1. Introduction

Welcome to the series of e-Learning module on mode, median and partition values part 2.

In part one session of this topic, we have understood the calculation of median and partition values for different statistical series. In our today's session, we will be able to calculate mode in case of:

- Individual series, discrete series, continuous series
- Inclusive series
- Cumulative frequency
- Unequal class interval
- Bi-modal distribution
- Undefined mode value
- Mode less than lower limit of modal class

Mode:
Mode is also a positional average which represents the most frequently occurring items of the series. It means it represents the item, which is repeated maximum number of items in the series. It is easy to calculate and it helps in determining the popularity of a commodity. It is denoted by the alphabet 'Z'.

Calculation of mode in individual series:
Problem 1
Calculate mode for the following data:
10, 27, 24, 12, 27, 27, 20, 18, 15, 30

Solution:
As this is an individual series we use the inspection method.
By observing the data, we find that the value 27 repeats the maximum number of times. It is repeated 3 times in the given data and so 27 is the mode for this data.

Calculation of mode in case of discrete series:
Problem 2:
Calculate mode for the following data.

Table 1

X	10	11	12	13	14	15
f	5	8	15	7	4	5

Solution
In this problem, we use the inspection method because there is a gradual rise or fall in the frequencies and the highest frequency. Also, the adjacent frequencies are not too close. Thus, the variable 12 with the highest frequency 15 is the mode for the given data.

Problem 3
For the following ages of different people, determine the modal age.

Table 2

Age	15	20	25	30	35	40	45	50	55
Persons	2	3	4	10	11	12	3	2	1

Solution:
In this data, we can see that the adjacent frequencies are close to each other. Hence, we will use the grouping and analysis table for calculating the mode.

First let us prepare the grouping table:

Table 3

Age	II	III	IV	V	VI	VII
15	2	5				
20	3		7	9		
25	4	14			17	
30	10		21			25
35	11	23		33		
40	12		15		26	
45	3	5				17
50	2		3	6		
55	1	-				

In this table, we have taken the variable age 15, 20… up to 55 in the first column.

The second column has the original frequencies 2, 3, 4, 10… up to 1 and the highest value 12 is encircled. In the third column, we add the frequencies in two that is 2 plus 3=5, 10 plus 4 =14, and so on and encircle the highest value 23.

In column four, we leave the first item that is 2 and then add the frequencies in two that is 3 plus 4 =7, 10 plus 11 = 21 and so on and the highest value 21 is encircled.

In column five, we add the frequencies in threes 2 plus 3plus 4 =9, 10 plus 11 plus 12 is equal to 33… And the highest value 33 is encircled.

In column six, we leave the first item 2, and add the frequencies in three that is 3 plus 4 plus 10 = 17, and so on and encircle the value 26.

In the last that is seventh column, we leave away the first two items, and add the remaining

frequencies in three that is 4 plus 10 plus 11 =25 and encircle the highest value 25. Thus, the grouping table is done.

Next, we will prepare the analysis table.

Table 4

Age	15	20	25	30	35	40	45	50	55
II						X			
III					X	X			
IV				X	X				
V				X	X	X			
VI					X	X	X		
VII			X	X	X				
Total	0	0	1	3	5	4	1		

Here, we have taken the age variables in horizontal and the column values in vertical. In the first row, we will take the highest value of the first column and mark a cross against the corresponding variable 40. In the second row, let us take the value of the second column 23, which is corresponding to the variable 35 and 40 and put a cross against these variables. Similarly, we do for all the values of column 3, 4, 5, 6 and 7. Then, take the totals. We see that the total of variable 35 is high and hence it is the modal age.

When we compare the mode in both inspection and grouping analysis methods, we will see that the mode through inspection is 40 and through grouping is 35. The position of the mode shifts with the weightage of the preceding and succeeding items.

2. Calculation of Mode in Case of Continuous Series

Calculation of mode in case of continuous series:
Problem 4
Calculate mode for the following data.

Table 5

X	10-20	20-30	30-40	40-50	50-60	60-70	70-80
F	7	9	10	20	12	11	9

Solution:
By inspection, we can identify that the modal class is 40-50 because this class has the highest frequency 20.

Table 6

X	0-9	10-19	20-29	30-39	40-49	50-59	60-69	70-79	80-89
F	2	5	3	4	10	6	2	3	1

Now, let us apply the formula for interpolation of the mode.
Z is equal to l1 plus (f1 minus f0) divided by (2f1 minus f0 minus f2) into c
Let us substitute the following values in the equation:
L1 = 40, f1 = 20, f0=10, f2=12 and c=10
Z is equal to 40 plus (20 minus 10) divided by (2 multiplied by 20) minus 10 minus 12) multiplied by 10
Z is equal to 40 plus 10 divided by 18 multiplied by 10
Z is equal to 40 plus 100 divided by 18 is equal to 40 plus 5.56 is equal to 45.56
Thus, the modal value is 45.56.

