# 1. Introduction

Welcome to the series of E-learning modules on Practical. Here we find the probabilities of more than one event, which are depend on each other using conditional probability and Bayes' theorem.

By the end of this session, you will be able to:

- Explain how to compute probabilities when there are two or more events which are not independent, i.e., conditional probability
- Explain the usage of Bayes' theorem in conditional probability

In an examination, 30 percent of the students have failed in Mathematics, 20 percent of the students have failed in chemistry and 10 percent have failed in both mathematics and chemistry. A student is selected at random. What is the probability that the student has failed in mathematics if it is known that he has failed in chemistry?

Let A denotes the selected students who have failed in mathematics and B denotes the selected students who have failed in chemistry.

We have given, p of A is equal to 30 percent is equal to zero point three,
P of B is equal to 20 percent is equal to zero point two and
P of A intersection B is equal to 10 percent is equal to zero point one.
Probability that the student has failed in mathematics if he has failed in chemistry is,
P of A given B is equal to p of A intersection B divided by p of B is equal to zero point 1 by zero point 2 is equal to half.

A box has 5 white and 2 red balls. Another box has 3 white and 4 red balls. One ball is randomly selected from the first box and it is transferred to the second box. After that, one ball is randomly drawn from the second box. Find the probability that it is red.

Let event A denotes, the transferred ball is white
B denotes the transferred ball is red, and
E denotes the ball drawn from the second box is red.

Then probability of A is equal to 5 by 7, and probability of B is equal to 2 by 7.

If one white ball is transferred to the $2^{nd}$ box, then there will be 4 white and 4 red balls.
Therefore p of E given A is equal to 4 by 8
If one red ball is transferred to the $2^{nd}$ box, then there will be 3 white and 5 red balls.
Therefore, p of E given B is equal to 5 by 8.

Event E can occur along with A and B.

Therefore,
Probability that ball from selected box is red
Is equal to probability of A intersection E union B intersection E

Is equal to probability of A intersection E plus B intersection E
Is equal to p of A into p of E given A plus p of B into p of E given B
Is equal to 5 by 7 into 4 by 8 plus 2 by 7 into 5 by 8
Is equal to 15 by 28 is equal to zero point five three five seven.

# 2. Illustrations 3 - 4

A dice is thrown. If it shows even numbers, two balls will be drawn from box A; otherwise two balls will be drawn from box B. Box A contains 4 black, 6 white and 2 blue balls, while box B contains 2 black, 4white and 2 blue balls. Find the probability that both the balls drawn are white.

Let A denotes box A is selected, B denotes box B is selected and E denote two white balls are selected.

Box A is selected if the number on the die is even. In a die chance of getting even number is half. Hence probability of A is equal to half.
Similarly, probability of B is equal to half.

After selecting the box two balls are drawn from the selected urn
Hence, probability to E given A is equal to 6 c 2 divided by 12 c 2
Is equal to 5 by 22.
And probability of E give B is equal to 4 C 2 divided by 8 C 2
Is equal to 3 by 14.

Therefore,
Probability that two white balls are drawn,
Is equal to probability of A intersection E union B intersection E
Is equal to probability of A intersection E plus B intersection E
Is equal to p of A into p of E given A plus p of B into p of E given B
Is equal to half into 5 by 22 plus half into 3 by 14
Is equal to 17 by 77 is equal to zero point two two zero eight.

Plant I of XYZ manufacturing organisation employs 5 production and 3 maintenance foremen; another plant II of same organisation employs 4 production and 5 maintenance foremen. From any one of these plants, a single selection of two foremen is made. Find the probability that one of them would be production and the other maintenance foremen.

Let A denotes, plant 1 is selected; B denotes plant 2 is selected and E denote in a selection of 2 persons, one is production and the other is maintenance foreman.

Since there are two plants, the selection of each being equally likely, we have
Probability of A is equal half and probability of B is equal to half.

Probability of selecting one production and one maintenance foreman in a selection of two foremen from the first plant we get, p of E given A is equal to 5 c 1 into 3 c 1 divided by 8 c 2 is equal to 15 by 28.
Similarly, p of E given B is equal to 4 c 1 into 5 c 1 divided by 9 c 2 is equal to 5 by 9.

