1. Introduction

Welcome to the series of E-learning modules on Practical. Here given the situation, we try to define the random variable and identify the distribution of that random variable and find the probability of the event. We also fit a distribution to the given frequency distribution.

By the end of this session, you will be able to:

- Find the probabilities of an event after identifying the distribution
- Fitting a given distribution for the given frequency distribution when parameters are known or unknown

A computer has 4 disks, numbered 0, 1, 2, and 3, one of which is chosen at random on boot to store temporary files. Find the probability that an odd-numbered disk is chosen.

Let x denote the number on the chosen disk.
Since the desks are numbered serially 0, 1, 2 and 3, all the disks have equal chance of being selected. Hence X is a discrete uniform distribution with n is equal to 4.
Therefore, the probability mass function is given by P of x is equal to 1 by n, x take values zero, one, 2 and 3.

Since disk is numbered from zero to 3, we have two odd numbers 1 and 3.
Therefore, probability that odd numbered disk is chosen
Is equal to p of x is equal to 1 plus p of x is equal to 3
Is equal to 1 by 4 plus 1 by 4 is equal to half.

The incidence of an occupational disease in an industry is such the workers have 25% chance of suffering from it. What is the probability that out of 5 workers, at the most 2 contract the disease?

Let X denotes the number of workers contracting the disease among 5 workers.
Then X is a binomial variate with parameters n is equal to 5 and p is equal to 25 percent is equal to zero point two 5.

The probability mass function is p of x is equal to n c x into p power x into q power n minus x, x take values zero, one, etc, n.
That is p of x is equal to 5 c x into zero point 2 five power x into zero point 7 five power 5 minus x.

The probability that at most 2 workers contract the disease is,
P of x less than or equal two
Is equal to p of zero plus p of 1 plus p of 2
Is equal to 5 c zero into zero point 2 five power zero into zero point 7 five power 5 plus 5 c one into zero point 2 five power one into zero point 7 five power 4 plus 5 c two into zero point 2 five power two into zero point 7 five power 3
Is equal to zero point 2 three 7 three plus zero point 3 nine 5 five plus zero point 2 six 3 seven.

Is equal to zero point 8 nine 6 five.

Seven unbiased coins are thrown 128 times. In how many throws would you find at least 3 heads?

Let X denote the number of heads obtained in a throw of 7 coins and hence x is binomial variate with parameters n is equal to 7 and p is equal to half.
Hence probability mass function, p of x is equal to n c x p power x into q power n minus x, where x take values zero, one, etc, n
That is p of x is equal to 7 c x into half power x into half power 7 minus x.

Here, the frequency of tosses with at least 3 heads is required to be found. Hence, first we find the probability of getting at least 3 heads and then we multiply this probability by the total frequency.

Hence probability that at least 3 heads appears
is equal to 1 minus probability that less than 3 heads appear
is equal to 1 minus p of zero plus p of 1 plus p of 2
substituting the values of x in p of x and simplifying, we get,
1 minus, 1 by 128 plus 7 by 128 plus 21 by 128
Is equal to 1 minus 29 by 128
Is equal to 99 by 128

Therefore, the number of tosses resulting in at least 3 heads is given by,
N into probability that at least 3 heads appear
Is equal to 128 into 99 by 128 is equal to 99.

2. Illustrations 4 - 6

In a binomial distribution, with parameters n is equal 7 and p, the first and second terms are zero point zero 8 two 4 and zero point 2 four 7 one respectively. Find p.
The probability mass function of binomial distribution is given by,
P of x is equal to 7 c x into p power x into q power 7 minus x where x take values zero one etc seven.
Here p of zero is equal to 7 c zero into p power zero into q power 7 is equal to zero point zero 8 two 4 and
p of one is equal to 7 c one into p power one into q power 6 is equal to zero point 2 four 7 one.

Therefore, by taking the ratio of above probabilities, we get
P of zero by p of one is equal to q power seven divided by 7 into p into q power 6 is equal to q by 7 into p is equal to zero point zero 8 two 4 by zero point 2 four 7 one.
Implies 1 minus p by 7 p is equal to zero point 3 three 3 five
That is 1 minus p is equal to 2 point 3 three 4 five into p
Implies p is equal to 1 by 3 point 3 three 4 five is equal to zero point 2 nine 9 nine.
Thus, probability mass function is given by, p of x is equal to 7 c x into zero point 2 nine 9 nine power x into zero point 7 zero zero 1 power 7 minus x, where x take values zero, one etc 7.

