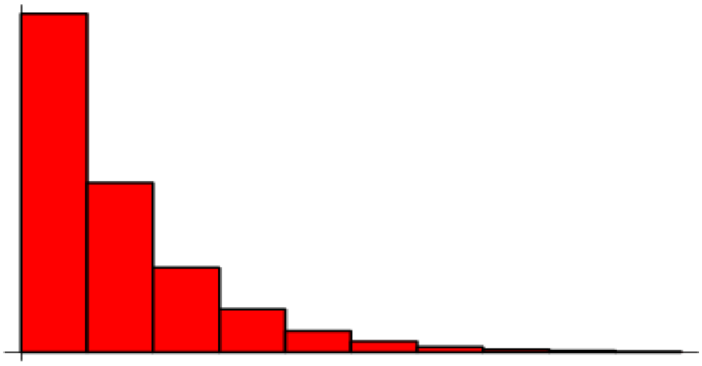
<u>**Frequently Asked Questions**</u>

1. Define geometric distribution.

Answer: A random variable x is said to follow a geometric distribution with parameter p, if its probability mass function is given by,
$p(x) = q^x p$; $x=0, 1, 2, \ldots,\ 0<p\leq 1$
$\quad 0$, otherwise
And we write $X \sim Geometric(p)$

2. Draw the graph of probability mass function of geometric distribution.

Answer:



3. What are the conditions to be satisfied for a random variable to have geometric distribution?

Answer:
1. An experiment consists of repeating trials until first success
2. Each trial has two possible outcomes
    - A success with probability p
    - A failure with probability q = 1-p
3. Repeated Trials are independent

4. Give examples of geometric distribution.

Answer:
1. Tossing a coin repeatedly until head occurs.
2. Observing single births until a girl.
3. Roll a die until it results in number 5.
4. Products are inspected until first defective is encountered.
5. If 1 percent of the bits transmitted through a digital transmission are received in error, then bits are transmitted until the first error.
6. If terminals on an on-line computer system are attached to a communication line to the central computer system, then the number of terminals polled the first ready terminal is located.

5. What is the relationship between mean and variance of geometric distribution?

Answer: For geometric distribution mean is less than that of variance.

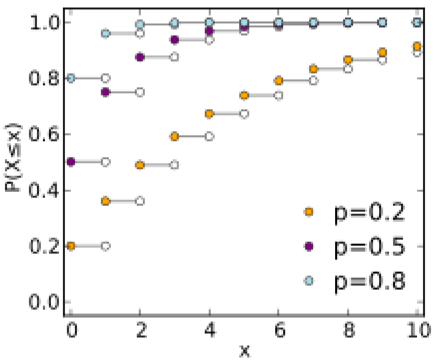
6. Find distribution function of geometric distribution.

Answer:

$$F(x) = \sum_{x=0}^{x} pq^x$$
$$= p(1+q+q^2+q^3+\ldots+q^x)$$
$$= p(1-q^{x+1})/(1-q)$$
$$=1-q^{x+1}$$

7. Draw the graph of cumulative distribution function of geometric distribution for different values of p.

Answer:



8. Find mean of geometric distribution.

Answer:

$$\mu_1' = E(X)$$
$$= \Sigma x.p(x)$$
$$= \sum_{x=0}^{\infty} x.pq^x$$
$$= pq\sum_{x=0}^{\infty} x.q^{x-1}$$
$$= pq(1-q)^{-2}$$
$$= q/p$$

9. Obtain variance of geometric distribution.

Answer:

$\mu_1' = E(X)$

$= \Sigma x.p(x)$

$= \sum_{x=0}^{\infty} x.pq^x$

$= pq\sum_{x=0}^{\infty} x.q^{x-1}$

$= pq(1-q)^{-2}$

$= q/p$

$\mu_2' = E(X^2)$

$= E[X(X-1)+X]$

$= \Sigma[X(X-1)+X].p(x)$

$= \sum_{x=0}^{\infty} [x(x-1)+x].pq^x$

$= 2pq^2\sum_{x=1}^{\infty}\left[\frac{x(x-1)}{2\times 1}\right].q^{x-2}$ $+q/p$

$= 2pq^2(1-q)^{-3} + q/p$

$= 2q^2/p^2 + q/p$

Variance is given by,

$\mu_2 = \mu_2' - \mu_1'^2 = 2q^2/p^2 + q/p - (q/p)^2$

$= q/p^2$

10. Explain the nature of the geometric distribution.

Answer: Coefficient of skewness is given by,

$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{2-p}{q}$

Hence the geometric distribution is always a skewed distribution

Coefficient of Kurtosis is given by,

$\beta_2 = \frac{\mu_4}{\mu_2^2} = 9 + \frac{p^2}{q}$, which is greater than 3.

Hence geometric distribution has leptokurtic curve.

