1. Introduction

Welcome to the series of e-Learning modules on Practical. Here we can verify the nature, behaviour and properties of the distribution by sketching the probability distributions.

By the end of this session, you will be able to:

- Describe how to draw the probability mass function of different distributions
- Identify the nature, behaviour and properties of the distribution

We have given that if x follows binomial distribution with parameters 5 and p, then sketch the curve for different values of the parameter p of this distribution and comment.

The probability mass function of the binomial distribution is given by,
P of x is equal to n c x into p power x into q power n minus x where x take values zero one two etc. n and p plus q is equal to 1
When n is equal to 5, p of x is equal to 5 c x into p power x into q power 5 minus x where x takes value zero, one etc. five.

Now we will find the probabilities of binomial distribution for different values of p namely, zero point 1, zero point two five, zero point 5 and zero point seven 5.

For example if p is equal to zero point 1 then
p of zero is equal to 5 c zero, into zero point one power zero into zero point nine power 5 minus zero is equal to zero point 5 nine.

Similarly, we calculate the value of p for different values of x as well as p and tabulate them as follows.

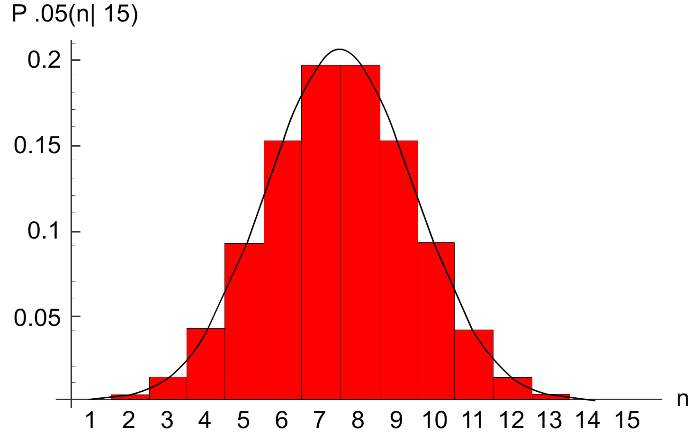
Figure 1

X	p = 0.1	p = 0.25	p = 0.5	P = 0.75
0	0.59	0.24	0.03	0.00
1	0.33	0.40	0.16	0.01
2	0.07	0.26	0.31	0.09
3	0.01	0.09	0.31	0.26
4	0.00	0.01	0.16	0.40
5	0.00	0.00	0.03	0.24

The actual graph of the binomial distribution is given as follows.

That is, since x takes discrete values, we have a histogram.

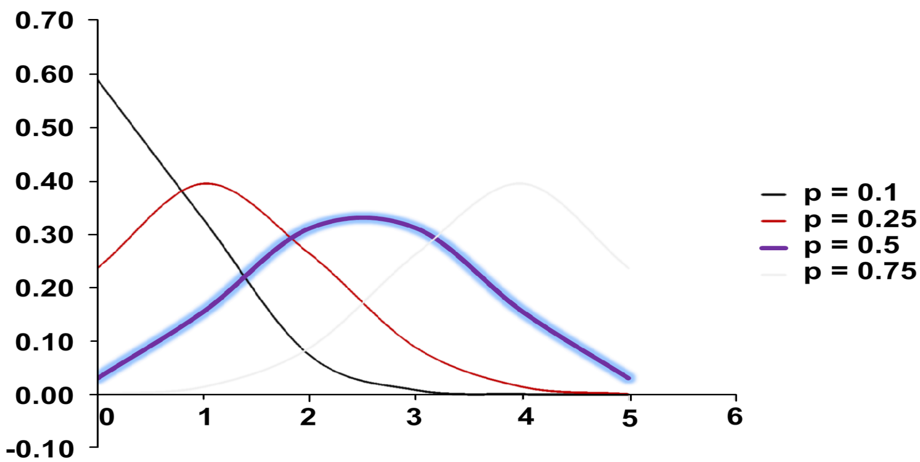
Figure 2



But for the sake of comparison, we join the top points and draw a smooth curve.

Now let us draw the smooth curve by joining the different points of p of x.

Figure 3



The black line shows the curve of the probability mass function when p is equal to zero point 1
The red line shows the curve of the probability mass function when p is equal to zero point two 5
The purple line shows the curve of the probability mass function when p is equal to zero point seven 5 and
The white line shows the curve of the probability mass function when p is equal to zero point seven 5.

