1. Introduction

Welcome to the series of e-Learning modules on Arithmetic Mean. In this module we are going to discuss about the basic concept of Arithmetic Mean.

At the end of this session, you will be able to:
- Explain what is Arithmetic Mean
- Explain what are the types of Arithmetic Mean
- Explain how to calculate Arithmetic Mean in:
    - Individual Series
    - Discrete Series
    - Continuous Series
- Explain the properties of Arithmetic Mean

Mean is the most widely used measures for representing the entire data. It is also known as Arithmetic Mean. Mean is divided into two types- Simple Arithmetic Mean and Weighted Mean.

Thus, arithmetic Mean is obtained by adding values of all the items and dividing their sum by the number of items.

Horace Sacrist defined Arithmetic Mean as "Arithmetic mean is the amount secured by dividing the sum of values of the items in a series by their number".

2. Calculation of Arithmetic Mean in Individual Series.

We all are now familiar with the meaning of arithmetic mean. Now we will move forward to the calculations of arithmetic mean.

The calculation of arithmetic mean in case of individual series is very simple.

Individual observations or series mean is the one where frequencies are not given. Here, all the different values of the items are added and are divided by the total number of items.

There are three different methods of calculating Arithmetic mean in individual series. Let us see what the methods are.
- Direct Method
- Short-Cut Method
- Step Deviation Method also known as Coding Method.

Let us see how to calculate the Arithmetic mean in Direct method.
We can get the arithmetic mean by applying this formula:

$$\bar{x} = \frac{\sum x}{n}$$ **(Formula 1)**

Where, , $\bar{x}$ = Arithmetic mean or sample mean.

$\sum x$ = sum of all the items or observations of the x.
n = the total number of items.

Let us make it easier by taking an example.
We have taken daily expenditure of five families in a city.
Expenditure in Rs. Are 30, 70, 40, 20, and 60. (These are termed as x)

Figure1

x	30	70	40	20	60

Formula:

$$\bar{x} = \frac{\sum x}{n}$$ **(Formula 2)**

Here, $\sum x$ is the total of all the expenditures given in the series i.e. 220.
n = total number of families i.e. 5.

$\overline{x}$ **= 220/5 = 44.**

Thus, the average daily expenditure is Rs. 44.

Now we will calculate the arithmetic mean using short cut method.

 The arithmetic mean can be calculated by using assumed mean. When deviations are taken from assumed mean then the formula for calculating arithmetic mean is:

$$\overline{x} = A + \frac{\sum dx}{n}$$ **(Formula 3)**

Where, A is the assumed mean.
dx stands for the deviation of the items from the assumed mean (x-A).

Let us take the same example of daily expenditure of five families.
Here, in this given series of expenditures we have assumed that A is 40.

We will get the value of dx by subtracting the individual daily expenditure with the assumed value A.

See this table: Here in the first row the expenditure of Rs. 30 is subtracted by the assumed value i.e. 40. And the result is -10. In this way we can get the value of $\sum dx$ . .

Formula:

$$\overline{x} = A + \frac{\sum dx}{n}$$ **(Formula 4)**

Here the value of is $\sum dx$ 20
n = 5, and A = 40.

$\overline{x}$ **= 40 + 20/5**
$\overline{x}$ **= 40 + 4 = 44**
Thus, the average daily expenditure is Rs. 44.

 Now we move forward to the step deviation method.

$$\overline{x} = A + \frac{\sum dx'}{n} \times C$$

Formula for this method is **(Formula 5)**

Where, A is the assumed mean,
dx is the deviation from x by applying the formula (x-A) and divide it by C.
C is the common factor.

n is the total number or items.

For this method also we will take the same example.

Here, in this given series of expenditures we have assumed 20 as A.

We will get the value of dx' by subtracting the individual daily expenditure with the assumed value A and then divide the result by C.

See this table: Here in the first row the expenditure of Rs. 30 is subtracted by the assumed value i.e. 20. And the result is -10 is then divided by C i.e. 20. In this way we can get the value of $\sum dx'$.

Formula:
$$\bar{x} = A + \frac{\sum dx'}{n} \times C$$
(Formula 6)
Here, A is 20, n is 5, dx' is 22, and C=5.
$$\bar{x} = 20 + \frac{24}{5} \times 5$$

$$\bar{x} = 20 + 24 = 44$$

Thus, the average daily expenditure is Rs. 44.

3. Calculation of Arithmetic Mean in Discrete Series.

Now we will discuss about the discrete series and how to calculate arithmetic mean in discrete series.

The series dealing with discrete variables is known as Discrete Series. Discrete series cannot be expressed in fractions.

