / Ei)	Joint Probabilities $P(Ei \cap A)$	Posterior Probabilities P(Ei / A)
E1	P(E1)	P(A/E1)	P(E1) P(A/E1)	P(E1/A)
E2				
E3				

7. Suppose that in answering a question on a multiple choice test, an examinee knows the answer with probability p or he guesses with probability 1-p.

Assume that the probability of answering a question correctly is unity for an examinee who knows the answer and 1/m for the examinee who guesses where m is the number of multiple choice alternatives. Compute the probability that an examinee knows the answer to a problem given that he has correctly answered it.

Answer:

Let,

E1: An Examinee knows the answer

E2: An Examinee guesses the answer

A : An Examinee answers correctly

Then, P(E1) = p; P(E2) =1- p; P(A/E1) = 1; P(A/E2) =1/m

Using Bayes' Theorem

$$P(E_1 / A) = \frac{P(E1)P(A/E1)}{\sum P(Ei)P(A/Ei)}; \quad i = 1 \text{ to } 2$$

$$= \frac{P(E1)P(A/E1)}{P(E1)P(A/E1) + P(E2)P(A/E2)}$$

$$= \frac{p..1}{p.1+(1-p)1/m}$$

$$= \frac{mp}{1+(m-1)p}$$

8. An oil company purchased an option on land in Alaska. Preliminary geologic studies assigned the following priori probabilities.
   P(high- quality oil) = 0.50
   P(medium-–quality oil) = 0.20
   P(no oil) = 0.30
   After 200 ft of drilling on the first well a soil test is made. The probabilities of finding a particular type of soil identified by the test are
   P(soil/high quality oil) = 0.20
   P(soil/medium quality oil) = 0.80
   P(soil/ no oil) = 0.20

   How should the firm interpret the soil test? What are the revised probabilities and what is the new probability of finding oil?

Answer:

Let, E1: High quality oil
E2: Medium quality oil
E3: No oil

A: Finding a particular type of soil

Then P (E1) = 0.50; P(E2) =0.20; P(E3) = 0.30
P (A/E1) = 0.20; P(A/E2) = 0.80; P(A/E3) = 0.20

Using Bayes' Theorem

$$P(E_1 / A) = \frac{P(E1)P(A/E1)}{\sum P(Ei)P(A/Ei)}; \quad i= 1 \text{ to } 3$$

$$= \frac{P(E1)P(A/E1)}{P(E1)P(A/E1)+P(E2)P(A/E2)+P(E3)P(A/E3)}$$

$$= \frac{(0.50)(0.20)}{(0.50)(0.20)+(0.20)(0.80)+(0.30)(0.20)}$$

$$= 0.3125$$

$$P(E2 / A) = \frac{P(E2)P(A/E2)}{\sum P(Ei)P(A/Ei)}; \quad i= 1 \text{ to } 3$$

$$= \frac{P(E2)P(A/E2)}{P(E1)P(A/E1)+P(E2)P(A/E2)+P(E3)P(A/E3)}$$

$$= \frac{(0.20)(0.80)}{(0.50)(0.20)+(0.20)(0.80)+(0.30)(0.20)}$$

$$= 0.5$$

$$P(E3 / A) = \frac{P(E3)P(A/E3)}{\sum P(Ei)P(A/Ei)}; \quad i= 1 \text{ to } 3$$

$$= \frac{P(E3)P(A/E3)}{P(E1)P(A/E1)+P(E2)P(A/E2)+P(E3)P(A/E3)}$$

$$= \frac{(0.30)(0.20)}{(0.50)(0.20)+(0.20)(0.80)+(0.30)(0.20)}$$

$$= 0.1875$$

From the posterior probability P (Ei/A), i=1,2,3 we have P(E2/A) = 0.5 which is highest . Hence if a particular type of soil is identified, the probability of having medium quality oil is 0.5.

The revised probabilities when the particular type of soil is identified are

P (high- quality oil) = 0.3125
P (medium-–quality oil) = 0.5
P (no oil) = 0.1875

9. The Wayne manufacturing company purchases a certain part from suppliers A, B and C.

Supplier A supplies 60% of the parts, B 30% and C 10%. The quality of parts varies among the suppliers with A, B and C parts having:

0.25%, 1% and 2% defective rates respectively. The parts are used in one of the company's major products

a) What is the probability that the company's major product is assembled with a defective part?
b) When a defective part is found which supplier is the likely source?