Hence, from the above curves we observe that,
Binomial distribution is positively skewed if p is less than zero point 5
Binomial distribution is negatively skewed if p is greater than zero point 5 and
Binomial distribution is symmetric if p is equal to zero point 5.

2. Illustrations 2 - 4

In this problem we have given that x follows binomial distribution with parameters n and zero point 5 and we need to sketch the curve for this distribution for different values of n and comment.

The probability mass function of the binomial distribution is given by,
P of x is equal to n c x into p power x into q power n minus x where x take values zero one two etc. n
When p is equal to zero point 5, we get
P of x is equal to n c x into zero point 5 power x into zero point five power n minus x where x takes value zero, one etc. hundred.

In this slide we have table which gives the value by using the probability mass function of the binomial distribution, p of x corresponding to different values taken by x when n is equal to 10, n is equal to 20 and n is equal to 30.

Figure 4

n=10

x	p(x)
0	0.00
1	0.01
2	0.04
3	0.12
4	0.21
5	0.25
6	0.21
7	0.12
8	0.04
9	0.01
10	0.00

n=20

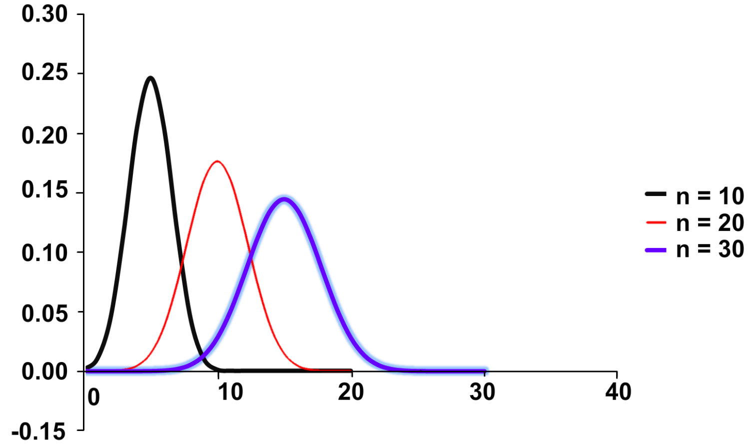
x	p(x)	x	p(x)
0	0.00	11	0.16
1	0.00	12	0.12
2	0.00	13	0.07
3	0.00	14	0.04
4	0.00	15	0.01
5	0.01	16	0.00
6	0.04	17	0.00
7	0.07	18	0.00
8	0.12	19	0.00
9	0.16	20	0.00
10	0.18		

n=30

x	p(x)	x	p(x)	x	p(x)
0	0.00	11	0.05	21	0.01
1	0.00	12	0.08	22	0.01
2	0.00	13	0.11	23	0.00
3	0.00	14	0.14	24	0.00
4	0.00	15	0.14	25	0.00
5	0.00	16	0.14	26	0.00
6	0.00	17	0.11	27	0.00
7	0.00	18	0.08	28	0.00
8	0.01	19	0.05	29	0.00
9	0.01	20	0.03	30	0.00
10	0.03				

Now we will draw a smooth curve by joining the different points of p of x.

