Frequently Asked Questions

1. A bag has 5 red and 3 blue balls. Two balls are randomly drawn. Using addition theorem, find the probability that they are of same colour.

Answer:
Let A denote drawing 2 red balls
B denote drawing 2 blue balls
Total No. of outcomes is ${}^8c_2$=(8.7)/(2.1)=28.
No. of outcomes for A is ${}^5c_2$=(5.4)/(2.1)=10.
No. of outcomes for B is ${}^3c_2$=(3.2)/(2.1)=3
P(A)=10/28 and P(B)=3/28

Since A and B are mutually exclusive,
P(AUB)=P(A)+P(B)
= 10/28 +3/28
=13/28

2. In a college, there are five lecturers. Among them, three are doctorates. In a committee consisting three lecturers is formed. What is the probability that at least two of them are doctorates?

Answer:
From the five lecturers, 3 lecturers can be selected in ${}^5c_3$ ways. Thus, there are 10 equally likely, mutually exclusive and exhaustive outcomes.
Let A: two of the selected lecturers are doctorates
B: all the three selected lecturers are doctorates.
Then even A has ${}^3c_2 x {}^2c_1$ =6 favourable outcomes and event B has ${}^3C_3$=1 favorable outcomes. Here events A and B are mutually exclusive,
P[at least two doctorates] = P[2 or 3 doctorates]
= P(AUB)
= P(A)+P(B)
= $\frac{3\times 2}{10}+\frac{1}{10}=\frac{7}{10}$

3. Three groups of children respectively consist of 3 boys and 1 girl; 2 boys and 2 girls; 1 boy and 3 girls. A child is selected at random from each group. What is the probability that the selected group consists of
   i. 2 boys and one girl
   ii. Only boys?

Answer:
i. A-Selecting a girl from 1st group, a boy from 2nd and 3rd group.
B-Selecting a girl from 2nd group, a boy from 1st and 3rd group.
C-Selecting a girl from 3rd group, a boy from 1st and 2nd group.
Total no. of possible selection = 4.4.4=64
P(A)=(1.2.1)/64 = 2/64
P(B)=(3.2.1)/64=6/64
P(C)=(3.2.3)/64=18/64
P(AUBUC)=P(A)+P(B)+P(C)

=2/64+6/64+18/64

= 26/64=13/32

ii. D – Selecting a boy from each group

P(D)=(3.2.1)/64 = 3/32

4. The odds favouring the event of a person hitting a target are 3 to 5. The odds against the event of another person hitting the target are 3 to 2. If each of them fire once at the target, find the probability that both of them hit it.

Answer:

Let A: the first person hits the target.

B: the second person hits the target

Then **odds favouring** event A are 3 to 5. This means if there are 3+5 = 8 cases 3 will be favourable to A and 5 will be against A. Therefore,

P(A)= $\frac{3}{3+5}=\frac{3}{8}$ and P(A') = $\frac{5}{8}$

The **odds against** event B are 3 to 2. This means if there are 3+2=5 case3s, 3 will be against B and 2 will be favourable to B. Therefore,

P(B)= $\frac{2}{3+2}=\frac{2}{5}$ and P(B')= $\frac{3}{5}$

P[both hit the target] = P(A∩B)

= P(A).P(B)

= $\frac{3}{8}\times\frac{2}{5}=\frac{6}{40}$

= 0.15

5. In a firm, 40% of employees are women. Among them it is found that $(1/8)^{th}$ of women and $(1/10)^{th}$ of the men wear spectacles. Find the probability that a randomly selected employee is a man or a person wearing spectacles.

Answer:

Let A - employee selected being man

B – employee selected being wearing spectacle.

P(A)=(100-40)%=0.6

P(B)=(1/8.40%)+(1/10)=0.11 and

P(A ∩B)=(1/10).60%=6/100=0.06

P(man or a person wearing spectacles)

=P(AUB)

=P(A)+P(B)-P(A ∩B)

=0.6+0.11-0.06

=0.46

6. One card is drawn from a pack of 52 cards, each of the 52 cards being equally likely to be drawn. Find the probability of

i. The card drawn is either red or a king,

ii. The card drawn is neither red nor a king.

