1. Introduction

Welcome to the series of E-learning modules on absolute and relative measures of dispersion (Part I).

By the end of this session, you will be able to:

- Calculate the following absolute and relative measures of dispersion for various statistical series.
    - Range and coefficient of range
    - Quartile deviation and coefficient of quartile deviation
    - Mean deviation and coefficient of mean deviation
    - Mean deviation when central values are in fraction

Introduction:
Dispersion indicates the measure of the extent to which the individual items differ. It indicates the lack of uniformity in the size of the items. These measures give an average of differences of various items from an average and hence they are termed as the averages of the second order.

Sometimes when the series are of the same population, we use the absolute measure for dispersion. When there is a set of series from different population, then we use the relative measures of dispersion in order to make the series comparable.

Let us calculate the Range and coefficient of range for individual, discrete and continuous series:

Problem 1–Individual series
Find the range and its coefficient of range from the following data of daily wages of workers.

Table 1

Wages(Rs.)	22	35	32	45	42	48	39

Solution:
Let us calculate the range of daily wages. The formula for calculation of range is the highest value minus the lowest value in the data. Here, the highest value is equal to 48 and the lowest value is equal to 22. Therefore, the range is equal to 48-22 is equal to 26. Thus, the absolute measure range of daily wages is equal to Rs.26.
Let us now calculate the relative measure of the coefficient of range. We use the formula coefficient of range is equal to the differences of the highest value & lowest value divided by the sum of the highest value and the lowest value. Here, the coefficient of range is equal to 48 minus 22 divided by 48 plus 22 is equal to 26 divided by 70 is equal to 0.37.

Problem 2 – Discrete series

Find the range and coefficient of range of the following distribution.

Table 2

Expenses (Rs.)	3	4	5	6	7	8	9	10
Students	35	30	20	10	6	3	2	1

Solution:
Let us calculate the range of expenses. The formula for calculation of range is the highest value minus the lowest value in the data. Here, the highest value is equal to 10 and the lowest value is equal to 3, therefore the range is equal to 10 minus 3, which is equal to 7. Thus, the absolute measure range of expenses is equal to Rs.7.

Let us now calculate the relative measure of the coefficient of range. We use the formula coefficient of range is equal to the differences of the highest value & lowest value divided by the sum of the highest value and the lowest value. Here, the coefficient of range is equal to 10 minus 3 divided by 10 plus 3 is equal to 7 divided by 13 is equal to 0.54 approximately.

Problem 3 – Continuous series
Calculate the range and its coefficient from the following distribution:

Table 3

Marks	5-10	10-15	15-20	20-25	25-30	30-35
Students	6	11	7	4	3	1

Solution:
Let us calculate the range of marks. The formula for calculation of range is the highest value minus the lowest value in the data. Here the highest value is equal to 35 and the lowest value is equal to 5, therefore, the range is equal to 35 minus 5 is equal to 30. Thus, the absolute measure range of marks is equal to 30.

Let us now calculate the relative measure the coefficient of range. We use the formula coefficient of range is equal to the differences of the highest value and lowest value divided by the sum of the highest value and the lowest value. Here the coefficient of range is equal to 35 minus 5 divided by 35 plus 5 is equal to 30 divided by 40 is equal to 0.75.

2. Quartile Deviation and Coefficient of Quartile Deviation – Part 1

Let us not calculate the Quartile deviation and coefficient of quartile deviation for individual, discrete and continuous series.

Problem 4 – Individual series.
Calculate the quartile deviation and its coefficient for the following data.

Table 4

Months	Jan	Feb	Mar	Apr	May	June	July	Aug	Sep	Oct	Nov	Dec
Sales (Rs.) in lakhs	55	60	70	90	90	110	120	130	145	145	155	170

Solution:
For the calculation of the quartile deviation, first we need to know the values of the lower quartile and the upper quartile of the data. So let us calculate the value of Q1 and Q3.