Figure 5



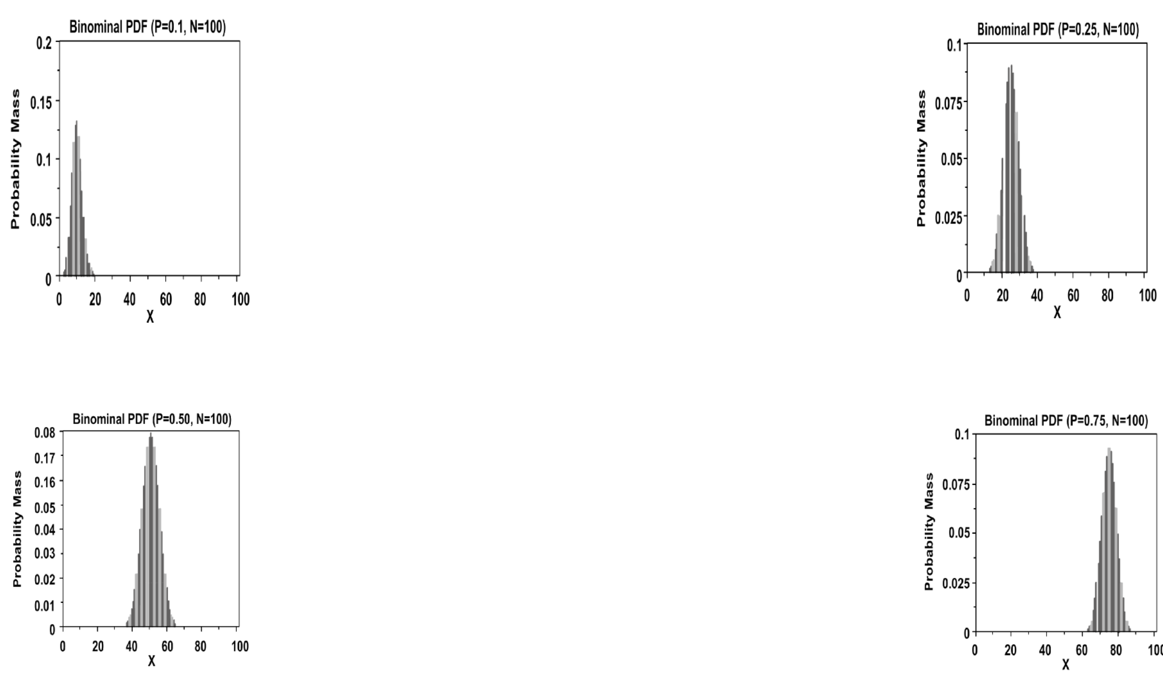
The black line shows the curve of the probability mass function when n is equal to 10.
The red line shows the curve of the probability mass function when n is equal to 20 and
The purple line shows the curve of the probability mass function when n is equal to 30
Observe that when n increases, the curve becomes more flat and bell shaped.

Now let us sketch the curve of binomial distribution when n is very large, that is 100 for different values of p and see how it changes.

The probability mass function of binomial distribution is given by,
P of x is equal n c x into p power x into q power n minus x, where x takes value zero, 1, etc., n
For n is equal to 100, the probability mass function becomes, p of x is equal to 100 c x into p power x into q power 100 minus x where x takes value zero one etc., 100.

Now we will draw the curve by joining the different points of p of x for different values of namely, p is equal zero point 1, p is equal to zero point 2 five, p is equal to zero point 5 and p is equal to zero point 7 five.

Figure 6



Observe the shape of the curve in all the four figures. For any value of p, when n is very large, the binomial distribution has more or less bell shaped and symmetric curve. Hence, binomial distribution tends to normal distribution for large n.

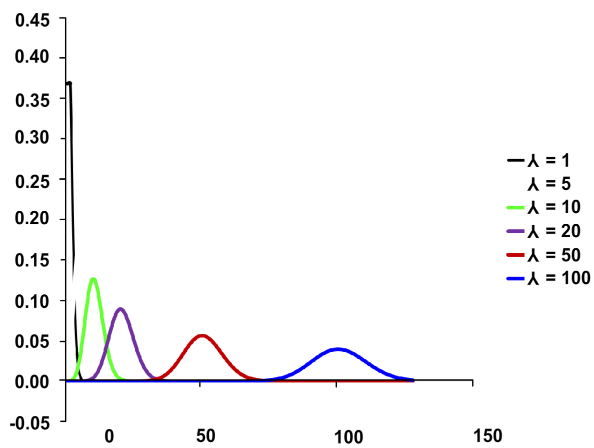
If x follows Poisson distribution with parameter lambda, then sketch the curve for different values of lambda and hence identify the nature of the distribution.

Since x follows Poisson distribution with parameter lambda, the probability mass function of the distribution is given by,
p of x is equal to e power minus lambda, lambda power x by x factorial where x take values zero one two etc. infinity.

And we find the values of p of x for different values of the parameter lambda namely, lambda is equal to 1, lambda is equal to 5, lambda is equal to 10, lambda is equal to 20, lambda is equal to 50 and lambda is equal to 100.

Let us draw a smooth curve by joining the different points of p of x to see the nature of the distribution that is how it varies with the value of the parameter.

Figure 7



In the figure, the black line indicates the curve of probability mass function of the Poisson distribution when lambda is equal to 1.
The white line shows the curve of probability mass function of the Poisson distribution when lambda is equal to 5.
The green line shows the curve of the probability mass function of the Poisson distribution when lambda is equal to 10.
The purple line shows the curve of probability mass function when lambda is equal to 20.
The red line shows the curve of probability mass function when lambda is equal to 50 and the blue line shows the curve of probability mass function of the Poisson distribution when lambda is equal to 100.
Observe that when lambda is very small, we have a curve, where as the beginning probability is very high and then the probability decreases drastically and then it decreases slowly. But as lambda increases, the curve becomes flatter and bell shaped. Hence, in general the Poisson distribution has leptokurtic curve but as lambda increases, the curve becomes mesokurtic. Hence, for large value of lambda, the Poisson distribution tends to normal distribution.

