Frequently Asked Questions

1. Define Negative Binomial distribution?

Answer: A random variable x is said to follow a Negative Binomial distribution with parameters r and p, if its probability mass function is given by,

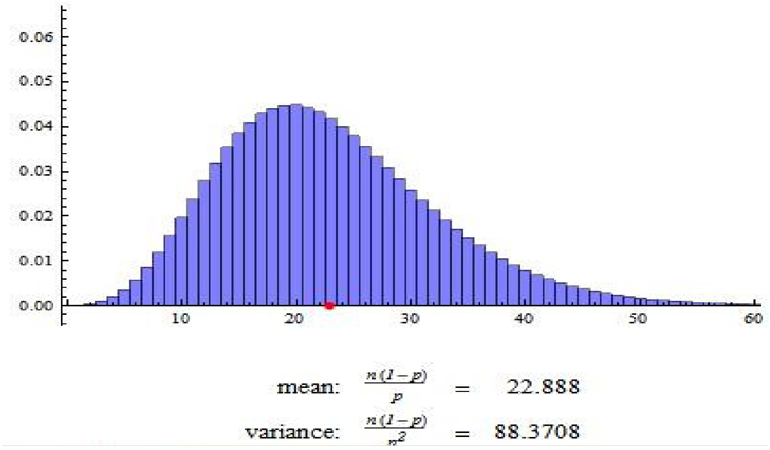
$$p(x)= \binom{-r}{x} p^r(-q)^x; x=0,1,2,\ldots\ldots$$

0,otherwise

And we write X~NB(r,p).

2. Draw the graph of probability mass function of Negative Binomial distribution.

Answer: The graph of probability mass function for n is 8 and p is 0.26 is,



3. What are the conditions to be satisfied for a random variable to have Negative Binomial distribution?

Answer:

(i) The trials are independent

(ii) The probability of success 'p' in a trial remains constant from trial to trial.

4. Give examples of Negative Binomial distribution

Answer:

1. In the game craps, you decide to play until you lose 5 games.
2. A health-related researcher is studying the number of hospital visits in past 12 months by senior citizens in a community based on the characteristics of the individuals and the types of health plans under which each one is covered.

3. School administrators study the attendance behavior of high school juniors at two schools. Predictors of the number of days of absence include the type of program in which the student is enrolled and a standardized test in mathematics.

5. What is the relationship between mean and variance of Negative Binomial distribution?

Answer: For Negative Binomial distribution mean is less than that of variance.

6. Derive moment generating function of Negative Binomial distribution.

Answer:
$M_X(t) = E(e^{tX})$

$$= \sum_{x=0}^{\infty} e^{tx} p(x)$$

$$= \sum_{x=0}^{\infty} e^{tx} \binom{-r}{x} Q^{-r} \left(-\frac{P}{Q}\right)^x$$

$$= \sum_{x=0}^{\infty} \binom{-r}{x} Q^{-r} \left(-\frac{Pe^t}{Q}\right)^x$$

$= (Q-Pe^t)^{-r}$

7. Derive characteristic generating function of Negative Binomial distribution.

Answer:
$\Phi_x(t) = E(e^{itx})$

$$= \sum_{x=0}^{\infty} e^{itx} p(x)$$

$$= \sum_{x=0}^{\infty} e^{itx} \binom{-r}{x} Q^{-r} \left(-\frac{P}{Q}\right)^x$$

$$= \sum_{x=0}^{\infty} \binom{-r}{x} Q^{-r} \left(-\frac{Pe^{it}}{Q}\right)^x$$

$= (Q-Pe^{it})^{-r}$

8. Find mean of Negative Binomial distribution using mgf.

Answer:

$$\mu_1' = \left(\frac{d}{dt} M_X(t)\right)_{t=0}$$

$$= [-r(-Pe^t)(Q-Pe^t)^{r-1}]_{t=0}$$

$= rP$

9. Obtain variance of geometric distribution using mgf.

Answer:

$$\mu_1' = \left(\frac{d}{dt}M_X(t)\right)_{t=0}$$

$$= [-r(-Pe^t)(Q-Pe^t)^{r-1}]_{t=0}$$

$$= rP$$

$$\mu_2' = \left(\frac{d^2}{dt^2}M_X(t)\right)_{t=0}$$

$$= [rPe^t(Q-Pe^t)^{r-1} + (-r-1)rPe^t(Q-Pe^t)^{r-2}(-Pe^t)]_{t=0}$$

$$= rP + r(r+1)P^2$$

$$\mu_2 = \mu_2' - \mu_1'^2 = r(r+1)P^2 + rP - r^2P^2$$

$$= rPQ$$

10. Derive Cumulants generating function of Negative Binomial distribution.

Answer:

Cumulants generating function (cgf) is obtained by taking logarithm of mgf and is denoted by $K_X(t)$.

ie., $K_x(t) = \log M_x(t)$

$\log(Q-Pe^t)^{-r}$

$$= -r\log(Q-Pe^t)$$

$$= -r\log\left[Q - P(1+t+\frac{t^2}{2!}+\frac{t^3}{3!}!+\frac{t^4}{4!}+\ldots)\right]$$

$$= -r\log\left[1 - P(t+\frac{t^2}{2!}+\frac{t^3}{3!}!+\frac{t^4}{4!}+\ldots)\right]$$

11. Write first four Cumulants of Negative Binomial distribution.

Answer:

$K_1 = \mu_1' = rP$

$K_2 = \mu_2 = rPQ$

$K_3 = \mu_3 = rP(1+3P+2P^2)$

$=rP(1+p)(1+2P)=rPQ(Q+P)$

$K_4 = rP(1+P)(1+6P+6P^2)$

$=rPQ(1+6PQ)$

12. Explain the nature of the Negative Binomial distribution.

Answer: Coefficient of skewness is given by,

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{(1+q)^2}{rq}$$

Hence the Negative Binomial distribution is always a skewed distribution
Coefficient of Kurtosis is given by,

$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{p^2 + 3q(r+2)}{rq}$, which is greater than 3.

