1. Introduction

Welcome to the series of E-learning modules on Practical. The first topic is computing probability using addition and multiplication theorems. Here, we will solve some problems to find the probability using the theorems when the events are mutually exclusive, dependent on each other and are independent of each other.

By the end of this session, you will be able:

- Explain the usage of addition theorem in finding probabilities for any two events and mutually exclusive events
- Explain the usage of Multiplication theorem in finding the probabilities for independent events

Let us start this session with the following problem.
A bag has 5 red and 3 blue balls. Two balls are randomly drawn. Using addition theorem, find the probability that they are of same colour.

Here we need to find the balls of same colour. Here we have balls of two colours red and blue. Hence let us define the events

A – drawing 2 red balls and B – drawing two blue balls
Since altogether there are 8 balls, we can select 2 balls in 8 c 2 ways, which is same as 28 ways.

Since there are 5 red balls, we can select 2 red balls in 5 c 2 ways. That is 10 ways and there are 3 blue balls. We can select 2 blue balls in 3 c 2 ways, same as 3 ways.

Therefore, Probability of event A is equal to 10 by 28 and probability of event b is equal to 3 by 28.

Since events A and B are mutually exclusive,
Probability of A union B is equal to P of A plus P of B
Is equal to 10 by 28 plus 3 by 28
Is equal to 13 by 23

There are three groups of children consisting of 3 boys and a girl in one group, 2 boys and 2 girls in another group and the last group has a boy and 3 girls. A child is selected at random from each group. What is the probability that the selected group consists of

i. 2 boys and one girl
ii. Only boys

Observe that in this problem, there are 3 groups and 1 child is selected at random from each group.
For 1^{st} sub question we need to find 2 boys and 1 girl
Observe that, we have following possibilities.
Event A denote Selecting a girl from 1^{st} group, a boy from 2^{nd} and 3^{rd} group

Event B denote Selecting a girl from 2nd group, a boy from 1st and 3rd group
Event C denote Selecting a girl from 3rd group, a boy from 1st and 2nd group
Total number of possible selection of children = 4 into 4 into 4 is equal to 64
Therefore probability of event A is equal to 1 into 2 into 1 divided by 64 is equal to 2 by 64
probability of event B is equal to 3 into 2 into 1 divided by 64 is equal to 6 by 64
Probability of event C is equal to 3 into 2 into 3 divided by 64 is equal to 18 by 64.

Since events A, B and c are mutually exclusive,
Probability of A union B union C is equal to p of A plus p of B plus p of c
Is equal to 2 by 64 plus 6 by 64 plus 18 by 64
Is equal to 26 by 64, same as 13 by 32

Under 2nd sub question we need to find only boys.
Let Event D denote selection boys from all the three groups
That is selecting a boy from 1st group, 2nd group and 3rd group.
Therefore P of D is equal to 3 into 2 into 1 divided by 64 is equal to 3 by 32.

2. Illustrations 3 - 4

In a firm, 40% of employees are women. Among them it is found that $(1/8)^{th}$ of women and $(1/10)^{th}$ of the men wear spectacles. Find the probability that a randomly selected employee is a man or a person wearing spectacles.

Here we need to find the probability that a randomly selected employee is a man or a person wearing spectacles.
Hence define the events,
A – Employee selected being man and B – employee selected being wearing spectacle.

From the given problem,
60 percent are women. Hence Probability of A is equal to 100 minus 40 percent
Is equal to zero point 6
Probability of B is equal to 1 by 8 into 40 percent plus 1 by 10 into 60 percent.
Is equal to zero point one one.
Also 1 by tenth of men wear spectacles.
Therefore, probability of A intersection B is equal to 1 by tenth of 60 percent
Is equal to 6 by hundred is equal to zero point zero six.

Here A and B are not mutually exclusive. Hence,
Probability of A union B is equal to probability of A plus probability of B minus probability of A intersection B.
Is equal to zero point 6 plus zero point one one minus zero point zero six
Is equal to zero point four six

One card is drawn from a pack of 52 cards, each of the 52 cards being equally likely to be drawn. Find the probability of

i. The card drawn is either red or a king
ii. The card drawn is neither red nor a king

Here we define two events A and B as follows.
Let A denote the card drawn is red and B denotes the drawn card is a king.
In a pack of cards, we have 52 cards, 26 of them are red, and there are 4 kings.