3. Illustrations 5 - 6

If x follows binomial distribution with parameters n and p then sketch the curve for this distribution for small values of p and large values of n. Use the Poisson approximation to binomial distribution. Vary whether the curves in both cases, matches each other or not.

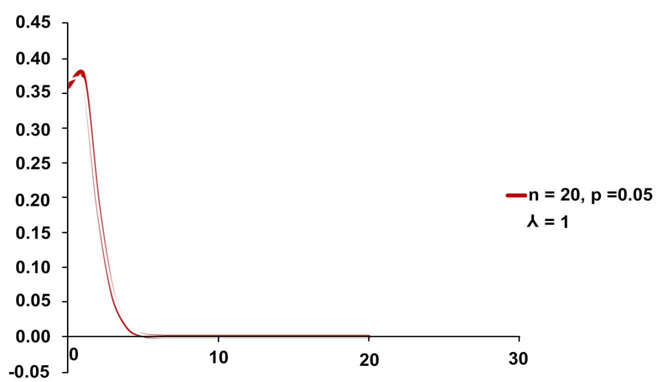
The probability mass function of the binomial distribution is given by,
P of x is equal to n c x into p power x into q power n minus x, where x take values, zero one etc., n.

We know that as n tends to infinity and p tends to zero, binomial distribution tends to Poisson distribution with parameter lambda is equal to n into p and the probability mass function of the Poisson distribution is given by,
P of x is equal to e power minus lambda, lambda power x divided by x factorial where x take values zero one two etc. Infinity.

Now we will draw the smooth curve by joining the different points of p of x for p is equal to zero point zero five and n is equal to 20 using probability mass function of binomial distribution and for lambda is equal to n into p is equal to 20 into zero point zero 5 is equal to 1 using the probability mass function of Poisson distribution.

In the graph, observe that red line shows the binomial curve with n is equal to 20 and p is equal to zero point zero five.

Figure 8



The white line shows the Poisson curve with lambda is equal to one.
And both the lines more or less coincide with each other.

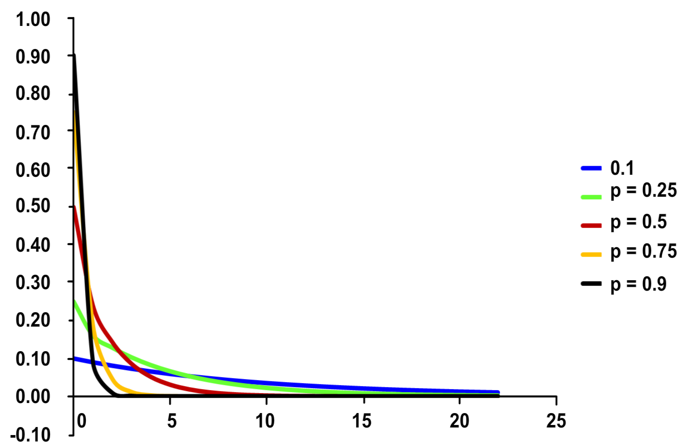
Hence, we can conclude that for small values of p and large values of n binomial distribution tends to Poisson distribution.

If x follows geometric distribution with parameter p, then draw the curve for the different values of p and comment.

Since x has geometric distribution with parameter p, the probability mass function of the distribution is given by,
P of x is equal to q power x into p where x take values zero, one, two etc., and p lies between zero and one.

Using above probability mass function we find p of x for different values of namely, p is equal to zero point 1, p is equal to zero point 2 five, p is equal to zero point 5, p is equal to zero point seven 5 and p is equal to zero point 9.

Figure 9



Observe that in the graph,
The blue line shows the curve of the geometric distribution when p is equal to zero point 1
The green line shows the curve of the geometric distribution when p is equal to zero point two 5
The red line shows the curve of the geometric distribution when p is equal to zero point 5
The yellow line shows the curve of the geometric distribution when p is equal to zero point seven 5 and
The black line shows the curve of the geometric distribution when p is equal to zero point 9.
From the graph observe that as p increases the curve becomes more and more steeper.