The values are already arranged in ascending order. So, first let us calculate the lower quartile Q1 is equal to size of N plus 1 divided by 4^{th} item is equal to 12 plus 1 divided by 4 is equal to 13 divided by 4 is equal to 3.25^{th} item, which is equal to the size of the 3^{rd} item plus one fourth of the (4^{th} item minus the third item) is equal to 70 plus one fourth of (90 minus 70) is equal to 70 plus 5 is equal to 75.

Next, we will calculate the value of the upper quartile Q3, which is equal to thrice N plus 1 by 4^{th} item, which is equal to 3 into 3.25 is equal to 9.75^{th} item, which is equal to the sum of the 9^{th} item plus three fourth of the 10^{th} item minus the 9^{th} item, is equal to 145 plus three fourth of 145 minus 145 is equal to 145 plus zero is equal to 145.

Now, by substituting the value in the equation we will get the quartile deviation is equal to Q3 minus Q1 divided by 2 is equal to 145 minus 75 by 2 is equal to 35.

Then, we will calculate the coefficient of quartile deviation is equal to the difference of the quartiles divided by the sum of the quartiles which is equal to 145 minus 75 divided by 145+75 is equal to 70 divided by 220 is equal to 0.318.

Problem 5 – Discrete series
Calculate the quartile deviation and the coefficient of quartile deviation from the following data.

Table 5

Age in Years	50	51	52	53	54	55	56
Teachers	10	12	15	10	14	18	6

Solution:
For the calculation of the quartile deviation, first we need to know the values of the lower quartile and the upper quartile of the data. So, let us calculate the value of Q1 and Q3.
Let us calculate the cumulative frequency first, we get, 10, 22, 37, 47, 61, 79 & 85.

Table 6

Age in Years	50	51	52	53	54	55	56
Teachers	10	12	15	10	14	18	6
Cumulative frequency	10	22	37	47	61	79	85

3. Quartile Deviation and Coefficient of Quartile Deviation – Part 2

Problem 6 – continuous series.
Calculate the quartile deviation and its coefficient from the following data.

Table 7

Marks	0-10	10-20	20-30	30-40	40-50
Students	4	15	28	16	7

Solution:
For the calculation of the quartile deviation, first we need to know the values of the lower quartile and the upper quartile of the data. So let us calculate the value of Q1 and Q3.
First, while calculating the cumulative frequency, we get 4, 19, 47, 63 and 70.

Table 8

Marks	0-10	10-20	20-30	30-40	40-50
Students	4	15	28	16	7
Cumulative Frequency	4	19	47	63	70

Let us calculate the lower quartile Q1 is equal to size of N divided by 4^{th} item is equal to 70 divided by 4 is equal to 17.5^{th} item which is equal to the cumulative frequency 19 and the respective class interval is 10-20.

Now, we will have to interpolate the value using the formula Q1 is equal l1 plus N by 4minus cumulative frequency divided by f where l1=10, N/4=17.5, cf=4, f=15, c=10 by which we will get Q1 is equal to 19.

Next, we will calculate the value of the upper quartile Q3, which is equal to 3 N by 4^{th} item, which is equal to 3 into 70 by 4 is equal to 52.5^{th} item, which is equal to the cumulative frequency 63 and the respective class interval is 30-40.

Now, we will have to interpolate the value using the formula Q3 is equal l1 plus 3N by 4minus cumulative frequency divided by f, where l1=30,3 N/4=52.5, cf=47, f=16, c=10 by which we will get Q3 is equal to 33.44.

Now, by substituting the value in the equation we will get the quartile deviation is equal to Q3 minus Q1 divided by 2 is equal to 33.44 minus 19 divided by 2 is equal to 7.22.

Then, we will calculate the coefficient of quartile deviation which is equal to the difference of the quartiles divided by the sum of the quartiles which is equal to 33.4 minus 19 divided by 33.4 plus 19 is equal to 14.44 divided by 52.44 is equal to 0.275.

4. Mean Deviation and Coefficient of Mean Deviation

Now calculate the Mean deviation and coefficient of mean deviation for individual, discrete and continuous series.

Problem 7 – Individual series

The following are the room rents charged by a hotel for various type of accommodation provided to the occupants: Rs. 300, 400, 700, 200, 600, 500, 100. Calculate the mean deviation and its coefficient.