Therefore, probability of event A is 26 by 52 and probability of event B is 4/52.
In red cards, there are two kings. Hence, the probability of A intersection B is equal to 2 by 52.

In first sub-question, we need to find the probability of card drawn is red or king.
That is probability of A union B
Is equal to p of A plus p of B minus p of A intersection B
Is equal to 26 by 52 plus 4 by 52 minus 2 by 52
Is equal to 28 by 52 same as 7 by 13

In second sub question, we need to find neither red nor a king.
That is probability of A complement intersection B complement.

Which can be written as 1 minus A union B
From 1st sub question, probability of A union B is equal to 7 by 13
On substitution, we get one minus 7 by 13, which is same as 6 by 13.

3. Illustrations 5 - 6

In a college, there are 1500 students, of them 200 play chess, 50 play cricket and 75 play volleyball. 35 students play chess and cricket, 23 play cricket and volleyball, 60 play chess and volleyball. 15 play all three games.

A student selected at random from this college. Find the probability that he plays

i. At least one game
ii. Chess and cricket but not volley ball

In this problem, we have given the college students play 3 games Cricket, Chess and Volley ball.

Let event A denotes selected students play chess
Event B denotes selected students play cricket
Event C denotes selected students play volleyball

In this problem, we have given number of students playing different games. Hence, we can find the probabilities of the above mentioned events.
Probability of A is equal to 200 by 1500
Probability of B is equal to 50 by 1500
Probability of C is equal to 75 by 1500
Probability of A intersection B is equal to 35 by 1500
Probability of B intersection C is equal to 23 by 1500
Probability of A intersection C is equal to 60 by 1500 and
Probability of A intersection B intersection C is equal to 15 by 1500

As A, B and C are not mutually exclusive,
Probability that the selected student plays at least one game
Is equal to probability of A union B union C
Is equal p of A plus p of B plus p of C minus p of A intersection B minus p of B intersection C minus p of A intersection C plus p of A intersection B intersection C
Is equal to 200 by 1500 plus 50 by 1500 plus 75 by 1500 minus 35 by 1500 minus 60 by 1500 minus 23 by 1500 plus 15 by 1500
Is equal to 222 by 1500

In second sub-question, we need to find the selected student plays chess or cricket but not volley ball. That is probability of A intersection B intersection C complement.

We can write
A intersection B is equal to A intersection B intersection C plus C complement
Is equal to A intersection B intersection C plus A intersection B intersection C complement.

Therefore taking probability, we get,
P of A intersection B is equal to probability of A intersection probability of B intersection probability of C plus probability of A intersection probability of B intersection probability of C complement.

Which implies, p of A intersection B inter section C complement is equal to p of A intersection B minus P of A intersection B intersection C
Is equal to 35 by 1500 minus 15 by 1500
Is equal to 20 by 1500 same as 1 by 75

The probability that a student X passing in Mathematics is 2 by 3 and the probability that he passes in statistics is 4 by 9. If the probability of passing at least one subject is 4 by 5, then what is the probability that X will pass both the subjects?

In this problem, we need to find the probability that Mr. X will pass in Mathematics and Statistics. Hence, we define two events A, the student will pass in Mathematics and B, and the student will pass in Statistics.

It is given that probability of A is equal to 2 by 3, probability of B is equal to 4 by 9 and the probability of passing at least one subject that is probability of A union B is equal to 4 by 5.
From addition theorem, we have probability of A union B is equal to p of A plus p of B minus p of A intersection B.
Implies probability of A intersection B is equal to p of A plus p of B minus p of a union B.
Hence the required probability that the student passes in both the subjects,
P of A intersection B is equal to p of A plus p of B minus p of A union B.
Is equal to 2 by 3 plus 4 by 9 minus 4 by 5
Is equal to 14 by 45

4. Illustrations 7- 8

In a C.A. (Foundation) Exam, the probability of A passing is ½, probability of B passing the exam is 1/3 and probability of neither A nor B passing is ¼. Find the probability of A and B passing the exam.