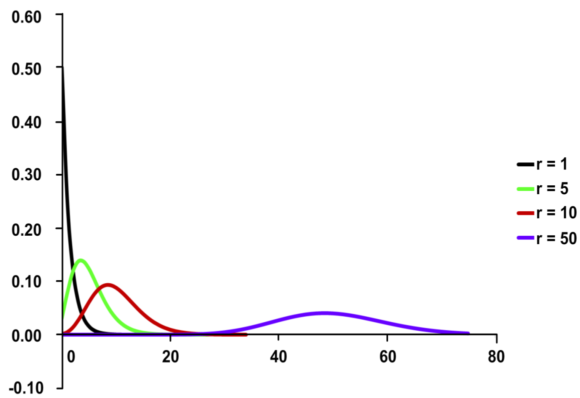
4. Illustrations 7 - 8

If x follows negative binomial distribution with parameters r and p then draw curve for fixed value of p and different values of r and Comment.

Since x has negative binomial distribution with parameters r and p, its probability mass function is given by,
P of x is equal to x plus r minus 1 c r minus 1 into p power r into q power x where x take values zero one two etc. And p lies between zero and 1.

Now we will draw the smooth curve by joining the different points of p of x for p is equal to zero point 5 and different values of r namely, r is equal to one, r is equal to 5, r is equal to 10 and r is equal to 50.

Figure 10



In the graph, the black line shows the curve of probability mass function of negative binomial distribution when r is equal to 1.
The green line shows the curve of probability mass function of negative binomial distribution when r is equal to 5.
The red line shows the curve of probability mass function of negative binomial distribution when r is equal to 10 and
The purple line shows the curve of probability mass function of negative binomial distribution when r is equal to 50.

Observe that when p is equal to zero point 5 and
r is equal to 1, the curve of negative binomial distribution is similar to that of geometric distribution. Hence we can say that when r is equal to one, the negative binomial distribution tends to geometric distribution.

For small value of r, the negative binomial distribution is positively skewed and has leptokurtic curve. As the value of r increases, the curve becomes more and more flat and symmetric. Hence, when p is equal to zero point 5 and for large values of r, the negative binomial distribution tends to normal distribution.

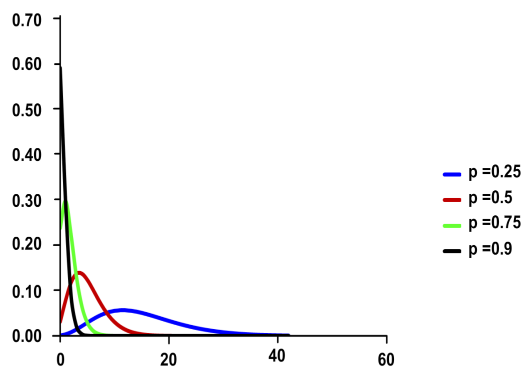
If x follows negative binomial distribution with parameters r and p then draw the curve for fixed value of r and different values of p. Comment on the behaviour of the distribution.

Since x has negative binomial distribution with parameters r and p, its probability mass function is given by,
P of x is equal to x plus r minus 1 c r minus 1 into p power r into q power x, where x take values zero one two etc., and p lies between zero and one.

Now we well draw the smooth curve for different values of p when r is small, that is, r is equal to 5 and when r is large, that is r is equal to 20.

Now let us draw a smooth curve by joining the different points of p of x when r is equal to 5 and p is equal to zero point two five, p is equal to zero point 5, p is equal to zero point 7 five and p is equal to zero point 9.

Figure 11

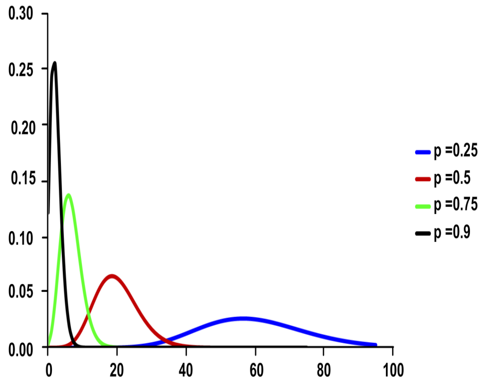


In the graph, the blue line shows the curve of probability mass function of negative binomial distribution when p is equal to zero point two 5
The red line shows the curve of probability mass function of negative binomial distribution when p is equal to zero point 5
The green line shows the curve of probability mass function of negative binomial distribution when p is equal to zero point seven 5 and
The black line shows the curve of probability mass function of negative binomial distribution when p is equal to zero point 9.