Solution:

As a first step, we will arrange the given data in ascending order we will get the data as 100, 200, 300, 400, 500, 600, 700. Now, we will take the deviations from the median of the data. The median is calculated as N plus 1 divided by 2 is equal to 7plus 1 by 2 is equal to 8 by 2 is equal to the 4^{th} item which is equal to 400. For calculation purpose, we take the deviation as the absolute d, which ignores the sign. Therefore, we get the following values for the absolute d they are 300, 200, 100, 0, 100, 200, 300 and its total is 1200.

Table 9

Rent (x)	d (signs ignored)
100	300
200	200
300	100
400	0
500	100
600	200
700	300
	Σ d = 1200

Now, let us calculate the mean deviation using the formula. Mean deviation is equal to the summation of absolute d, divided by N, which is equal to 1200 divided by 7 is equal to 171.43. The coefficient of mean deviation is equal to the mean deviation divide by median which is equal to 171.43 divided by 400 is equal to 0.429.

Problem 8 – Discrete series

Calculate mean deviation and its coefficient from the following data:

Table 10

Height	61	64	67	70	73
Persons	5	18	42	27	8

Solution:
First, calculate the cumulative frequency of the given data so that we will get the values 5, 23, 65, 92 and 100. Then we will calculate the median by taking the formula N plus 1 divided by 2 is equal to 100 plus 1 divided by 2 is equal to 101 divided by 2 is equal to 50.5^{th} item. Hence, the median variable is 67 having a cumulative frequency of 65.

Table 11

Height (x)	Persons (f)	Cumulative Frequency (cf)
61	5	5
64	18	23
67	42	65
70	27	92
73	8	100

The next step is to calculate the mean deviation and its coefficient for which we need to calculate the deviation absolute d from the median and multiply it by the frequency f. We will get the value of summation f absolute d. Here the values of absolute d are 6, 3, 0, 3, 6, and the f absolute d values are 30, 54, 0, 81 and 48 and the value of summation f absolute d is equal to 213.

Table 12

Height (x)	Persons (f)	Cumulative Frequency (cf)	|d|	f|d|
61	5	5	6	30
64	18	23	3	54
67	42	65	0	0
70	27	92	3	81
73	8	100	6	48
				$\Sigma f\|d\| = 213$

So the mean deviation is equal to summation absolute d divided by N is equal to 213 divided by 100 is equal to 2.13 and the coefficient of mean deviation is equal to mean deviation divided by median is equal to 2.13 divided by 67 is equal to 0.32.

Problem 9 – Continuous series.
Calculate the mean deviation and its coefficient both from mean and median for the following.

Table 13

Marks	0-20	20-40	40-60	60-80	80-100
Students	10	16	30	32	12

Solution:
The need of this problem is we have to calculate both the mean and the median and then find the mean deviation from both the values. So to calculate the mean we will first take the mid values of the class interval, we will get 10, 30, 50, 70 and 90. In the next step, we will calculate the deviation dx dash by using the formula x minus A divided by c, we will get minus 2, minus 1, 0, 1 and 2. The next step is to calculate the value of fdx dash we will get minus 20, minus 16, 0 32 and 24. Calculate the mean for the data we get mean is equal to A plus summation fdx dash divided by N into C. by substituting the values we will get mean is equal to 50 plus 20 divided by 100 into 20 is equal to 54.

Table 14

Marks	Students	Mid Value	dx′	fdx′
0-20	10	10	-2	-20
20-40	16	30	-1	-16
40-60	30	50	0	0
60-80	32	70	+1	32
80-100	12	90	+2	24
	N=100			Σfdx′= 20

The next step is to calculate the mean deviation and the coefficient of mean deviation, for which we will calculate the absolute d from the mean and f absolute d to get the value of summation f absolute d. The absolute d are 44, 24, 4, 16 and 36. The values of f absolute d are 440, 384, 120, 512 and 432. The value of summation f absolute d is 1888.