Five coins are tossed 3200 times. Find the frequencies of the distribution of number of heads and tabulate the result.

Let X is the number of heads obtained in a toss of the five fair coins. Then x is binomial variate with n is equal to 5 and p is equal to half.
The probability mass function is
P of x is equal to 5 c x into half power n where x take values zero one etc 5.

The frequency function is
E x is equal to N into p of x
Is equal to 3200 into 5 c x into half power n

Now, by substituting the values of x as zero 1, 2 etc. 5, we get expected frequencies E not, E one, E two etc E 5

Hence, the theoretical distribution of number of heads is given in the following table.

Figure 1

No. of heads	No. of tosses
0	100
1	500
2	1000
3	1000
4	500
5	100

A survey of 100 families each having 5 children revealed the following distribution

Fit a binomial distribution for the given data.

Let X denotes the number of male children in a family having 5 children. Here n is equal to 5. The parameter p has to be estimated from the given data. For this, the mean of the data is equated with the theoretical mean. The mean of the data is
X bar is equal to summation f into x by N,
By substituting f, x and N, and simplifying, we get 2
Thus n into p is equal to 2 implies, p is equal to 2 by n
Substituting n is equal to 5, we get p is equal to zero point 4 and q is equal to 1 minus zero point 4 is equal to zero point 6.
Thus probability mass function is
P of x is equal to 5 c x into zero point 4 power x into zero point 6 power 5 minus x, where x take values zero one etc 5.

First expected frequency is given by,
E not is equal to N into p of zero is equal to 100 into zero point six power 5 is equal to seven point 7 seven 6.
Now, instead of using the direct probability mass function, we can use recurrence relation to find the expected frequency.
E x is equal to n minus x plus 1 divided by x into p by q into e x minus 1
That is E x is equal to 5 minus x plus one divided by x into zero point 4 by zero point 6 into E x minus 1 is equal to 6 minus x divided by x into 2 by 3 into E x minus 1
Putting x is equal to 1 2, etc. 5, we get E 1, E 2 etc E 5. We write observed and theoretical distribution in a tabular form. Expected frequencies are approximated such a way that the total comes to 100.

Figure 2

No. of male Children	No. of families	Theoretical frequency	
		Actual	Approximated
0	9	7.776	8
1	24	25.92	26
2	35	34.56	34
3	24	23.04	23
4	6	7.68	8
5	2	1.024	1
Total	100		100

3. Illustrations 7 – 8

On an average, a typist makes 3 mistakes while typing one page. What is the probability that a randomly observed page is free of mistakes? Among 200 pages, in how many pages would you expect mistakes?

Let X denotes the number of mistakes in a page.
Then x is a Poisson variate with parameter lambda is equal to 3.
The probability mass function is given by,
P of x is equal to e power minus lambda into lambda power x divided by x factorial
Is equal to e power minus 3 into 3 power x divided by x factorial, where x take values zero one two etc infinity.

Probability that a page is free of mistakes is equal to p of zero
Is equal to e power minus 3 into 3 power zero divided by zero factorial
Is equal to e power minus 3 is equal to zero point zero 4 nine 8.

To find number of pages among 200 pages, expected mistakes, first we find probability that a page has mistake.
Probability that a page has mistake is equal to 1 minus probability that a page has no mistakes
Is equal to 1 minus zero point zero 4 nine 8.
Is equal to zero point 9 five zero 2.
Therefore, among 200 pages, the expected number of pages containing mistakes is N into probability that a page has mistakes
Is equal to 200 into zero point 9 five zero 2 is equal to 190.

2 percent of the fuses manufactured by a firm are expected to be defective. Find the probability that a box containing 200 fuses contains 3 or more defective fuses.
2 percent of the fuses are defective. Therefore, probability that a fuse is defective is zero point zero 2.
Let X denotes the number of defective fuses in the box of 200 fuses. Then X is a binomial variate with parameters n is equal to 200 and p is equal to zero point zero 2.
Here p is very small and n is very large. Therefore X can be treated as Poisson variate with parameter lambda is equal to n into p is equal to 200 into zero point zero two is equal to 4.

