Frequently Asked Questions

1. What do you mean by a probability?

Answer:
When the event is not certain to take place or in other words there is an uncertainty about happening of the events in question, we try to present conditions under which we can make sensible numerical statements about uncertainty. Probability is a numerical measure of the likelihood that an event will occur. If probabilities were available we could determine the likelihood of each event occurring. The probability of a given event is an expression of likelihood or chance of occurrence of an event. Hence probability is nothing but a numerical measure associated with the occurrence or non occurrence of an event.

2. Write a note on Exhaustive events.

Answer:
The total number of possible outcomes of a random experiment is called exhaustive cases for the experiment
Example:
i) In a toss of a single coin there are two exhaustive cases {H, T}
If two coins are tossed the exhaustive cases are $2^2 = 4$: { HH, HT, TT, TH}
Similarly for 3 coins $2^3 = 8$ cases
Therefore for n coins it is 2^n
ii) Exhaustive cases for a toss of a single die $= 6^1 = 6$
Exhaustive cases for a toss of a 2 dice $= 6^2 = 36$
Exhaustive cases for a toss of three dice $= 6^3 = 216$
iii) In drawing two cards from a pack of cards the exhaustive number of cases is $52_{c2.}$

3. Define equally likely events.

Answer:
The outcomes of a random experiment is said to be equally likely if after taking into consideration all relevant evidence, none of them can be expected in preference to another.
For Example:
i) If a coin is unbiased the two outcomes head and tail are equally likely
ii) Drawing of 2 cards from a pack of fifty-two cards, the outcomes are equally likely

4. How the probability is an important tool in all disciplines?

Answer:
Although historically, the probability theory originated from the games of chance played by tossing of coins, throwing dice, drawing cards etc, and its importance has enormously increased in recent years. Today the notion of probability finds important applications in almost all disciplines – Physics, Chemistry, Biology, Psychology, Education, Economics, Business, industry, engineering etc. For e.g.: Business decisions are often based on an analysis of uncertainties such as the following:

What are the chances that sales will decrease if we increase the prices?
What is the likelihood that a new assembly method will increase productivity?
How likely is it that the project will be completed on time?

What are the odds in favor of a new investment being profitable?
Probability is important in decision making because it provides the way to measure, express and analyze the uncertainties associated with future events

5. Can probability take any possible value? Explain

Answer:
Probability values are always assigned on a scale between zero and 1. A probability near zero indicates that an event is unlikely to occur; a probability near 1 indicates that an event is almost sure to occur. Other probabilities between 0 and 1 represent varying degrees of likelihood that an event will occur. Probability cannot exceed the value 1 and it cannot be less than 0.

6. What are the two basic requirements of probability that has to be satisfied in assigning probabilities to experimental outcomes?

Answer:
In assigning probabilities to experimental outcomes 2 basic requirements of probability must be satisfied.
The probability values assigned to each experimental outcome (sample point) must be between 0 and 1. If we let E_i denote the i^{th} experimental outcome and $P(E_i)$ indicate the probability of this experimental outcome, we must have $0 \leq P(E_i) \leq 1$ for all i.

The sum of all of the experimental outcome probabilities must be 1. For example if a sample space has k experimental outcomes, we must have $P(E_1) + P(E_2) + \ldots + P(E_k) = 1$

Any method of assigning probability values to the experimental outcomes that satisfies these two requirements and results in reasonable numerical measures of the likelihood of outcomes is acceptable.

7. State the classical approach for defining a probability.

Answer:
The classical approach of probability, given by a French mathematician Laplace and generally adopted by disciples of the classical school runs as follows: Probability is the ratio of number of favourable cases to the total number of equally likely cases.

If an experiment results in n mutually exclusive, equally likely and exhaustive outcomes and m of them are favourable to an event A and if the probability of occurrence of A is denoted by P (A) then by classical approach we have P (A)= Number of favourable cases / Exhaustive number of equally likely cases = m/n

8. What are the requirements for the computation of probability using classical definition of probability?

Answer:
For calculating probability using a classical approach we have to find out two things
Number of favourable cases
Total number of equally likely and exclusive cases

9. Using classical definition of probability find the probability of non-occurrence of an event.

Answer:

If an event A can happen in 'm' ways out of a total of 'n' equally likely and mutually exclusive ways then the probability of the occurrence of the event is denoted by p=P(A)= m/n. Then (n-m) of the outcomes are favourable for the non-occurrence of an event A. The probability of the non-occurrence of the event is given by

q= P (A^c) or P(A') = (n-m)/n

= m'/n =1-m/n = 1-P(A)=1-p

P (A) +P (A^c) =1

Therefore P (A^c) = 1- P (A)

10. What are the limitations of classical approach to probability?

Answer:

The classical definition of probability suffers from certain limitations. The definition cannot be applied whenever it is not possible to make a simple enumeration of equally likely and mutually exclusive cases. For example: How does it apply to probability of rain? We might think that there are two possibilities 'rain' or 'no rain'. But at any given time it will not usually be agreed that they are equally likely.