Answer

Let, E1: Parts are supplied by Supplier A

E2: Parts are supplied by Supplier B

E3: Parts are supplied by Supplier C

A: Event of obtaining a defective part supplied either by Suppliers A,or B or C respectively

Then P(E1) = 0.60; P(E2) =0.30; P(E3) = 0.10
P(A/E1) = 0.0025; P(A/E2) = 0.01; P(A/E3) = 0.02

The tabular Approach for Bayes' rule

(1)	(2)	(3)	(4)	(5)
Events E_i	Prior Probabilities $P(E_i)$	Conditional probabilities P(A / Ei)	Joint Probabilities $P(Ei \cap A)$ P(Ei) P(A/Ei)	Posterior Probabilities P(Ei / A)
E1	0.60	0.0025	0.001500	0.2308
E2	0.30	0.01	0.003000	0.4615
E3	0.10	0.02	0.002000	0.3077
		Total	0.006500	

a)Probability that the company's major product is assembled with a defective part is 0.0065

b)A supplier who is likely to supply a defective part is Supplier B

10. Small cars get better gas mileage, but they are not as safe as bigger cars. Small cars accounted for 18% of the vehicles on the road, but accidents involving small cars led to 11,898 fatalities during a recent year . Assume the probability a small car is involved in an accident is 0.18.The probability of an accident involving a small car leading to a fatality is 0.128 and the probability of an accident not involving a small car leading to a fatality is 0.05. Suppose you learn of an accident involving a fatality. What is the probability a small car was involved?

Answer:

Let E1: Small cars involved in an accident

E2: Vehicles other than small cars involved in an accident

A : Accident leading to a fatality

Then $P(E1) = 0.18$; $P(E2) = 0.82$;

$P(A/E1) = 0.128$; $P(A/E2) = 0.05$

The tabular Approach for Bayes' rule

(1)	(2)	(3)	(4)	(5)
Events E_i	Prior Probabilities $P(E_i)$	Conditional probabilities P(A / Ei)	Joint Probabilities $P(Ei \cap A)$ P(Ei) P(A/Ei)	Posterior Probabilities P(Ei / A)

E1	0.18	0.128	0.023040	0.36
E2	0.82	0.05	0.041000	0.64
		Total	0.0640	

From the posterior probability column we have P(E1/A) = 0.36. So if an accident leads to fatality the probability a small car was involved is 0.36.

11. Distinguish between Priori probability and Posterior probability

Answer:

Probabilities before revision by Bayes' rule are called a priori or simply prior probability because they are determined before the sample information is taken into account. A probability which has undergone revision in the light of sample information (using Bayes' rule) is called posterior probability. Since it represents the probability calculated after this information is taken into account.

Posterior probabilities are also called revised probabilities because they are obtained by revising the prior probabilities in the light of the additional information gained. Posterior probabilities are always conditional probabilities, the conditional event being the sample information. Thus a prior probability which is unconditional becomes a posterior probability which is a conditional probability by using Bayes' rule.

Hence the probabilities P(E1), P(E2),....,P(En) which are already given or known before conducting an experiment are termed as prior probabilities. The conditional probabilities P(E1/A), P(E2/A),....,P(En/A) which are computed after conducting the experiment i.e., occurrence of A are termed as posterior probability

12. There are three candidates for the post of Principal of a college. They are Mr.Bhat, Mr.D'Souza and Mr. Ibrahim. Their chances of appointment as

Principal are in the ratio 4:2:3. The probabilities of the three candidates introducing Computer science in the college respectively are 0.3,0.5 and 0.8. If later it is found that computer science is introduced in the college, who might be the principal?

Answer:

Let, E1: Mr. Bhat gets appointed as a principal
E2: Mr. 'Souza gets appointed as a principal
E3: Mr. Ibrahim gets appointed as a principal

A: Introduction of Computer Science in the college

Then, P (E1) = 4/9 = 0.44; P(E2) = 2/9 = 0.22; P(E3) =3/9= 0.33
P (A/E1) = 0.30; P(A/E2) = 0.50; P(A/E3) = 0.80

Using Bayes' Theorem

$$P(E_1 / A) = \frac{P(E1)P(A/E1)}{\sum P(Ei)P(A/Ei)}; \quad i = 1 \text{ to } 3$$

$$= \frac{P(E1)P(A/E1)}{P(E1)P(A/E1) + P(E2)P(A/E2) + P(E3)P(A/E3)}$$

$$= \frac{(0.44)(0.30)}{(0.44)(0.30)+(0.22)(0.50)+(0.33)(0.80)}$$

$$= 0.2609$$

$$P(E2 / A) = \frac{P(E2)P(A/E2)}{\sum P(Ei)P(A/Ei)}; \quad i = 1 \text{ to } 3$$