Here we define two events A and B. A is an event when the candidate A is passing in the exam and B, when the candidate B passing the exam.
It is given in the problem that probability of A is equal to half, probability of B is equal to 1 by 3 and probability of neither A nor B passing the exam is p of A complement intersection B complement is equal to 1 by four.

We know that 1 minus probability of A complement intersection B complement is equal to probability of A union B
Implies, probability of A union B is equal to 1 minus 1 by 4 is equal to 3 by 4.
We know that probability of A union B is equal to probability of A plus probability of B minus probability of A intersection B,
Implies p of A intersection B is equal to p of A plus p of B minus p of A union B
Is equal to half plus 1 by 3 minus 3 by 4
Is equal to 1 by 12

A problem in statistics is given to the three students A, B and C whose chances of solving it are half, three by four and one by four respectively. What is the probability that the problem is solved if all of them try independently?

Let A, B, C denote the events that the students A, B, C respectively solve the problem. Then probability of A is equal to half, probability of B is equal to three by four and probability of c is equal to one by four.

The problem will be solved if at least one of them solves the problem. Thus, we have to calculate the probability of occurrence of at least one of the three events A, B, C. That is probability of A union B union C.

Using addition theorem of probability, probability of A union B union C is equal to p of A plus p of B plus p of C minus p of A intersection B minus p of B intersection c minus p of A intersection C plus p of A intersection B intersection C.

Since event A, B and C are independent, we get
probability of A union B union C is equal to p of A plus p of B plus p of C minus p of A into p of B minus p of B into p of C minus p of A into p of C plus p of A into p of B into p of C.

Now by substituting the values in the above equation we get,
Half plus 3 by 4 plus 1 by four minus half into 3 by 4 minus 3 by 4 into 1 by 4 minus half into 1 by 4 plus half into 3 by 4 into 1 by 4 is equal to 29 by 32.

Or same problem we can do using complement event
That is probability of A union B union C is equal to 1 minus probability of A complement

intersection B complement intersection C complement
Since events A, B and C are independent, we get,
1 minus p of A complement into p of B complement into p of C complement
Is equal to 1 minus 1 minus half into 1 minus 3 by 4 into 1 minus 1 by 4
Is equal to 29 by 32

5. Illustrations 9- 10

A can solve 30 percent of the problems in a text, B can solve 40 percent and C can solve 50percent of them. If a randomly selected problem is given to them, what is the probability that is solved?

Let us define the events, A as A solves the problem B as B solves the problem and C as C solves the problem.

From the problem we have,
P of A is equal to 30 percent is equal to zero point 3 and p of A complement is equal to zero point 7
P of B is equal to 40 percent is equal to zero point 4 and p of B complement is equal to zero point 6
P of C is equal to 50 percent is equal to zero point 5 and p of C complement is equal to zero point 5

Assuming events A, B and C are independent, The probability that problem is solved is equal to 1 minus probability that problem is not solved
Implies, probability of A union B union C is equal to 1 minus probability of A complement intersection B complement intersection C complement
Is equal to 1 minus p of A complement into p of B complement into p of C complement
Is equal to 1 minus zero point 7 into zero point6 into zero point 5
Is equal to zero point seven nine

The probability that India wins a cricket match is zero point five two. If India plays three matches, find the probability that it wins

i. At least one match
ii. All the three matches

Let us define the events A as India wins the first match, B as India wins the second match and C as India wins the third match.
Then p of A is equal to p of B is equal to p of C is equal to zero point five two and p of A complement is equal to p of B complement is equal to p of C complement is equal to zero point four eight.

Consider the first sub question
Probability that India wins at least one match is equal to 1 minus wins none
Is equal to 1 minus probability of A complement intersection B complement intersection C complement
Is equal to 1 minus p of A complement into p of B complement into p of C complement
Is equal to 1 minus zero point four eight into zero point four eight into zero point four eight.
Is equal to zero point eight eight nine four.

For second sub question,
Probability that India will wins all the three matches is equal to probability of A intersection B intersection C

Is equal to zero point five two into zero point five two into zero point five two
Is equal to zero point one four zero six.

Here's a summary of our learning in this session:

- Use of addition theorem in finding probabilities for any two events and mutually exclusive events
- Multiplication theorem for independent events in finding the probabilities