When r is small that is 5, the curve is positively skewed for any value of p but as p increases, the curve becomes steeper.

Now we will draw the smooth curves by joining the different points of p of x when r is equal to 20 and different values of p namely, P is equal to zero point 2 five, P is equal to zero point 5, p is equal to zero point seven 5 and p is equal to zero point 9.

Figure 12



In the graph, the blue line shows the curve of probability mass function of negative binomial distribution when p is equal to zero point 2 five.
The red line shows the curve of probability mass function of negative binomial distribution when p is equal to zero point five
The green line shows the curve of probability mass function of negative binomial distribution when p is equal to zero point seven 5 and
The black line shows the curve of probability mass function of negative binomial distribution when p is equal to zero point 9.

When r is large that is 20, Observe that for smaller values of p the curve is more flat compared to large values of p. And also the curve is more or less symmetric compared to that of smaller value of r.

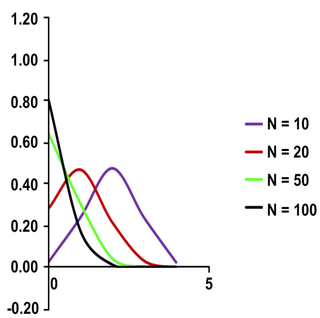
5. Illustrations 9 - 10

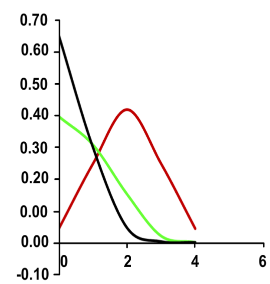
If x follows Hypergeometric distribution with parameters N, M and n, then draw the curve for different values of N when M and n are fixed. Also comment.

Since x has Hypergeometric distribution with parameters N, M and n, the probability mass function is given by, p of x is equal to M c x into N minus M c n minus x divided by N c n where x take values zero one two etc., minimum of n and M.

We draw the smooth curve by joining the different points of p of x of Hypergeometric distribution by taking different values of N namely, N is equal to 10, N is equal to 20, N is equal to 50 and N is equal to 100 when M = 5 and n = 4.
We also plot the smooth curve for Hypergeometric distribution for same values of N, when M is equal to 10 and n is equal to 4.

Figure 13





In the graph, the purple line shows the curve of
probability mass function of Hypergeometric distribution when N is equal to 10
The red line shows the curve of probability mass function of Hypergeometric distribution when N is equal to 20
The green line shows the curve of probability mass function of Hypergeometric distribution when N is equal to 50 and
The black line shows the curve of probability mass function of Hypergeometric distribution when N is equal to 100.

Observe that as N increases, the curve becomes positively skewed.
From the two graphs we can observe that when M divided N is equal to half, the curve will be symmetric.

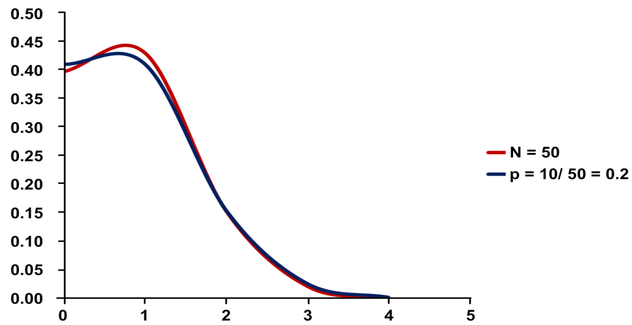
If x follows Hypergeometric distribution with parameters N, M and n, by plotting the curve, verify whether Hypergeometric distribution tends to binomial for large N.

The probability mass function of Hypergeometric distribution is given by, p of x is equal to M c x into N minus M c n minus x divided by N c n where x take values zero one two etc., minimum of n and M.

Theoretically, we know that as N tends to infinity, the Hypergeometric distribution tends to binomial with parameter p is equal to M by N.

We know that probability mass function of binomial distribution is given by, p of x is equal to n c x into p power x into q power n minus x where x takes values zero, one etc. n.
Now we will draw a smooth curve by joining points of p of x under both Hypergeometric and binomial case and verify whether the curves will match or not.

Figure 14



In the graph, the red line shows the curve of Hypergeometric distribution and
The black line shows the curve of binomial distribution.
Observe that both the curves more or less coincide with each other. Hence for large N, Hypergeometric distribution tends to binomial distribution.

Here's a summary of our learning in this session:

- To draw the probability mass function of different distributions.
- To identify the nature, behaviour and properties of the distribution.