Frequently Asked Questions

1. A random variable X has the following probability function

X :	-2	-1	0	1	2	3
p(x):	0.1	k	0.2	2k	0.3	k

Find the value of k, expectation of X and variance.

Answer:

Since given is probability function, $\Sigma p(x)=1$
Therefore
0.1+k+0.2+2k+0.3+k=1
4k+0.6=1
k=(1-0.6)/4=0.1
Hence the probability function can be written as,

x :	-2	-1	0	1	2	3
p(x):	0.1	0.1	0.2	0.2	0.3	0.1

Expectation of X is given by,
$E(X)=\Sigma x.p(x)$
=-2x0.1+(-1x0.1)+0x0.2+1x0.2+2x0.3+3x0.1
=-0.2-0.1+0+0.2+0.6+0.3=0.8
Variance is given by
$V(X)=E(X^2) – [E(X)]^2$
Let us find $E(X^2) = \Sigma x^2\, p(x)$
$=(-2)^2 x0.1+(-1)^2 x0.1+0^2\, x0.2+$
$1^2\, x0.2+2^2\, x0.3+3^2\, x0.1$
= 2.8
Therefore $V(X) = 2.8 – (0.8)^2$
= 2.16

2. The following is the distribution function of a discrete random variable X.

x :	-3	-1	0	1	2	3	4	5
p(x):	0.1	0.2	0.15	0.05	0.25	0.15	0.05	0.05

Find P(X=even), $P(1\le X\le 8)$

Answer:
In the problem we have 2 even numbers 2 and 4.
P(X=even)=P(X=2 or 4)
= 0.25 + 0.05 = 0.3
$P(1\le X\le 8)$=P(X=1,2,3,4,5)
= 0.05+0.25+0.15+0.05+0.05
=0.55

3. A bag has 5 pink and 4 yellow marbles. A boy draws two marbles at random. Find the expectation of drawing yellow marbles.

Answer:
Let X denote the number of yellow marbles drawn. Since 2 marbles are drawn, X can take values 0, 1 and 2 and the respective probabilities can be found as follows.
$P(X=0) = {}^5C_2 / {}^9C_2$ =5/18
$P(X=1) = 5x4/ {}^9C_2$ =5/9

$P(X=2) = {}^4C_2 / {}^9C_2 = 1/6$

$E(X)=\Sigma x.p(x)$

$= 0x\ 5/18 + 1 \times 5/9 + 2 \times 1/6$

$=8/9$

4. Ramesh asks his son suresh to open the front door of his house by handing over the key bunch consisting of 5 keys identical in size. Suresh does not know which key opens the door and so he selects a key at random without replacement. Ramesh tells Suresh "you will receive 20 rupees less thrice the number of attempts you required to open the door. Find the expected sum received by Suresh.

Answer:

Let X be the number of attempts made by Suresh to open the door. Then values of X are 1, 2, 3, 4 and 5 and the respective probabilities are,

$P(X=1) = 1/5$

$P(X=2) = (4/5).(1/4)=1/5$

$P(X=3) = (4/5).(3/4).(1/3) = 1/5$

Similarly, $P(X=4)=P(X=5)=1/5$

Hence $E(X)=\Sigma x.p(x)$

$=1x1/5+2x1/5+3x1/5+4x1/5+5x1/5$

$= 3$

It is given that the sum received by Suresh is 20 rupees less than thrice the number of attempts made by him to open the door

ie., $20 – 3X$

Therefore expected sum received by Suresh = $E(20 – 3X)$

$= 20 – 3E(X)$

$= 20 – 3x3$

=11 Rs.

5. A bag has 2 one-Rupee coins and 3 ten-Paise coins. A boy picks a coin at random from the bag. What is the expectation of the amount he has picked?

Answer:

Let X denote the amount that he has picked. Then X take values 100(Paise) and 10 (Paise)with respective probabilities 2/5 and 3/5. Therefore

$E(X)=\Sigma x.\ p(x) = 100x2/5+10x3/5=46$ Paise.

6. A person, by paying Rs.5 enters into a game of shooting a target. With one shot if he hits the target, he gets Rs.25 otherwise, he gets nothing. If his probability of hitting the target is 1/7 find his expected net loss.

Answer:

Let X denote the net amount (after deducting entrance fee) that the person receives.

Then X takes values -5 and 20 with probabilities

$p(x=-5)$=P(he does not hit)= 6/7

$p(x=20)$=P(he hits) = 1/7

Therefore his expected net amount is

$E(X)=\Sigma x.\ p(x)$

$= (-5)x6/7 + 20x1/7$

= -1.43
= Rs.1.43(loss)

7. An importer is offered a shipment of pearls for Rs.15000 and the probabilities that he will be able to sell them for Rs.17000, Rs.16000, Rs.15000, or Rs.14000 are respectively 0.20, 0.49, 0.24 and 0.07. If he buys pearls, what is his expected gross profit.

