1. Introduction

Welcome to the series of e-learning modules on Practicals. Here we shall verify whether a given distribution is a probability distribution, find the expectation and variance of the distribution, quartiles and quartile deviation, and try to find the nature of the distribution with the help of skewness and kurtosis.

At the end of this session, you will be able to:

- Verify if a given probability is PMF or not
- Find the expectation and variance of a distribution
- Find the quartiles and quartile deviation for a given distribution
- Compute the skewness and kurtosis for a given distribution

In the first problem we have been given the values taken by X and its corresponding probabilities.
It is also told that given the probability function, we need to find the value of the k, which is unknown, and the expectation of X and its variance from the given data.

First, we shall find the value of k. Since we have been given the probability function, sum of probabilities that is sum of p of x is equal to 1.
Therefore, zero point 1 plus k plus zero point two plus two k plus zero point 3 plus k is equal to 1.
Implies, four k plus zero point 6 is equal to 1.
On simplification, we get, k is equal to zero point 1.
Hence by substituting for k in the table given in the question, we get the values of p of x for different values of x as zero point 1, zero point 1, zero point 2, zero point 2, zero point 3, and zero point 1.

Now, the expectation of X is given by,
E of X is equal to summation of X into p of X.
Is equal to minus 2 into zero point 1 plus minus 1 into zero point 1 plus zero into zero point 2 plus 1 into zero point 2 plus 2 into zero point 3 plus 3 into zero point 1.
On simplification we get, the expectation of X as zero point eight.

Variance is given by v of x is equal to expectation of x square minus expectation of x whole square.
First let us find, expectation of x square is equal to summation x square into p of x
Is equal to minus 2 square into zero point 1 plus minus 1 square into zero point 1 plus zero square into zero point 2 plus 1 square into zero point 2 plus 2 square into zero point 3 plus 3 square into zero point 1
On simplification we get, two point eight.
Therefore v of x is equal to 2 point 8 minus zero point 8 square.
Is equal to 2 point one 6.

2. Problems (Part 1)

In the second problem we have been given the distribution of a discrete random variable x with corresponding probabilities. We need to find the probability that x is equal to even and probability that 1 less than or equal to x less than or equal to eight.

In the given problem, we have 2 even numbers 2 and 4.
Hence p of x is equal to even is equal probability that x is equal to 2 or 4.
Corresponding to 2 we have probability, zero point two 5 and corresponding to 4 we have probability zero point zero five
Hence we get,
Zero point two 5 plus zero point zero five is equal to zero point three.
Probability that 1 less than or equal to x less than or equal to 8
Is equal probability that x takes values 1, 2, 3, 4 or 5
Is equal to zero point zero five plus zero point 2 five plus zero point one 5 plus zero point zero five plus zero point zero five.
Is equal to zero point 5 five.

In this problem we need to find the expectation of drawing 2 yellow marbles drawn at random from a bag that contains 5 pink and 4 yellow marbles.

Let X denote the number of yellow marbles drawn. Since 2 marbles are drawn, we may get no yellow marbles or 1 yellow and 1 pink or both yellow marbles. Hence x take values zero, one or 2 and their respective probabilities can be found as follows.
Probability that no yellow marbles, p of x is equal to zero is equal to 5 c 2 divided by 9 c 2 is equal to 5 by 18.
Probability of drawing one pink and 1 yellow marble, p of x is equal to 1 +is equal to 5 into 4 divided by 9 c 2 is equal to 5 by 9.
Probability of drawing both yellow marbles, p of x is equal to 2 is equal to 4 c 2 divided by 9 c 2 is equal to 1 by 6.
Therefore, the expectation of X is equal to summation x into p of x.
Is equal to zero into 5 by 18 plus 1 into 5 by 9 plus 2 into 1 by 6.
Is equal to 8 by 9.

In this problem we need to find the expected sum received by Suresh when he tries to open the front door using a key bunch of 5 keys of identical size. He receives Rs. 20 less thrice the number of attempts he makes.

We define x as the number of attempts made by Suresh to open the door. Hence he may succeed in 1^{st} attempt or 2^{nd} or 3^{rd} or 4^{th} or 5^{th} attempt. Hence x take values 1, 2, 3, 4 and 5 with respective probabilities,
Probability that he succeeds in 1^{st} attempt, P of x is equal to 1 is equal to 1 by 5
Probability that he fails in first attempt and succeeds in second attempt with remaining 4 keys, p of x is equal to 2 is equal to 4 by 5 into 1 by 4 is equal to 1 by 5
Probability that he fails in first 2 attempts and succeeds in 3^{rd} attempt, p of x is equal to 3 is equal to 4 by 5 into 3 by 4 into 1 by 3 is equal to again 1 by 5
Similarly we get p of x is equal to 4, which is same as p of x is equal to 5 is 1 by 5

Hence expectation of x is equal to summation x into p of x
Is equal to 1 into 1 by 5 plus 2 into 1 by 5 plus 3 into 1 by 5 plus 4 into 1 by 5 plus 5 into 1 by 5
Is equal to 3.