The required event of selecting one production and one maintenance foreman in a selection of 2 persons can materialise in following ways.
probability of A intersection E union B intersection E

Is equal to probability of A intersection E plus Probability of B intersection E
Is equal to p of A into p of E given A plus p of B into p of E given B
Is equal to half into 15 by 28 plus half into 5 by 9
Is equal to 275 by 504 is equal to zero point five four five six.

# 3. Illustrations 5 - 6

Suppose that a product is produced in three factories X, Y and Z. It is known that factory X produces thrice as many items as factory Y, and that factories Y and Z produces the same number of products. Assume that 3 percent of the items produced by each of the factories X and Z are defective while 5 percent of those manufactured by Y are defective. All the items produced in the three factories are stocked, and an item of product is selected at random. What is the probability that this item is defective?

Let the number of items produced by each of the factories Y and Z be ‘n’. Then the number of items produced by the factory X is 3n. Let A, B and c denote the events that the items are produced by factory X, Y and Z respectively and let E be the event of the item being defective.

Then we have P of A is equal to 3 n by 3 n plus n plus n is equal to zero point 6
P of B is equal to n by 5 n is equal to zero point 2 and
P of C is equal to n by 5 n is equal to zero point 2.
Also it is given that, p of E given A is equal to p of E given C is equal to zero point zero three and p of E given B is equal to zero point zero 5.

The probability that an item selected at random from the stock is defective is given by,
P of E is equal to p of A intersection E union B intersection E union C intersection E
Using addition theorem of probability, we get, P of A intersection E plus p of B intersection E plus p of C intersection E.

Which is equal to p of A into p of E given A plus p of B into p of E given B plus p of C into p of E given C
Is equal to zero point six into zero point zero three plus zero point two into zero point zero five plus zero point 2 into zero point zero three
Is equal to zero point zero three four.

A manufacturing firm produces pipes in two plants 1 and 2 with daily production of one thousand five hundred, and two thousand pipes respectively. The fraction of defective pipes produced by two plants are zero point zero zero six and zero point zero zero eight respectively. If a pipe selected at random from the day’s production is found to be defective, what is the probability that it has come from plant 1 and plant 2?

Let E1 denotes that pipe is manufactured in plant 1
E2 denotes that pipe is manufactured in plant 2 and
E denotes the defective pipe drawn.
Hence, probability of E1 is equal to one thousand 5 hundred divided by one thousand 5 hundred plus 2 thousand is equal to 3 by 7
Probability of E2 is equal to 2 thousand divided by one thousand 5 hundred plus 2 thousand is equal to 4 by 7.

Further it is given that,
P of E given E1 is equal to zero point zero zero six and p of E given E2 is equal to zero point zero zero eight.

P of E1 intersection E and p of E2 intersection E are the joint probabilities that the items are produced by the 1st and 2nd plant and are defective. Also
P of E1 intersection E is equal to p of E1 into p of E given E1 is equal to zero point zero zero six into 3 by 7 is equal to zero point zero one 8 by 7
P of E2 intersection E is equal to p of E2 into p of E given E2 is equal to zero point zero zero 8 into 4 by 7 is equal to zero point zero three two by seven.

Using Bayes' theorem, Probability of defective has come from plant 1 is given by,
P of E1 given E is equal to p of E1 intersection E divided by p of E1 intersection E plus p of E2 intersection E
Is equal to zero point zero one 8 by 7 divided by zero point zero one eight by 7 plus zero point zero three two by 7
Is equal to zero point three six

Similarly,
P of E2 given E is equal to p of E2 intersection E divided by p of E1 intersection E plus p of E2 intersection E
Is equal to zero point zero three two by 7 divided by zero point zero one eight by 7 plus zero point zero three two by 7
Is equal to zero point six four.
Since p of E2 given E is greater than p of E1 given E, it is most probable that the defective pipe has been drawn from the output of the second plant.

# 4. Illustrations 7-8

The probabilities of X, Y and Z becoming managers are 4 by 9, 2 by 9 and 1 by 3 respectively. The probabilities that the Bonus Scheme will be introduced if X, Y and Z becomes managers are 3 by 10, half and 4 by 5 respectively. What is the probability that the manager appointed was X when the Bonus Scheme was introduced?

Let E1, E2 and E3 denote the events that X, Y and Z become managers respectively and E denote the event that 'Bonus Scheme' is introduced.