Answer:

Let X denote the gross profit of an importer after deducting the purchase price of Rs.15000. hence X takes values

2000 with probability 0.20, 1000 with probability 0.49, zero with probability 0.24 and -1000 with probability 0.07

Hence $E(X) = \Sigma x.P(X) = 0.20x2000+0.49x1000+0.24x0+(-1000x0.07)$ = Rs.820

8. In a lottery, there are 1000 tickets costing Re.1 each. There is one first prize worth Rs.100, two second prizes worth Rs.20 each and ten third prizes worth Rs.10 each. Find the expected loss in buying one ticket.

Answer:

Let X denote the amount (after deducting the purchase cost) that one lottery ticket fetches. Then X takes values 99, 19, 9 and -1 with respective probabilities

p(99)=P[I prize] = 1/1000=0.001

P(19)=P[II prize] = 2/1000=0.002

P(9)=P[III prize] = 10/1000=0.01

p(-1)=P[no prize] = 987/1000=0.987

Thus, expectation of the net amount is

$E(X)=\Sigma x.\ p(x)$

$=99x0.001 + 19x0.002 + 9x0.01$

$+ (-1)x0.987$

= -0.76

= Rs.0.76 (loss)

9. Determine the quartile deviation from the following frequency distribution for grouped measurement of weight of bullocks. Also find its coefficient.

Weight (lbs)	No. of bullocks	Weight (lbs)	No. of bullocks

850-900	2	1100-1150	140
900-950	24	1150-1200	66
950-1000	45	1200-1250	42
1000-1050	120	1250-1300	20
1050-1100	110	1300-1350	15

Answer:

Weight (lbs)	No. of bullocks	Cum. Freq.
850-900	2	2
900-950	24	26
950-1000	45	71
1000-1050	120	191
1050-1100	110	301
1100-1150	140	441
1150-1200	66	507
1200-1250	42	549
1250-1300	20	569
1300-1350	15	584

$$QD = \frac{Q_3 - Q_1}{2}$$

Quartile deviation is given by

$$Q_1 = L + \frac{N/4 - m}{f} \times h$$

Where

N/4=584/4=146th item which lies in the 1000-1050 class. Hence

$$Q_1 = 1000 + \frac{146 - 71}{120} \times 50 = 1031.25$$

3.N/4=3.584/4=438th item which lies in the 1100-1150 class.

$$Q_3 = L + \frac{3N/4 - m}{f} \times h = 1100 + \frac{438 - 301}{140} \times 50 = 1148.93$$

$$QD = \frac{Q_3 - Q_1}{2} = \frac{1148.93 - 1031.25}{2} = 58.84$$

Coefficient of quartile deviation is given by

$$\frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{1148.93 - 1031.25}{1148.93 + 1031.25} = 0.035$$

10. An analysis of production rejects resulted in the following figures. Compute Karl Pearson's coefficient of skewness and comment.

No. of rejects per operator	No. of operators	No. of rejects per operator	No. of operators
21-25	5	41-45	15
26-30	15	46-50	12
31-35	28	51-55	3
36-40	42		

Answer:
Consider the following table

No. of rejects per operator	No. of operators (f)	Mid value X	u= (X-A)/h	fu	fu^2
20.5-25.5	5	23	-3	-15	45
25.5-30.5	15	28	-2	-30	60
30.5-35.5	28(f_1)	33	-1	-28	28
35.5-40.5	42(f)	38	0	0	0
40.5-45.5	15(f_2)	43	1	15	15
45.5-50.5	12	48	2	24	48
50.5-55.5	3	53	3	9	27
Total	100			-25	223

$$S_K = \frac{\overline{X} - Z}{\sigma}$$

Karl Pearson's coefficient of skewness is given by,
Hence first we find , Z and σ from the given data.

$$\overline{X} = A + \frac{\Sigma fu}{N} \times h = 38 + \frac{(-25)}{100} \times 5 = 36.75$$

Mean

$$Z = L + \frac{f - f_1}{2f - f_1 - f_2} \times h = 35.5 + \frac{42 - 28}{2 \times 42 - 28 - 15} \times 5 = 37.207$$

Mode,

$$\sigma = h\sqrt{\frac{\Sigma fu^2}{N} - \left(\frac{\Sigma fu}{N}\right)^2} = 5\sqrt{\frac{223}{100} - \left(\frac{-25}{100}\right)^2} = 5 \times \sqrt{2.23 - 0.0625} = 7.36$$

Standard deviation,
Hence Karl Pearson's coefficient of skewness is given by,

$$S_K = \frac{\overline{X} - Z}{\sigma} = \frac{36.75 - 37.207}{7.36} = 0.0621$$

Since S_k is negative, the given distribution is negatively skewed.

11. For a distribution, Bowley's coefficient of skewness is -0.56, Q_1=16.4 and median = 24.2. What is the coefficient of Quartile Deviation?

Answer:
Here we are given, S_K=-0.56, Q_1=16.4 and M = 24.2.
Substituting the given values in Bowley's coefficient of skewness

$$S_K = \frac{Q_3 + Q_1 - 2M}{Q_3 - Q_1}$$

$$-0.56 = \frac{Q_3 + 16.4 - 2 \times 24.2}{Q_3 - 16.4}$$

Implies

Implies, $Q_3 = 26.4$

Therefore coefficient of quartile deviation is given by

$$QD = \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{26.4 - 16.4}{26.4 + 16.4} = 0.035$$

12. The coefficient of skewness for a certain distribution based on the quartiles is -0.8. If the sum of the upper and lower quartiles is 100.7 and median is 55.35, find the distribution on the basis of the upper and lower quartile.