Calculation of mode in case of inclusive series:
Problem 5
Calculate mode for the following data.

Table 7

X	0-9	10-19	20-29	30-39	40-49	50-59	60-69	70-79	80-89
F	2	5	3	4	10	6	2	3	1

Solution:

First, convert the inclusive series to exclusive series by adding the correction factor 0.5 to the upper limit and deducting it from the lower limit. We will get the new set of class intervals as below9.5, 9.5-19.5, 19.5-29.5, 29.5-39.5, 39.5-49.5, 49.5-59.5, 59.5-69.5, 69.5-79.5 and 79.5-89.5. By inspection, we can say that mode lies in 40-49 but the real class limit of this class are 39.5-49.5.

Table 8

X	Below 9.5	9.5-19.5	19.5-29.5	29.5-39.5	39.5-49.5	49.5-59.5	59.5-69.5	69.5-79.5	79.5-89.5
F	2	5	3	4	10	6	2	3	1

Now, let us apply the formula for interpolation of the mode.
Z is equal to l1 plus (f1 minus f0) divided by (2f1 minus f0 minus f2) into c
Let us substitute the following values in the equation:
L1 = 39.5, f1 = 10, f0=4, f2=6 and c=10
Z is equal to 39.5 plus (10 minus 4) divided by (2 multiplied by 10) minus 4 minus 6) multiplied by 10
Z is equal to 39.5 plus 6 divided by 10 multiplied by 10
Z is equal to 39.5 plus 60 divided by 10 is equal to 39.5 plus 6 is equal to 45.5.
Thus, the modal value is 45.5.

3. Calculation of Mode in Case of Cumulative Frequency

Calculation of mode in case of cumulative frequency:
Problem 6
Calculate mode for the following data.

Table 9

x	10	20	30	40	50	60	70	80	90	100	110	120
f	100	97	92	85	75	63	48	36	30	28	20	10

Solution:
This is a cumulative frequency distribution, and we convert it into simple frequency distribution. We will get the class intervals as 10-20, 20-30, 30-40, 40-50, 50-60, 60-70, 70-80, 80-90, 90-100, 100-110 and 110-120. The frequencies are 3, 5, 7, 10, 12, 15, 12, 6, 2, 8, and 10. By inspection, we can say that mode lies in 60-70.

Table 10

x	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100	100-110	110-120
f	3	5	7	10	12	15	12	6	2	8	10

Now, let us apply the formula for interpolation of the mode.
Z is equal to l1 plus (f1 minus f0) divided by (2f1 minus f0 minus f2) into c
Let us substitute the following values in the equation:
L1 = 60, f1 = 15, f0=12, f2=12 and c=10
Z is equal to 60 plus (15 minus 12) divided by (2 multiplied by 15) minus 12 minus 12) multiplied by 10
Z is equal to 60 plus 3 divided by 6 multiplied by 10
Z is equal to 60 plus 30 divided by 6 is equal to 60 plus 5 is equal to 65.
Thus, the modal value is 65.

Calculation of mode in case of unequal class interval data:
Problem 7
Calculate mode for the following data.

Table 11

X	0-10	10-20	20-30	30-40	40-50	50-60	60-70
F	278	520	464	398	324	178	44

Solution:
By inspection, we can identify that the modal class is 10-20 because this class has the highest frequency.

Table 12

X	0-10	10-20	20-30	30-40	40-50	50-60	60-70
F	278	520	464	398	324	178	44

Now, let us apply the formula for interpolation of the mode.
Z is equal to l1 plus (f1 minus f0) divided by (2f1 minus f0 minus f2) into c
Let us substitute the following values in the equation:
L1 = 10, f1 = 520, f0=278, f2=464 and c=10
Z is equal to 10 plus (520 minus 278) divided by (2 multiplied by 520) minus 278 minus 464) multiplied by 10.
Z is equal to 10 plus 242 divided by 298 multiplied by 10
Z is equal to 10 plus 2420 divided by 298 is equal to 10 plus 8.12 is equal to 18.12
Thus, the modal value is 18.12.

Calculation of mode in case of Bimodal distribution:
Problem 8
For the following data, calculate the modal value.

Table 13

Weight	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90
No. of People	4	6	20	32	33	17	8	2

Solution
By inspection, it is difficult to say which the modal class is. Hence, we resort to the grouping analysis method of calculating the mode.

Table 14

Weight	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90
No. of People	4	6	20	32	33	17	8	2

First, let us prepare the grouping table:

Table 15

Weight	II	III	IV	V	VI	VII
10-20	4	10		30		
20-30	6		26		58	
30-40	20	52				85
40-50	32		65	82		
50-60	33	50			58	
60-70	17		25			27
70-80	8	10				
80-90	2					

In this table, we have taken the variable weights 10-20, 20-30 and so on in the first column.