Hence Negative Binomial distribution has leptokurtic curve.

13. Obtain recurrence relation for central moments of Negative Binomial distribution.

Answer

Let X is a Negative Binomial variate with probability mass function

$$p(x) = \binom{x+r-1}{x} p^r q^x, x = 0,1,2,...$$

Mean = rq/p

μ_k = E[X-(rq/p)^k]

$$\mu_k = \sum_{x=0}^{\infty} \left(x - \frac{rq}{p} \right)^k p(x)$$

$$= \sum_{x=0}^{\infty} \left(x - \frac{rq}{p} \right)^k \binom{x+r-1}{x} p^r q^x$$

Differentiating w.r.t. q, we get,

$$\frac{d\mu_r}{dq} = \sum_x \left[k\left(x - \frac{rq}{p} \right)^{k-1} \times \left\{ \frac{d}{dq}\left(x - \frac{rq}{p} \right) \right\} \binom{r+x-1}{x} q^x p^r \right] + \sum_x \left[\left(x - \frac{rq}{p} \right)^k \binom{r+x-1}{x} \left\{ xq^{x-1} p^r + q^x .rp^{r-1} . \frac{dp}{dq} \right\} \right]$$

But $\frac{dp}{dq} = \frac{d}{dq}(1-q) = -1$

And $\frac{d}{dq}\left[x - \frac{rq}{p} \right] = \frac{d}{dq}\left[x - r\left(\frac{1}{p} - 1 \right) \right] = \frac{r}{p^2} . \frac{dp}{dq} = -\frac{r}{p^2}$

Therefore

$$\frac{d\mu_k}{dq} = -\frac{kr}{p^2} \sum_x \left(x - \frac{rq}{p} \right)^{k-1} .p(x) + \sum_x \left(x - \frac{rq}{p} \right)^k \binom{r+x-1}{x} .q^{x-1} p^r \left(x - \frac{rq}{p} \right)$$

$$= \frac{kr}{p^2} \mu_{k-1} + \frac{1}{q} \sum_x \left(x - \frac{rq}{p} \right)^{k+1} p(x)$$

$$= \frac{kr}{p^2} \mu_{k-1} + \frac{1}{q} \mu_{k+1}$$

$$\Rightarrow \mu_{k+1} = q\left[\frac{d\mu_k}{dq} + \frac{kr}{p^2} \mu_{k-1} \right]; k = 1,2,3...$$

14. Find the limiting distribution of Negative Binomial distribution.

Answer:

Negative Binomial distribution tends to Poisson distribution as P tends to 0 and r tends to ∞ such that rP = λ(finite). Proceeding to the limits, we get,

$$\lim p(x) = \lim \binom{x+r-1}{r-1} p^r q^x$$

$$= \lim \binom{x+r-1}{x} Q^{-r} \left(\frac{P}{Q}\right)^x$$

$$= \lim_{r \to \infty} \frac{(x+r-1)(x+r-2)...(r+1)r}{x!} (1+P)^{-r} \left(\frac{P}{1+P}\right)^x$$

$$= \lim_{r \to \infty} \left[\frac{1}{x!} \left(1+\frac{x-1}{r}\right)\left(1+\frac{x-2}{r}\right)...\left(1+\frac{1}{r}\right).1.r^x (1+P)^{-r} \left(\frac{P}{1+P}\right)^x \right]$$

$$= \frac{1}{x!} \lim_{r \to \infty} (1+P)^{-r} \left(\frac{rP}{1+P}\right)^x$$

$$= \frac{\lambda^x}{x!} \lim_{r \to \infty} \left[1+\frac{\lambda}{r}\right]^{-r} \lim_{r \to \infty} \left[1+\frac{\lambda}{r}\right]^{-x}$$

$= \frac{\lambda^x}{x!}.e^{-\lambda}.1 = \frac{e^{-\lambda}\lambda^x}{x!}$, the pmf of Poisson distribution with parameter λ

15. State and prove additive property of Negative Binomial distribution.

Answer:
Statement: Let the two independent random variables X_1 and X_2 have the same geometric distribution. Show that the conditional distribution of X_1 given $X_1 + X_2 = n$ is discrete uniform.

Proof
Consider the mgf of ΣX_i

$$M_{\sum X_i}(t) = \prod M_{X_i}(t)$$

$$= \prod p^{r_i}(1-qe^t)^{-r_i}$$

$$= p^{r_1}(1-qe^t)^{-r_1} p^{r_2}(1-qe^t)^{-r_2} ... p^{r_n}(1-qe^t)^{-r_n}$$

$= p^{\sum r_i}(1-qe^t)^{-\sum r_i}$, which is the mgf of Negative Binomial distribution. Hence by uniqueness theorem of mgf., ΣX_i has Negative Binomial distribution with parameters Σr_i and p

Characteristic generating function is given by
$\Phi_x(t) = E(e^{itx})$

$$= \sum_{x=0}^{\infty} e^{itx} p(x)$$

$$= \sum_{x=0}^{\infty} ei^{tx} pq^x$$

$$= p\sum_{x=0}^{\infty} (e^{it} q)^x$$

$= p(1-qe^{it})^{-1}$
$= p/(1-qe^{it})$