Answer:

Bowley's coefficient of skewness is given by,

$$S_K = \frac{Q_3 + Q_1 - 2M}{Q_3 - Q_1}$$

$$-0.8 = \frac{100.7 - 2 \times 24.2}{Q_3 - Q_1}$$

Implies

Implies, Q_3-Q_1=12.5

Also Q_3+Q_1=100.7

By solving two equations we get,

Q_1=44.1 and Q_3=56.6

13. Following figures relate to the size of capital of companies. Find the coefficient of skewness with the help of Bowley's measure of skewness.

Capita (in lakhs of Rs.)	No. of companies	Capita (in lakhs of Rs.)	No. of companies
1-5	20	21-25	48
6-10	27	26-30	53
11-15	29	31-35	70
16-20	38		

Answer: Consider the following table

Capita (in lakhs of Rs.)	No. of companies	Cumulative Frequency
0.5-5.5	20	20
5.5-10.5	27	47
10.5-15.5	29	76
15.5-20.5	38	114
20.5-25.5	48	162
25.5-30.5	53	215
30.5-35.5	70	285

$$S_K = \frac{Q_3 + Q_1 - 2M}{Q_3 - Q_1}$$

Bowley's Coefficient of skewness is given by,

where Q_1 Q_3 and M are calculated as follows.

Here N = 285. N/4=285/4=71.25. Hence 1st quartile class is 10.5-15.5. Therefore

$$Q_1 = L + \frac{N/4 - m}{f} \times h = 10.5 + \frac{71.25 - 47}{29} \times 5 = 14.68$$

3.N/4 = 213.75. Hence third quartile class is, 25.5-30.5. Therefore

$$Q_3 = L + \frac{3N/4 - m}{f} \times h = 25.5 + \frac{213.75 - 162}{53} \times 5 = 30.382$$

To find Median, N/2 = 142.5. Hence median class is 20.5-25.5. Therefore

$$M = L + \frac{N/2 - m}{f} \times h = 20.5 + \frac{142.5 - 114}{48} \times 5 = 23.47$$

Hence Bowley's coefficient of skewness is,

$$S_K = \frac{Q_3 + Q_1 - 2M}{Q_3 - Q_1} = \frac{30.382 + 14.68 - 2 \times 23.47}{30.382 - 14.68} = -0.12$$

14. The first four moments of a distribution about the value 5 of the variable are 2, 20, 40 and 50. Find the mean variance and β_2.

Answer:

With usual notation we are given, A=5, μ_1' =2, μ_2' =20, μ_3' = 40 and μ_4' = 50

Mean = μ_1' +A =2+5=7

$\mu^2 = \mu^{2\prime} - (\mu^{1\prime})^2$ =20-4=16

$\mu^3 = \mu^{3\prime} - 3\,\mu^{1\prime}\mu^{2\prime} + 2\,(\mu^{1\prime})^3$

=40-3.20.2+2.8=-64

$\mu^4 = \mu^{4\prime} - 4\,\mu^{1\prime}\mu^{3\prime} + 6(\mu^{1\prime})^2\,\mu^{2\prime} - 3(\mu^{1\prime})^4$

= 50-4.40.2+6.20.4-3.16=162

$$\beta_2 = \frac{\mu_4}{\mu_2^{\,2}} = \frac{162}{(16)^2} = 0.63$$

, which is less than 3. Hence the distribution has platikurtic curve.

.

15. The first 4 moment of a distribution about the value 5 are 7, 70, 140 and 75. Calculate β_1 and β_2. Comment on the nature of the distribution.

Answer:

Let μ'_r be the r^th order raw moment about the value 5 and μ_r be the r^th order central moment. Given

μ_1' =7, μ_2' =70, μ_3' = 140 and μ_4' = 175.

$\mu^2 = \mu^{2\prime} - (\mu^{1\prime})^2$ =70-49 = 21

$\mu^3 = \mu^{3\prime} - 3\,\mu^{1\prime}\mu^{2\prime} + 2\,(\mu^{1\prime})^3$

$=140-3.7.70+2.7^3 = -644$

$\mu^4 = \mu^{4\prime} - 4\,\mu^{1\prime}\mu^{3\prime} + 6(\mu^{1\prime})^2\,\mu^{2\prime} - 3(\mu^{1\prime})^4$

$= 175 - 4.7.140 + 6.7^2 .70 - 3.7^4 = 9632$

Therefore

$$\gamma_1 = \sqrt{\beta_1} = \frac{\mu_3}{\mu_2^{3/2}} = \frac{-644}{(21)^{3/2}} = -6.692 \qquad \beta_2 = \frac{\mu_4}{\mu_2^{2}} = \frac{9632}{(21)^2} = 21.8431$$

and

Observe that Υ_1 is negative and β_2 is greater than 3, the distribution is negatively skewed an has leptokurtic curve