Table 15

Marks	Students	Mid Value	|d|	f|d|
0-20	10	10	44	440
20-40	16	30	24	384
40-60	30	50	4	120
60-80	32	70	16	512
80-100	12	90	36	432
	N=100			$\Sigma f\|d\| =$ 1888

The mean deviation is equal to 1888 divided by 100 is equal to 18.88 or 19 marks. The coefficient of mean deviation is equal to mean deviation by mean is equal to 18.88 divided by 54 is equal to 0.349.

The second part of the question asks us to calculate the mean deviation from the median for which first we need to calculate the cumulative frequency of the data we will get the values as 10, 26, 56, 88 and 100. Now, we will calculate the median by using the formula N divided by 2 is equal to 100 divided by 2 is equal to 50, which is equal to 40-60 group as the median class.

Now we need to interpolate the value of the median by using the formula median is equal to the lower limit plus N divided by 2 minus the cumulative frequency of the preceding class divided by the frequency of the median class. So we will get 40 plus 50 minus 26 divided by 30 into 20 is equal to 40 plus 16 is equal to 56.

Table 16

Marks	Students	cf
0-20	10	10
20-40	16	26
40-60	30	56
60-80	32	88
80-100	12	100
	N=100	

Next, we calculate the absolute d from the median as 46, 26, 6, 14, 34 and then we calculate the value of f absolute d is equal to 460, 416, 180, 448 and 408, and the value of summation of f absolute d is equal to 1912.

Table 17

Marks	Students	|d|	f|d|
0-20	10	46	460
20-40	16	26	416
40-60	30	6	180
60-80	32	14	448
80-100	12	34	408
	N=100		Σf|d|= 1912

Therefore, the mean deviation is equal to 1912 divided by 100 is equal to 19.12 or 19 and the coefficient of mean deviation is equal to mean deviation divided by median is equal to 19.12 divided by 56 is equal to 0.341.

5. Mean Deviation from Median or Mean or Mode

Mean deviation from median or mean or mode when this value is in a fraction:
Problem 10- Mean as a fraction
From the following, find the mean deviation from its mean and its coefficient.

Table 18

Wages	10-20	20-30	30-40	40-50	50-60	60-70
workers	1	3	5	20	10	5

Solution:
To calculate the mean deviation from the mean we will first take the mid values of the class interval, we will get 15, 25, 35, 45, 55 and 65. Next we will calculate summation fx by multiplying the variable x with the frequency we will get 15, 75, 175, 900, 550 and 325. The value of summation fx is equal to 2040, the value of mean is equal 2040 divided by 44 is equal to 46.36.

Table 19

Wages	Workers	Mid Value	fx
10-20	1	15	15
20-30	3	25	75
30-40	5	35	175
40-50	20	45	900
50-60	10	55	550
60-70	5	65	325
	N=44		Σfx=2040

As the mean is a fraction value, we will calculate the mean deviation by dividing the data into two halves. The first half will have around four values and named B the second half will have 2 values and named as A. The total of the frequency of these halves are taken separately as summation fB is equal to 29 and summation fA is equal to 15. Similarly, the product of f and x is also taken as summation of fxB as 1165 and summation of fxA as 875.

Table 20

Wages	Workers		Mid Value	fx	
10-20	1		15	15	
20-30	3		25	75	
30-40	5	Σ fB = 29	35	175	ΣfxB=1165
40-50	20		45	900	
50-60	10		55	550	
60-70	5	ΣfA= 15	65	325	ΣfxA=875
	N=44			Σfx=2040	

Now, these values are substituted in the equation mean deviation, which is equal to summation fxA minus summation fxB minus of Summation fA minus summation fB into mean divided by N is equal to 875 minus 1165 – of 15 minus 29 into 46.36 divided by 44 is equal to minus 290 plus 649.04 divided by 44 is equal to 359.04 divided by 44 is equal 8.16. The coefficient of mean deviation is equal to mean deviation divided by mean is equal 8.16 divided by 46.36 is equal to 0.176.

In this session We have understood the calculation of the:

- Range and coefficient of range
- Quartile deviation and coefficient of quartile deviation
- Mean deviation and coefficient of mean deviation
- Mean deviation when central values are in fraction