Answer:
A – The card is red
B – The card is a king
$P(A) = 26/52$, $P(B) = 4/52$ $P(A \cap B)=2/52$

i. P(card drawn is red or king)

$=P(AUB)$

$=P(A)+P(B)-P(A \cap B)$

$=26/52+4/52-2/52$

$=28/52=7/13$

ii. P(card drawn is neither red nor king)

$= P(A' \cap B')$

$= 1 – P(AUB)$

$= 1 – 7/13 = 6/13.$

7. In a college, there are 1500 students. Of them 200 play chess, 50 play cricket and 75 play volley ball. 35 play chess and cricket, 23 play cricket and volley ball, 60 play chess and volley ball. 15 play all three games. A student selected at random from this college. Find the probability that he plays

i. At least one game

ii. Chess and cricket but not volleyball

Answer:

i. A- Selected student playing chess
B- Selected student playing cricket
C - Selected student playing volley ball.
it is given,
$P(A)=200/1500$, $P(B)=50/1500$, $P(C)=75/1500$, $P(A \cap B)=35/1500$, $P(B \cap C)=23/1500$, $P(A \cap C)=60/1500$, $P(A \cap B \cap C)=15/1500$.
$P(AUBUC)=P(A)+P(B)+P(C)-P(A \cap B)- P(B \cap C)-P(A \cap C)+P(A \cap B \cap C)$
$=200/1500+50/1500+75/1500- 35/1500 - 23/1500-60/1500+15/1500$
$= 222/1500$

ii. We should find $P(A \cap B \cap C')$

We can write $A \cap B= A \cap B \cap (C+C')= A \cap B \cap C+A \cap B \cap C'$
Therefore, $P(A \cap B)=P(A \cap B \cap C)+P(A \cap B \cap C')$
Implies, $P(A \cap B \cap C') = P(A \cap B) – P(A \cap B \cap C)$
$= 35/1500 – 15/1500$
$= 20/1500=1/75$

8. The probability that a student Mr. X passed Mathematics is 2/3; the probability that he passes statistics is 4/9. If the probability of passing at least one subject is 4/5, what is the probability that Mr. X will pass both the subjects?

Answer:
Let A and B denote the events that the student will pass in Mathematics and Statistics.
$P(A)=2/3$, $P(B)=4/9$ and $P(AUB)=4/5$

From addition theorem of probability
$P(AUB) = P(A) + P(B) - P(A\cap B)$
Implies, $P(A\cap B)= P(A) + P(B) - P(AUB)$
$= 2/3 + 4/9 – 4/5$
$= 14/15$

9. In a C.A. (Foundation) Exam, the probability of A passing is ½, probability of B passing the exam is 1/3 and probability of neither A nor B passing is ¼. Find the probability of A and B passing the exam.

Answer:

Let A – Candidate A passing the exam

B – Candidate B passing the exam.

$P(A)=1/2$, $P(B) = 1/3$, $P(A'\cap B') = 1/4$

$1- P(A'\cap B') = P(AUB)$

Implies, $P(AUB) = 1- 1/4 =3/4$

We know that, $P(AUB) = P(A) + P(B) - P(A\cap B)$

Implies, $P(A\cap B)= P(A) + P(B) - P(AUB)$

$= 1/2 + 1/3 -3/4$

$= 1/12$

10. Out of the numbers 1 to 100, one is selected at random. What is the probability that it is divisible 7 or 8.

Answer:
The sample space in the random experiment is,
S={1, 2, 3, ..., 99, 100}
The number of elements in the sample space is 100
A : the number chosen is divisible by 7.
B : the number chosen is divisible by 8.
Sample points of A is given by, A = {7, 14, ..., 98} i.e., 98/7=14 elements and
Sample points of B is given by, B = {8, 16, ..., 96} i.e., 96/8=12 elements.
Also, $A\cap B=\{56\}$ i.e., 1 element.
Hence, $P(AUB)=P(A)+P(B)-P(A\cap B)$

$$= \frac{14}{100}+\frac{12}{100}-\frac{1}{100}=\frac{25}{100}=\frac{1}{4}$$

11. A problem in statistics is given to the three students A, B and C whose chances of solving it are ½, ¾ and ¼ respectively. What is the probability that the problem is solved if all of them try independently?