In discrete series, arithmetic mean may be calculated applying either:
Direct method, Short cut method and Step deviation method.

In case of direct method, the formula for calculating mean is:

$$\bar{x} = \frac{\sum fx}{n}$$ **(Formula 7)**

Where, $\bar{x}$ = Arithmetic mean or sample mean,
f is the frequency,
x is the value of the variable,
n is the sum of the frequency or n =

Figure 2

Wages (Rs.)	No. of Persons
20	3
10	4
50	2
80	5
60	3
50	5
90	3

Example: Wages of persons working in a factory.

This table shows the wages column and the no. of person's column. Let's find the Arithmetic mean.

We have created a separate table to calculate fx.

Figure 3

Wages (Rs.) x	No. of Persons f	fx
20	3	60
10	4	40
50	2	100
80	5	400
60	3	180
50	5	250
90	3	270
	N or Σf = 25	1300

Here, the wages is x and no. of persons is f, on multiplying the wages and no. of persons i.e. (x*f) we will get fx.

Formula:

$$\bar{x} = \frac{\sum fx}{n}$$ **(Formula 8)**

Here, $\sum fx$ = 1300(wages *no. of persons),
n= 5 (total no. of persons).

$\bar{x}$ = 1300/25 = 52
Hence average wages is Rs. 52.

According to short cut method the formula is:

$$\bar{x} = A + \frac{\sum fdx}{n}$$ **(Formula 9)**

Where, A is assumed mean
dx is a deviation of items from assumed mean (x-A)
f is a frequency
n = total number of observations.

We have taken the same example of wages of persons. This table helps us in finding the dx and fdx values.

Figure 4

Wages (Rs.) x	No. of Persons f	$dx=(x-A)$	fdx
20	3	20-50 = -30	-90
10	4	10-50 = -40	-160
50	2	50-50 = 0	0
80	5	80-50 = +30	+150
60	3	60-50 = +10	+30
50	5	50-50 = 0	0
90	3	90-50 = +40	+120
Totals	N or Σf = 25		50

As we know that that on subtracting x to A we will get dx.
Here, in the first row of this example x(wages) is 20 and A(Assumed mean) is 50, thus, dx is -30. Similarly all the other items in the series are calculated.

In order to get fdx, we have to multiply the calculated dx with f(frequency). We will take the first row of this table. Here, dx is -30 and f is 3. Thus, fdx is -90. Similarly all the other items in the series are calculated to get fdx.

$$\overline{x} = A + \frac{\sum fdx}{n}$$ **(Formula 10)**

Formula:

Here, n or $\quad$ = 25
A is 50,
$\sum fdx$ = 50.

$$\overline{x} = 50 + \frac{\sum 50}{25}$$

Thus,

$$\overline{x} = 50 + 2$$

$$\overline{x} = 52$$

Hence average wages is Rs. 52.

In case of step deviation method the formula is: $\bar{x} = A + \dfrac{\sum fdx'}{n} \times C$ **(Formula 11)**

Where, A is assumed mean
f is a frequency
C is a common factor
dx' is deviation of items from assumed mean (x-A) and divide by common factor C.
fdx' is calculated by multiplying dx' with frequency

In this table dx' and fdx' is calculated.

Figure 5

Wages (Rs.) x	No. of Persons f	$dx'=(x-A)/C$	fdx'
20	3	20-20/5 = 0	0
10	4	10-20/5 =-2	-8
50	2	50-20/5 =+6	+12
80	5	80-20/5 =+12	+60
60	3	60-20/5 =+8	+24
50	5	50-20/5 =+6	+30
90	3	90-20/5 =+14	+42
Total	n = 25		fdx' = 160

Here, in the first row of this example x(wages) is 20 and A(Assumed mean) is 20 and the result is divided by C 5 (i.e. common factor), thus, dx' is 0. Similarly all the other items in the series are calculated.

In order to get fdx', we have to multiply the calculated dx with f(frequency). We will take the first row of this table. Here, dx is 0 and f is 3. Thus, fdx is 0. Similarly all the other items in the series are calculated to get fdx.

As per the formula:

$\bar{x} = A + \dfrac{\sum fdx'}{n} \times C$ **(Formula 12)**

We will get the value of n or $\sum f$ that is 25,
A is 20,
$\sum fdx'$ = 160 and C is 5

$$\bar{x} = 20 + \frac{160}{25} \times 5$$

$$\bar{x} = 20 + 32$$

$$\bar{x} = 52$$

Hence average wages is Rs. 52.

4. Calculation of Arithmetic Mean in Continuous Series.

Now we will see how to calculate the arithmetic mean in case of continuous series.