We have given, p of E1 is equal to 4 by 9, p of E2 is equal to 2 by 9 and p of E3 is equal to 1 by 3.
Probability that bonus scheme will be introduced given that X becomes the manager; p of E given E1 is equal to 3 by 10.
Similarly for manager Y and Z is given by,
P of E given E2 is equal to half and p of E given E3 is equal to 4 by 5.

The event E can materialise in the following mutually exclusive ways:

i. Mr. X becomes manager and bonus scheme is introduced. That is E1 intersection E
ii. Mr. Y becomes manager and bonus scheme is introduced. That is E2 intersection E
iii. Mr. Z becomes manager and bonus scheme is introduced. That is E3 intersection E

Therefore, p of E is equal to p of e1 intersection E plus p of E2 intersection E plus p of E3 intersection E
Is equal to p of E1 into p of E given E1 plus p of E2 into p of E given E2 plus p of E3 into p of E given E3.
Is equal to 4 by 9 into 3 by 10 plus 2 by 9 into half plus 1 by 3 into 4 by 5
Is equal to 23 by 45.
Using Bayes' theorem, the probability that X becomes the manager and Bonus Scheme is introduced, is p of E1 given E is equal to P of E1 into p of E given E1 divided by p of E is equal to 12 by 90 divided by 23 by 45 is equal 6 by 23.

An insurance company insured 2 thousand scooter drivers, 4 thousand car drivers and 6 thousand truck drivers. The probability of accident is zero point zero 1, zero point zero three and zero point one five respectively. One of the insured persons meets an accident. What is the probability that he is a scooter driver?

Let E1, E2 and E3 denote the events that the insured person is a scooter driver, a car driver and a truck driver respectively and E denote the event that an insured person meets an accident.

We are given the following probabilities.
Altogether we have 2 thousand plus 4 thousand plus 6 thousand is equal to 12 thousand insured persons.

Hence, p of E1 is equal to 2 thousand by 12 thousand is equal to 1 by 6
P of E2 is equal to 4 thousand by 12 thousand is equal to 1 by 3 and

P of E3 is equal to 6 thousand by 12 thousand is equal to half.

Further we have given p of E given E1 is equal to zero point zero 1
P of E given E2 is equal to zero point zero three and
P of E given E3 is equal to zero point one five

Using Bayes' theorem, the required probability that an insured person is a scooter driver given that he meets an accident is given by,
P of E1 given E is equal to p of E1 intersection E divided by p of E,

Where p of E is equal to p of E1 intersection E plus p of E2 intersection E plus p of E3 intersection E
Is equal to p of E1 into P of E given E1 plus p of E2 into p of E given E2 plus p of E3 into p of E given E3
Is equal to 1 by six into zero point zero one plus 1 by 3 into zero point zero three plus half into zero point 1 five
Is equal to 13 by 150
Hence, p of E1 given E is equal to 1 by six into zero point zero one divided by 13 by 150
Is equal to 1 by 52.

# 5. Illustration 9

By examining the chest X-ray probability that T.B. is detected when a person actually suffering is zero point nine nine. The probability that the doctor diagnose incorrectly that a person has T.B on the basis of X-ray is zero point zero zero one. In a certain city 1 in one thousand persons suffers from T.B. A person is selected at random is diagnosed to have T.B. What is the chance that he actually has T.B.?

Let E1 and E2 denote the events that a person suffers and do not suffers from T.B. respectively and E the event that doctor diagnose correctly. We are given,
P of E1 is equal to 1 by 1 thousand is equal to zero point zero zero one so that p of E2 is equal to 1 minus zero point zero zero one is equal to zero point nine nine nine.

Also p of E given E1 is equal to zero point nine nine and p of E given E2 is equal to zero point zero zero one.

Using Bayes' theorem, the required probability that the person actually has T.B.
is given by,
P of E1 given E is equal to p of E1 intersection E divided by p of E,

Where p of E is equal to p of E1 intersection E plus p of E2 intersection E
Is equal to p of E1 into P of E given E1 plus p of E2 into p of E given E2
Is equal to 0.001 into 0.99 plus 0.999 into 0.001
Is equal to 0.001989

Therefore, p of E1 given E is equal to 0.001 into 0.99 divided by 0.001989
Is equal to 0.4977.

Here's a summary of our learning in this session:

- How to compute probabilities when there are two or more events which are not independent, i.e. conditional probability
- Usage of Bayes' theorem in conditional probability