Answer:
Let A, B, C denote the events that the problem is solved by the students A, B, C respectively.
Then P(A)=½, P(B)= ¾ and P(C)= ¼.

The problem will be solved if at least one of them solves the problem. Thus we have to calculate the probability of occurrence of at least one of the three events A, B, C i.e., P(AUBUC)

P(AUBUC)=P(A)+P(B)+P(C)-P(A∩B) - P(B∩C)-P(A∩C)+P(A∩B ∩C)

Implies, P(AUBUC)=P(A)+P(B)+P(C)-P(A)P(B)- P(B)P(C)-P(A)P(C)+P(A)P(B)P(C)

$= \frac{1}{2}+\frac{3}{4}+\frac{1}{4}-\frac{1}{2}.\frac{3}{4} - \frac{3}{4}.\frac{1}{4} - \frac{1}{2}.\frac{1}{4}+\frac{1}{2}.\frac{3}{4}.\frac{1}{4}$

= 29/32

OR

P(AUBUC)=1- P(A'∩B'∩C')

= 1- P(A').P(B').P(C')

$= 1- (1-\frac{1}{2}).(1-\frac{3}{4}).(1-\frac{1}{4})$

= 29/32

12. A can solve 30% of the problems in a text, B can solve 40% and C can solve 50% of them. If a randomly selected problem is given to them, what is the probability that is solved?

Answer

Let A – A solves the problem

B – B solves the problem

C – C solves the problem

P(A) = 30% = 0.3 and P(A') = 0.7

P(B) = 40% = 0.4 and P(B') = 0.6

P(C) = 50% = 0.5 and P(C') = 0.5

Assuming events A, B and C are independent

P(problem is solved)=1-P(problem is not solved)

Implies,

P(AUBUC) = 1- P(A'∩B'∩C')

= 1- P(A').P(B').P(C')

= 1- 0.7x0.6x0.5

= 0.79

13. The probability that India wins a cricket match is 0.52. if India plays three matches, find the probability that it wins

i. At least one match

ii. All the three matches

Answer

Let A – India wins the 1st match

B – India wins the 2nd match

C – India wins the 3rd match

P(A) = P(B) = P(C) = 0.52 and

P(A') = P(B') = P(C') = 0.48

As A, B and C are independent,

i. P[India wins at least one match]

=1 – P[wins none]

=1- P(A'∩B'∩C')

$= 1 - P(A')\cdot P(B')\cdot P(C')$
$= 1 - 0.48 \times 0.48 \times 0.48$
$= 0.8894$

ii. P[wins all matches] $= P(A \cap B \cap C)$
$= P(A)P(B)P(C)$
$= 0.52 \times 0.52 \times 0.52$
$= 0.1406$

14. If birth of a male child and birth of a female child are equi-probable, what is the probability that at least one of the three children born to a couple is male?

Answer

Let A – the first child is male
B – The second child is male
C – The third child is male
$P(A)=P(A')=P(B)=P(B')=P(C)=P(C') = ½$
As A, B and C are independent,

i. P[At lest one male child]
$=1 - P[\text{No male child}]$
$=1 - P(A' \cap B' \cap C')$
$= 1 - P(A')\cdot P(B')\cdot P(C')$
$= 1 - ½ \times ½ \times ½$
$= 7/8$

15. The probability that a bomb dropped from Bomber A hits the target is 0.2. The probability that a bomb dropped from Bomber B hits the target is 0.3. Both these bombers drop a bomb each on a bridge. Find the probability that the bridge is hit.

Answer:

Let A: bomb from Bomber A hits the bridge and
B: bomb from Bomber B hits the bridge.
$P(A)=0.2$ and $P(A')=0.8$. $P(B)=0.3$ and $P(B')=0.7$
Here events A and B are independent,
P[bridge is hit] = 1 – P[bridge is not hit]
$=1 - P(A' \cap B')$
$= 1 - P(A')\cdot P(B')$
$= 1 - 0.8 \times 0.7 = 0.44.$