The series dealing with the continuous variable is called continuous series. The continuous variable is one which can assume any conceivable value within a range. For example: income, weight, length etc.

In continuous series, arithmetic mean may be calculated by applying Direct method, short cut method and step deviation method.

In case of direct method, the formula for calculating mean is:

$$\bar{x} = \frac{\sum fx}{n}$$ **(Formula 13)**

In this method of calculating arithmetic mean, we have to find out the mid value of each class.
Then find n which is the sum of the frequency or n =
f is the frequency,
x is the value of the variable.

This time we have taken an example of a frequency distribution of marks of students

Figure 6

Marks	No. of Students
0–10	5
10–20	10
20–30	8
30–40	12
40–50	10
50–60	20

This table shows the marks and the no. of students. Let us calculate the mean by direct

method.
We have created a separate table to calculate the mid value i.e. x and fx.

Figure 7

Marks	No. of Students (f)	Mid–Value (x)	(fx)
0–10	5	5	25
10–20	10	15	150
20–30	8	25	200
30–40	12	35	420
40–50	10	45	450
50–60	20	55	1100
	n = 65		Σfx = 2345

To find out the mid value we have to add the lower class limit and upper class limit and the result is divided by 2.

Let's take the 3rd row of this table and calculate the mid value i.e x.
Here, lower class limit is 20
Upper class limit is 30
By adding both the classes we will get 20+30=50.
Then 50 is divided by 2, so that we will get the value of x i.e 25. Similarly all the other items in the series are calculated.

To find out fx, we have to multiply the calculated mid value (x) with f(frequency). We will take the third row of this table. Here, x is 25 and f is 8. Thus, fx is 200. Similarly all the other items in the series are calculated to get fx.

As per the formula:

$$\bar{x} = \frac{\sum fx}{n}$$ **(Formula 14)**

As per the formula:

$\sum fx$ = 2345 and n= 65 (which is the total number of students)

$\bar{x}$ = 2345/65

$\bar{x}$ = 36.07

Hence, the average marks of students are 36.07.

In case of short cut method, we apply this formula to calculate arithmetic mean.

$$\overline{x} = A + \frac{\sum fdx}{n}$$ **(Formula 15)**

Where, A is assumed mean
dx is deviation of mid-points from assumed mean (x-A)
n is the total number of observations
f is the frequency.

In the same example of calculating the marks of the students we have to calculate dx and fdx in short cut method.

Figure 8

Marks	No. of Students (f)	Mid–Value (x)	dx=(x-A)	(fdx')
0-10	5	5	5-25 = -20	-100
10-20	10	15	15-25 = -10	-100
20-30	8	25	25-25 = 0	0
30-40	12	35	35-25 = +10	+120
40-50	10	45	45-25 = +20	+200
50-60	20	55	55-25 = +30	+600
	n = 65			= 720

As we know that that on subtracting x to A we will get dx.
This time we will take the last row of this table, where x(is the mid value) is 55 and A(Assumed mean) is 25, thus, dx is +30. Similarly all the other items in the series are calculated.

In order to get fdx, we have to multiply the calculated dx with f(frequency). We will take the last row of this table. Here, dx is +30 and f is 20. Thus, fdx is +600. Similarly all the other items in the series are calculated to get fdx.

As per the formula:

$$\bar{x} = A + \frac{\sum fdx}{n}$$ **(Formula 16)**

As per the formula:

Here, n is 65, A is 25, and fdx is 720.
$$\bar{x} = 25 + \frac{720}{65}$$

$$\bar{x} = 25 + 11.07$$

$$\bar{x} = 36.07$$

Hence, the average marks of students are 36.07.

In step deviation method, the calculation of short cut method is further simplified. In this method we take the common factor from the data and then multiply the result with the common factor, the formula is:

$$\bar{x} = A + \frac{\sum fdx'}{n} \times C$$ **(Formula 17)**

Where, A = assumed mean
C = common factor
f = frequency
dx' = deviation of items from assumed mean (x-A) and divide by common   factor C.
fdx' = by multiplying dx' with frequency.

In this table dx' and fdx' is calculated.

Figure 9

Marks	No. of Students (f)	Mid-Value (x)	dx'=(x-A)/C	(fdx')
0-10	5	5	5-35/10 = -3	-15
10-20	10	15	15-35/10 = -2	-20
20-30	8	25	25-35/10 = -1	-8
30-40	12	35	35-35/10 = 0	0
40-50	10	45	45-35/10 = +1	+10
50-60	20	55	55-35/10 = +2	+40
	n = 65			fdx' = 7

Here, in the fifth row of this example x(that is the mid value) is 45 and A(Assumed mean) is 35 and the result is divided by C, 10 (i.e. common factor), thus, dx' is +1. Similarly all the other items in the series are calculated.