11. Derive moment generating function of geometric distribution.

Answer:

$M_X(t) = E(e^{tX})$

$= \sum_{x=0}^{\infty} e^{tx} p(x)$

$= \sum_{x=0}^{\infty} e^{tx} pq^x$

$$= p\sum_{x=0}^{\infty}(e^t q)^x$$

$= p(1-qe^t)^{-1}$

$= p/(1-qe^t)$

12. State and prove Morkov property of geometric distribution.

Answer

The geometric distribution is said to lack memory in a certain sense. Suppose an event E can occur at one of the times t=0,1,2,… and the occurrence (waiting) time X has geometric distribution. Thus

$P(X=t)=q^t p$; t=0,1,2…

Suppose we know that the event E has not occurred before k, ie., X≥k. Let Y=X-k. Thus Y is the amount of additional time needed for E to occur. We can show that

$P(Y=t/X\geq k)=P(X=t)=pq^t$

which implies that the additional time to wait has the same distribution as initial time to wait.

Since the distribution does not depend upon k, it, in a sense 'lacks memory' of how much we shifted the time origin. If 'B' were waiting for the event E and is relieved by 'C' immediately before time k, then the waiting time distribution of 'C' is the same as that of 'B'.

Proof

We have

$$P(X\geq r)=\sum_{s=r}^{\infty} pq^s$$

$$= p(q^r+q^{r+1}+\ldots)$$

$$= \frac{pq^r}{1-q}$$

$$= q^r$$

$$P(Y\geq t/X\geq k) = \frac{P(Y\geq t \cap X\geq k)}{P(X\geq k)}$$

$$= \frac{P(X-k\geq t \cap X\geq k)}{P(X\geq k)}$$

$$= \frac{P(X\geq k+t)}{P(X\geq k)}$$

$$= \frac{q^{t+k}}{q^k} = q^t$$

Therefore $P(Y=t/X\geq k)= P(Y\geq t/X\geq k)- P(Y\geq t+1/X\geq k)$

$= q^t - q^{t+1} = q^t(1-q) = q^t p=P(X=t)$

Hence the proof.

13. Derive Cumulant generating function of geometric distribution.

Answer:
Cumulants generating function (cgf) is obtained by taking logarithm of mgf and is denoted by $K_X(t)$.

ie., $K_x(t) = \log M_x(t)$
$= \log(p/(1-qe^t)$
$= \log p - \log(1-qe^t)$
$= \log p - [-qe^t - (qe^t)^2/2 - (qe^t)^3/3 - (qe^t)^4/4 - \ldots]$

$$= \log p + \left[\begin{array}{l} q(1+t+\frac{t^2}{2!}+\frac{t^3}{3!}!+\frac{t^4}{4!}+\ldots)+\frac{q^2}{2}(1+t+\frac{t^2}{2!}+\frac{t^3}{3!}!+\frac{t^4}{4!}+\ldots) \\ \frac{q^3}{3}(1+t+\frac{t^2}{2!}+\frac{t^3}{3!}!+\frac{t^4}{4!}+\ldots)+\frac{q^4}{4}(1+t+\frac{t^2}{2!}+\frac{t^3}{3!}!+\frac{t^4}{4!}+\ldots)+\ldots \end{array} \right]$$

14. Derive Characteristic generating function of geometric distribution.

Answer:
Characteristic generating function is given by
$\Phi_x(t) = E(e^{itx})$

$$= \sum_{x=0}^{\infty} e^{itx} p(x)$$

$$= \sum_{x=0}^{\infty} ei^{tx} pq^x$$

$$= p\sum_{x=0}^{\infty} (e^{it}q)^x$$

$= p(1-qe^{it})^{-1}$
$= p/(1-qe^{it})$

15. Let the two independent random variables X_1 and X_2 have the same geometric distribution. Show that the conditional distribution of X_1 given $X_1 + X_2 = n$ is discrete uniform.

Answer:

$P(X_1=k) = P(X_2=k)=pq^k$; $k=0,1,2,\ldots$

$$P[X_1=r/(X_1+X_2=n)] = \frac{P(X_1 = r \cap X_1 + X_2 = n)}{P(X_1 + X_2 = n)}$$

$$= \frac{P(X_1 = r \cap X_2 = n-r)}{P(X_1 + X_2 = n)}$$

$$= \frac{P(X_1 = r \cap X_2 = n-r)}{\sum_{s=0}^{n} \left[P(X_1 = s) \cap (X_2 = n-s) \right]}$$

$$= \frac{P(X_1 = r).P(X_2 = n-r)}{\sum_{s=0}^{n}\left[P(X_1 = s).P(X_2 = n-s)\right]}$$ (since x1 and x2 are independent)

Therefore

$$P[X_1=r/(X_1+X_2=n)] = \frac{pq^r.pq^{n-r}}{\sum_{s=0}^{n}\left[pq^s.pq^{n-s}\right]}$$

$$= \frac{p^2q^n}{\sum_{s=0}^{n}\left[p^2q^n\right]}$$

$$= \frac{p^2q^n}{(n+1)p^2q^n}$$

$$= \frac{1}{(n+1)}$$

and r=0, 1, 2,…n, which is the probability mass function of discrete uniform distribution. Hence the conditional distribution of X_1 given $X_1 + X_2 = n$ is discrete uniform.