The phrase "equally likely" appearing in the classical definition is synonymous with "equally probable". How do we know whether the probability is equal before we can measure them? If a person jumps from the top of Qutab Minar the probability of his survival will not be 50 percent since survival and death, the two mutually exclusive outcomes are not equally likely. The definition has only limited applications in coin tossing, die throwing and similar games of chance.

The classical approach also fails to answer the questions like

i) What is the probability that a male will die before the age 60? Or

ii) A bulb will burn less than two thousand hours? Etc.

The definition fails when the number of possible outcomes is infinitely large.

11. Give the relative frequency approach for the probability.

Answer:

If an event occurs m times out of n, its relative frequency is m/n; the value which is approached by m/n when n becomes infinity is called the limit of the relative frequency. In the relative frequency definition, which is also known as an empirical definition, the probability is the value which is approached by m/n when n becomes infinity.

Symbolically

$$P(A)= \lim_{n \to \infty} \frac{m}{n}$$

12. Differentiate between classical approach and relative frequency approach of probability?

Answer:

The two approaches classical and relative frequency or empirical though seemingly same differ widely. In the former P (A) and m/n were practically equal when n was large whereas in the latter we say that P (A) is the limit m/n as n tends to infinity.

In the second approach thus the probability itself is the limit of the relative frequency as the number of observations increases indefinitely.

The probability obtained by following relative frequency definition is called a posterior or empirical probability as distinguished from a priori probability obtained by the classical approach.

13. Briefly explain Priori and empirical probability.

Answer:

A clear distinction between a priori and an empirical probability is quite important for the proper understanding of the concept of probability. Priori probability is normally encountered in problems dealing with games of chance- for example: Dice and cards. Normally we think of a priori probability as being deductive in nature i.e., from cause to effect and based on theory instead of evidence of experience and experimentation.

Probability derived from the past experience is called an empirical probability and is used in many practical problems. The classic example being the preparation of Insurance mortality tables which obviously are based on past experience. But in the analysis of most of the practical business problems these two kinds of probability concepts are widely used.

A priori probability may be determined rather easily whenever a complete enumeration is available on the various ways an event may occur. For example: the probability of drawing a King from a deck of 52 cards is 4/52 or 1/13. Similarly the probability of getting 3 in a single throw of a dice is 1/6.

14. Explain the limitations of Relative Frequency Approach.

Answer:

The relative frequency approach though useful in practice has difficulties from the mathematical point of view. Since an actual limiting number may not really exist. Quite often people use this approach without evaluating a sufficient number of outcomes. For example: Mr. Kohli pointed out that his father and mother (aged 75 and 70) both had a serious heart problem in Jan-Feb 1999 and hence in winter people above 70 have high probability of heart attack. His friends took it seriously and started to give special attention to their parents. On deep thinking we may find that there is not enough evidence of establishing a relative frequency of occurrence of probability.

It may be observed that the empirical probability can never be obtained and one can only attempt at a close estimate of P (A) by taking n sufficiently large. For this reason modern probability theory has been developed axiomatically in which probability is an undefined concept much the same as point and line are undefined in Geometry.

15. What is the chance that that a leap year selected at random will contain 53 Sundays?

Answer:
In a leap year (which consists of 366 day) there are 52 complete weeks and 2 days over. The following are the possible combinations for these two over days
i) Sunday and Monday
ii) Monday and Tuesday
iii) Tuesday and Wednesday
iv) Wednesday and Thursday
v) Thursday and Friday
vi) Friday and Saturday and
vii) Saturday and Sunday
In order that a leap year selected at random should contain 53 Sundays one of the two over days must be Sunday. Since out of the above 7 possibilities, 2 viz., (i) and (vii) are favorable to this event
Required probability = 2/7