It is given that the sum received by Suresh is 20 rupees less than thrice the attempts made by him to open the door
That is 20 minus 3 into x.
Therefore expected sum received by Suresh is given by expectation of 20 minus 3 x
Is equal to 20 minus 3 into expectation of x
By substituting for expectation of x is equal to 3 we get, 20 minus 3 into 3
Is equal to 11 rupees.

3. Problems (Part 2)

In this problem we need to find the expectation of the amount when a boy picks a coin at random from a bag having 2 one rupee coins and 3 ten paise coins.

Let us define x as the amount (in paise) that a boy picked. Then x takes the values of 100 paisa and 10 paisa with respective probabilities 2 by 5 (since 2 coins are 1 rupee out of 5 coins) and 3 by 5 (since 3 coins out of 5 are 10 paise).
Hence expectation of x is equal to summation x into p of x
Is equal to 100 into 2 by 5 plus 10 into 3 by 5
Is equal to 46 paise.

In this problem we need to find the expected loss of a person who enters into a game of shooting a target by paying rupees 5. If he hits the target he gets rupees 25 otherwise he gets nothing.

Let x denote the net amount that a person receives after deducting the entrance fees.
Hence x takes value -5 if he does not hit the target and 20 that is 25 minus 5, the entrance fee, and the respective probabilities are 6 by 7 and 1 by 7
Therefore his expected net amount is given by
Expectation of x is equal to summation x into p of x
Is equal to minus 5 into 6 by 7 plus 20 into 1 by 7
Is equal to minus 1 point four 3
That is a loss of rupees 1 point four 3.

In this problem we need to find the expected loss in buying a lottery ticket from one thousand tickets costing rupee 1 each. There is a first prize of rupees one hundred, 2 second prizes worth rupees 20 each and ten 3rd prizes worth rupees ten each.

Let us define the random variable x as the amount that one lottery ticket fetches after deducting the purchase cost of rupee 1.
Since we deduct 1 rupee from each prize, x takes values 99, 19, 9 and -1, if there is no prize.
And the respective probabilities are given by,
P of 99 is equal to p of first prize is equal to 1 by one thousand is equal to zero point zero zero 1 as there is one first prize
P of 19 is equal to p of second prize is equal to 2 by one thousand is equal to zero point zero zero two, as there are 2 second prizes
P of 9 is equal to p of 3rd prize is equal to ten by one thousand is equal to zero point zero one and
P of minus 1 is equal to p of no prize is equal to nine hundred and eighty seven divided by one thousand is equal to zero point nine eight seven, as there are altogether 13 prizes and 987 tickets do not fetch any prize.

Thus expectation of x is equal to x into p of x
Is equal to ninety nine into zero point zero zero one plus 19 into zero point zero zero two plus nine into zero point zero 1 plus minus 1 into zero point 9 eight 7
Is equal to minus zero point seven 6

Since answer is negative, a person will have a loss of rupees zero point seven six.

4. Problems (Part 3)

In this problem we have been given the weight of 584 bullocks in pounds which vary from 850 pounds to 1 thousand 3 hundred and fifty pounds. We need to find quartile deviation and the coefficient of quartile deviation.

To find quartiles we need to find cumulative frequencies that is First we write 2, then 2 plus 24 is equal to 26, 26 plus 45 is equal to 71 and so on we get last entry in cumulative frequency column 584, which is nothing but the total frequency.

We know that the quartile deviation is given by,
Q D is equal to Q 3 minus Q 1 whole divided by 2 where Q1 and Q 3 are first and third quartiles respectively.

First quartile, Q 1 is found using the formula,
Q 1 is equal to L plus N by 4 minus m whole divided by f into h
Where L is the lower limit of the quartile class
m is the cumulative frequency preceding to quartile class
f is the frequency of the quartile class
h is the width of the quartile class and
N is the total frequency.
Hence first we identify the quartile class which is found as follows.
So, N by 4 is equal to 584/4 is equal to 146^{th} item, which lies in the class interval 1000 to 1050
Hence L is 1000, m is 71, and h is 50
Therefore Q 1 is equal to 1000 plus 146 minus 71 whole divided by 120 into 50
On simplification, we get,
1 thousand thirty one point two 5.

To find third quartile Q 3 we first identify the third quartile class
That is 3 into N by 4 is equal to 3 into 584 divided by 4 is equal to 438^{th} item which lies in the class interval 1100 to 1150
Hence Q 3 is given by,
Q 3 is equal to L plus 3 into N by 4 minus m whole divided by f into h
Is equal to 1100 plus 438 minus 301 divided by 140 into 50
On simplification we get, 1148 point nine 3
Therefore, quartile deviation is given by,
Q 3 minus Q 1 whole divided by 2 is equal to 1148 point nine 3 minus 1031 point two 5 whole divided by 2
Is equal to 58 point eight four.
Coefficient of quartile deviation is given by,
Q 3 minus Q 1 whole divided by Q 3 plus Q 1
Is equal to 1148 point nine 3 minus 1031 point two 5 whole divided by 1148 point nine 3 plus 1031 point two 5 is equal zero point zero three five.