The probability mass function of the distribution is given by,
P of x is equal to e power minus lambda, into lambda power x divided by x factorial
Is equal to e power minus 4 into 4 power x by x factorial where x take values zero one two etc., infinity.
Now let us find the probability that a box has 3 or more defective fuses
Is equal to 1 minus probability that a box has less than 3 defective fuses
Is equal to 1 minus p of zero plus p of 1 plus p of 2
Is equal to e power minus four into 1 plus 4 plus 8
Is equal to 1 minus zero point 2 three 7 nine
Is equal to zero point seven six 2 one

4. Illustrations 9 - 10

For the following data regarding births occurring in a hospital, fit a Poisson distribution.

Let X denotes the number of births occurring in a hospital. Then X is a Poisson variate. The parameter is, lambda is equal to x bar is equal to summation f into x by N

By substituting x, f and N as sum of frequencies and simplifying, we get
1 point 2.
The probability mass function is e power minus 1 point 2 into 1 point 2 power x by x factorial, where x take values zero 1, 2 etc infinity.
The frequency function is given by,
E x is equal to 50 into e power minus 1 point 2 into 1 point 2 power x by x factorial where x take values zero one two etc.
E not is equal to e power minus 1 point 2 is equal to 15 point zero six.
The theoretical frequencies are obtained by using the following recurrence relation
E x is equal to lambda by x into e x minus 1 is equal to 1 point 2 divided by x into e x minus 1

Hence, by substituting x is equal to 1 2 etc, we get E 1 E 2 etc and we write these in the frequency table as follows.

Figure 3

No. of births	No. of days	Theoretical frequency	
		Actual	Approximated
0	22	15.06	15
1	13	18.07	18
2	5	10.84	11
3	5	4.336	5
4	3	1.301	1
5	2	0.3122	0
Total	50		50

The theoretical frequencies are approximated in such a way that the total comes to 50.

If the probability that a target is destroyed on any one shot is zero point 5, what is the probability that it would be destroyed on 6^{th} attempt?

Let x denote the number of failures before the first success, which is destroying the target in this problem.
Therefore x follows geometric distribution with parameter p is equal to zero point 5 and the target is destroyed on sixth attempt that is there are 5 failures.

The probability mass function of the distribution is given by,
P of x is equal to q power x into p where x take values zero one two etc and p lies between zero and one
We have p is equal to zero point 5 and q is equal to 1 minus p is equal to zero point 5
Probability that target is destroyed on 6^{th} attempt), that is five failures
Is equal to p of x is equal to 5
Is equal to zero point 5 power 5 into zero point five
Is equal to zero point 5 power 6

5. Illustrations 11 - 12

An item is produced in large numbers. The machine is known to produce 5 percent defectives. A quality control inspector is examining the items by taking them at random. What is the probability that at least 4 items are to be examined in order to get 2 defectives?

Let X denote the number of defective items produced by the machine and the quality control inspector need to get 2 defective, for which he need to examine at least 4 items. Hence X has negative binomial distribution.

The probability mass function is given by
P of x is equal to x plus r minus 1 c r minus 1 p power r into q power x, where x take values zero 1, 2, etc and p lies between zero and 1.
If 2 defectives are to be obtained, it can happen in 2 or more trials. The probability of success is zero point zero five for every trial. Hence required probability is
P of x is equal to 4 plus p of x is equal to five plus etc
Is equal to 1 minus p of x is equal to 2 plus p of x is equal to 3
Is equal to zero point zero five square plus 2 into zero point zero five square into zero point nine five.
Is equal to zero point 9 nine 5.

A taxi cab company has 12 Ambassadors and 8 Fiats. If 5 of these taxi cabs are in the shop for repairs and Ambassador is as likely to be in for repairs as a fiat, what is the probability that at least 3 of them are Ambassadors?

Let X denote the number of ambassadors come for repair. Hence X has Hypergeometric distribution with parameters, N=12 plus 8 is equal to 20, M is equal to 12 and n is equal to 5.

Therefore probability mass function is given by,
P of x is equal to M c x into N minus M c n minus x divided by N c n, where x take values zero one etc minimum of M and n
Is equal to 12 c x into 8 c 5 minus x divided by 20 c 5.

Therefore the required probability is given by
Probability that there are at least 3 ambassadors
Is equal to p of x is equal to 3 plus p of x is equal to 4 plus p of x is equal to 5
By substituting value of x in p of x and simplifying, we get,
Zero point 3 nine 7 three plus zero point 2 five 5 four plus zero point zero 5 one 1
Is equal to zero point 7 zero 3 eight

Here's a summary of our learning in this session:

- Finding the probabilities of an event after identifying the distribution
- Fitting a given distribution for the given frequency distribution when parameters are known or unknown