In order to get fdx', we have to multiply the calculated dx with f(frequency). We will take the first row of this table. Here, dx is +1 and f is 10. Thus, fdx is +10. Similarly all the other items in the series are calculated to get fdx.

As per the formula:

$$\bar{x} = A + \frac{\sum fdx'}{n} \times C$$ **(Formula 18)**

As per the formula:

A is 35, n is 65 and fdx' is 7 and C is 10.

$$\bar{x} = 35 + \frac{7}{65} \times 10$$

$$\bar{x} = 35 + 0.107 \times 10$$

$$\bar{x} = 35 + 1.07$$

$$\bar{x} = 36.07$$

Hence, the average marks of students are 36.07.

Hope, you have understood how to calculate the arithmetic mean in three different methods.

5. Properties of Arithmetic Mean

Now let us discuss about the properties of arithmetic mean.

Important mathematical properties of arithmetic mean are:

Property 1: The sum of deviation of the items from the actual mean is always zero. That means   .

$$\sum(x - \bar{x}) = 0$$ **(Formula 19)**

This can be made clear with the help of the example:

Figure 10

x	$(x\text{-}\bar{x})$
2	2-4 = -2
3	3-4 = -1
4	4-4 = 0
5	5-4 = +1
6	6-4 = +2
Σx= 20	$\Sigma(x\text{-}\bar{x})$= 0

In this example x is given. Like 2, 3, 4, 5 and 6. Thus $\sum x$ is 20.

Now we will calculate $\bar{x}$.

$$\bar{x} = \frac{\sum x}{n} = \frac{20}{5} = 4$$ **(Formula 20)**

As per the formula we will calculate $(x\text{-}\bar{x})$

Like: 2-4 = -2, 3-4 = -1, 4-4 = 0, 5-4=1, 6-4=2.

Thus if the sum of the deviation from the actual mean i.e 4 is taken, it comes out to be zero.

Property 2: The sum of squared deviations of the items is minimum when deviation is from the actual mean. That means

$$\sum (x - \bar{x})^2$$ **(Formula 22)**

Figure 11

X	$(x - \bar{x})$	$(x - \bar{x})^2$
2	2 - 4 = - 2	4
3	3 - 4 = - 1	1
4	4 - 4 = 0	0
5	5 - 4 = + 1	1
6	6 - 4 = + 2	4
$\Sigma x = 20$	$\Sigma (x - \bar{x}) = 0$	$\Sigma (x - \bar{x})^2 = 10$

For example in this table, we have the value of x that is 20.
Value of (x-$\bar{x}$) that is 0,
And after calculating we will get the value of that is 10.
Thus, the sum of squared deviation is equal to 10 which is minimum to the actual mean that is 20.

Property 3: The means of all the sums and differences of corresponding observations in two series is equal to the sum or difference of means of the two series.

$$\sum x = n\bar{x}$$ **(Formula 23)**

 For example: If the average wages in a firm is Rs. 1000 and the number of workers employed is 100. In that case we can know the total wages bill by the formula,

$$\sum x = n\bar{x}$$ **(Formula 24)**

Thus, $\sum x$ = 100(1000) = Rs. 100000.

This is the total wage bill.

Property 4: If we have arithmetic mean and the number of items of two or more than two related groups, then the combined average of these groups is calculated by applying the formula:

$$\bar{x}_{123...n} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2 + n_3 \bar{x}_3 ... + n_n \bar{x}_n}{n_1 + n_2 + n_3 + ...n_n}$$ **(Formula 25)**

For example: The average salary of male employee is in a firm was Rs.5200 and that of females was Rs.4200. The mean salary of the entire employee was Rs.5000. Find the percentage of male and female employees.

Let n1 and n2 denote respectively the number of the male and female employees in the firm, 1 and 2 denote respectively their average salaries in Rs. Let denote the average salary of the entire worker in firm.

We know,   =

5000(n1+n2)   = 5200n1+4200n2
(5200-5000)n1= (5000-4200)n2
   20n1 = 80n2
        = 4
Therefore, the percentage of male employees in the firm is 80% and the percentage of female employees in the firm is 20%.

Here's a summary of our learning in this session:
   • Explain what are the types of Mean
   • Explain what is Arithmetic Mean
   • Explain how to calculate Arithmetic Mean in:
            • Individual Series
            • Discrete Series
            • Continuous Series
   • Explain the properties of Arithmetic Mean