In this problem we have given the data on production rejects and we need to find Karl Pearson's coefficient on skewness.

Since the given distribution is of inclusive type, first we convert into exclusive type that is by subtracting zero point 5 from lower limit and adding zero point 5 to upper limit.

Karl Pearson's coefficient of skewness is given by, S k is equal to x bar minus Z divided by sigma, where x bar is mean, Z is mode and sigma is standard deviation.
Hence we find the different totals needed to compute mean, mode and standard deviation.

Since we shift the origin that is A to 38 and scale, h as 5, that is u is equal to X minus A by h,
Mean X bar is equal to
A plus summation of f into u divided by N into h
Is equal to 38 plus minus 25 divided by 100 into 5
On simplification, we get 36 point seven 5.
To find mode, first we identify the modal class which has highest frequency that is 35 point 5 to 40 point 5
Mode is given by, Z is equal to L plus f minus f 1 divided by 2 into f minus f1 minus f2 into h
Is equal to 35 point 5 plus 42 minus 28 divided by 2 into 42 minus 28 minus 15 into 5
Is equal to 37 point 2 zero 7.

Standard deviation is given by,
Sigma is equal to h into square root of summation of f u square by N minus summation f u by N the whole square.
On substituting and simplifying we get, 7 point 3 six.
Hence Karl Pearson's coefficient of skewness is given by,
S k is equal to x bar minus Z divided by sigma is equal to 36 point 7 five minus 37 point 2 zero 7 divided by 7 point 3 six is equal to zero point zero 6 two 1.
Since S k is negative, the given distribution is negatively skewed.

5. Problems (Part 4)

In this problem we have to find the Bowley's measure of skewness.

Bowley's measure of skewness depends on quartiles and median. Hence first we find first and 3rd quartile and median.
Since given data is of inclusive type we convert into exclusive type and find the cumulative frequencies.
Bowley's coefficient of skewness, S k is equal to Q3 plus Q1 minus 2M divided by Q3 minus Q1
To find q 1 first we identity the 1st quartile class. That is N by 4 is equal to 285 by 4 is equal to 71 point two 5. Hence first quartile class is 10 point 5 to 15 point 5.
Therefore Q1 is equal to L plus N by 4 minus m divided by f into h
Is equal to 10 point 5 plus 71 point two 5 minus 47 divided by 29 into 5
Is equal to 14 point six 8.

3 into N by 4 is equal to 213 point seven 5. Hence third quartile class is 25 point 5 to 30 point 5
Therefore Q3 is equal to L plus 3 into N by 4 minus m divided by f into h
Is equal to 25 point 5 plus 213 point seven 5 minus 162 divided by 53 into 5 is equal to 30 point 3 eight 2.

To find median, we find N by 2 is equal to 142 point five. Hence median class is 20 point 5 to 25 point 5.
Therefore M is equal to L plus N by 2 minus m divided by f into h is equal to 20 point 5 plus 142 point five minus 114 divided by 48 into 5 Is equal to 23 point 4 seven.
Hence Bowley's coefficient of skewness, S k is equal to Q3 plus Q1 minus 2M divided by Q3 minus Q1 on substituting and simplifying the values, we get,
Minus zero point 1 two.

In this problem we have given first 4 raw moments and we need to find the nature of the distribution. From raw moment we shall try to find the central moments.

Given mew 1 dash is equal to 7, mew 2 dash is equal to 70, mew 3 dash equal to 140 and mew 4 dash is equal to 175.
Mew two is equal to mew 2 dash minus mew 1 dash square
On substitution and simplification, we get, 21
Mew 3 is equal to mew 3 dash minus 3 mew 1 dash mew 2 dash plus 2 mew 1 dash cube
On substitution and simplification, we get minus 644.
Mew 4 is equal to mew 4 dash minus 4 mew 1 dash into mew 3 dash plus 6 mew 1 dash square into mew 2 dash minus 3 mew 1 dash to the power 4
On substitution and simplification, we get nine 6 three 2.

Therefore gamma 1 is equal to square root of beta 1 is equal to mew 3 by mew 2 to the power 3 by 2 is equal to minus 6 point 6 nine 2
And beta 2 is mew 4 by mew 2 square is equal to 21 point 8 four 3 one.
Observe that gamma one is negative and beta 2 is greater than 3, the distribution is

negatively skewed and has a leptokurtic curve

Here’s a summary of our learning in this session. In this session we solved problems on how to:

- Verify if a given probability is PMF or not
- Find the expectation and variance of a distribution
- Find the quartiles and quartile deviation for a given distribution
- Compute the skewness and kurtosis for a given distribution