$$= \frac{P(E2)P(A/E2)}{P(E1)P(A/E1) + P(E2)P(A/E2) + P(E3)P(A/E3)}$$

$$= \frac{(0.22)(0.50)}{(0.44)(0.30)+(0.22)(0.50)+(0.33)(0.80)}$$

$$= 0.2174$$

$$P(E3/A) = \frac{P(E3)P(A/E3)}{\sum P(Ei)P(A/Ei)}; \quad i = 1 \text{ to } 3$$

$$= \frac{P(E3)P(A/E3)}{P(E1)P(A/E1) + P(E2)P(A/E2) + P(E3)P(A/E3)}$$

$$= \frac{(0.33)(0.80)}{(0.44)(0.30)+(0.22)(0.50)+(0.33)(0.80)}$$

$$= 0.521739$$

Since P (E3 / A) is greater than P(E1 / A) and P(E2 / A), Mr. Ibrahim has the maximum chance of introducing computer science in the college and hence Mr, Ibrahim might be the Principal of the college.

13. Martin Coleman, credit manager for Beck's knows that the company uses three methods to encourage collection of delinquent accounts. From past collection records he learns that 70% of the accounts are called on personally, 20% are phoned and 10% are sent a letter. The probabilities of collecting an overdue amount from an account with the three methods are 0.75, 0.60 and 0.65 respectively. Mr. Coleman has just received a payment from a past-due account. What is the probability that this account

a) Was called on personally?
b) Received a phone call?
c) Received a letter?

Answer:

Let, E1: Accounts are called on personally
E2: Received a phone call
E3: Received a letter

A: Collection of an overdue amount

Then P(E1) = 0.70; P(E2) = 0.20; P(E3) =0.10

$P(A/E1) = 0.75; \quad P(A/E2) = 0.60; \quad P(A/E3) = 0.65$

Using Bayes' Theorem

$$P(E_1 / A) = \frac{P(E1)P(A / E1)}{\sum P(Ei)P(A / Ei)}; \quad i= 1 \text{ to } 3$$

$$= \frac{P(E1)P(A / E1)}{P(E1)P(A/E1)+ P(E2)P(A/E2)+P(E3)P(A/E3)}$$

$$= \frac{(0.70)(0.75)}{(0.70)(0.75)+(0.20)(0.60)+(0.10)(0.65)}$$

$$= 0.7394$$

$$P(E2 / A) = \frac{P(E2)P(A/E2)}{\sum P(Ei)P(A / Ei)}; \quad i= 1 \text{ to } 3$$

$$= \frac{P(E2)P(A/E2)}{P(E1)P(A/E1)+ P(E2)P(A/E2)+P(E3)P(A/E3)}$$

$$= \frac{(0.20)(0.60)}{(0.70)(0.75)+(0.20)(0.60)+(0.10)(0.65)}$$

$$= 0.1690$$

$$P(E3 / A) = \frac{P(E3)P(A/E3)}{\sum P(Ei)P(A / Ei)}; \quad i= 1 \text{ to } 3$$

$$= \frac{P(E3)P(A/E3)}{P(E1)P(A/E1)+ P(E2)P(A/E2)+P(E3)P(A/E3)}$$

$$= \frac{(0.10)(0.65)}{(0.70)(0.75)+(0.20)(0.60)+(0.10)(0.65)}$$

$$= 0.09155$$

Probability that the account was called personally is 0.7394
Probability that the account was called through a phone call is 0.1690

Probability that the account was called through a letter is 0.09155

14. State Bayes' theorem for another future event C along with an arbitrary event A.

Answer:
The probability of the materialization of another event C given by

P(C/ A ∩ $E1$), P(C/ A ∩ $E2$), P(C/ A ∩ $E3$), ………., P(C/ A ∩ En) is

$$P(C / A) = \frac{\sum_{i=1}^{n} P(Ei)\, P(A/Ei)\, P(C/A \cap Ei)}{\sum_{i=1}^{n} P(Ei)\, P(A / Ei)}$$

15. What are the strengths and weaknesses of Bayes' theorem?

Strengths:

- Very useful when dealing with statistical data
- Can be applied to qualitative data
- Output is more estimative than frequentist statistics output, thus more easily understood
- Instinctive Bayesian Approach can be used to influence analyst recommendations by understanding where they fall between gullible and stubborn and vacillating, indecisive and overly cautious.
- Can be applied to various fields including law enforcement, business and healthcare
- When used, Bayes takes into account analysts' biases

Weaknesses:

- It is difficult to learn and explain to others
- It requires a good understanding of statistics and probability
- It can be time consuming
- It can be confusing and counter-intuitive
- It requires a computer application or software to implement on complex problems for it to be accurate

- Can be hard to communicate it to the decision makers