The second column is the original frequencies 4, 6, 20 and so on and the highest value 33 is encircled. In the third column, we add the frequencies in two, that is 4 plus 6=10, 20 plus 32 =52, and so on and encircle the highest value 52.

In the column four, we leave the first item that is 4 and then add the frequencies in two, that is 6 plus 20 = 26, 32 plus 33 = 65 and so on and the highest value 65 is encircled.

In the column five, we add the frequencies in threes 4 plus 6plus 20 =30, 32 plus 33 plus 17 is equal to 82, and the highest value 82 is encircled.

In the column six, we leave the first item, and add the frequencies in three, that is 6 plus 20 plus 32 = 58, 33 plus 17 plus 8 =58 and encircle the value 58.

In the last column that is the seventh column, we leave away the first two items, add the remaining frequencies in three, that is 20 plus 32 plus 33 = 85, 17 plus 8 plus 2 = 27, and encircle the highest value 85. Thus, the grouping table is done.

Next, we will prepare the analysis table.

Table 16

Weight	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90
II					X			
III			X	X				
IV				X	X			
V				X	X	X		
VI		X	X	X	X	X	X	
VII			X	X	X			
Total		1	3	5	5	2		

Here, we have taken the weights variable in horizontal and the column values in vertical. In the first row, we will take the highest value of the first column and mark a cross against the corresponding variable 50-60. In the second row, let us take the value of the second column 52, which is corresponding to the variable 30-40, 40-50 and put a cross against these variables. Similarly, we do for all the values of column 3, 4, 5, 6 and 7. Then, take the totals. We see that the total of variable 40-50 and 50-60 is same and hence this is a data with bimodal value.

Hence, mode has to be determined indirectly by applying the empirical formula
For which we need to calculate the median and the mean.

Median
Let us calculate the cumulative frequency. We will get 4, 10, 30, 62, 95, 112, 120, and 122. Now, identify the median class using the formula N divided by 2 is equal to 122 divided by 2 is equal to 61st item, which lies in the group 40-50. Thus, the median class is equal to 40-50.

Table 17

Weight c	Frequency	(Cf)
10-20	4	4
20-30	6	10
30-40	20	30
40-50	32	62
50-60	33	95
60-70	17	112
70-80	8	120
80-90	2	122

Now, we will interpolate the median value by using the formula:
Median is equal to l1 plus N divided by 2 minus (c into f) divided by f into c.
Where,
l1 is the lower limit of the class interval which is equal to 40
N/2 is the size of the item 61th item
cf is the cumulative frequency of the preceding class, which is equal to 30
f is the frequency of the median class, which is equal to 32
c is the width of the class interval, which is equal to 10.
When these values are substituted and calculated, we get the median as 49.688.

Mean
In this method, we will calculate dx' by dividing the deviation further by the common factor 'c' to simplify the calculation and later multiply the result with the common factor. Here, we will get the total value of sigma fdx' as minus 67 and the mean value as 55 plus minus 67 divided

by 122 multiplied by 10 is equal to 49.508.

Table 18

Weights (x)	(f)	Mid value	dx′ = x-A/c	fdx′
10-20	4	15	-4	-16
20-30	6	25	-3	-18
30-40	20	35	-2	-40
40-50	32	45	-1	-32
50-60	33	55	0	0
60-70	17	65	1	17
70-80	8	75	2	16
80-90	2	85	3	6
Total	122			-67

Thus by substituting the value of the mean and the median in the formula, Mode is equal to 3 median – 2 mean, we will get
Mode is equal to 3 into 49.688 minus 2 into 49.508, which is equal to 149.064 minus 99.016, which is equal to 50.048.

4. Calculation of Mode in Case of Undefined Mode Value

Calculation of mode in case of undefined mode value:
Problem 9
Calculate mode for the following data.

Table 19

Class Interval	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100	100-110
Frequency	4	6	5	10	20	22	24	6	2	1

Solution:
By the grouping & analysis method, we will calculate the mode.
First, let us prepare the grouping table:

Table 20

Class Interval	II	III	IV	V	VI	VII
10-20	4	10		15		
20-30	6		11		21	
30-40	5	15				35
40-50	10		30	52		
50-60	20	42			66	
60-70	22		46			52
70-80	24	30		32		
80-90	6		8		9	
90-100	2	3				
100-110	1					

In this table, we have taken the variable class interval 10-20, 20-30 and so on in the first column.
The second column has the original frequencies 4, 6, 5 and so on and the highest value 24 is encircled.

In the third column, we add the frequencies in two that is 4 plus 6=10, 5 plus 10 = 15, and so on and encircle the highest value 42.

In column four, we leave the first item that is 4 and then add the frequencies in two that is 6 plus 5 =11, 10 plus 20 = 30 and so on and the highest value 46 is encircled.

In column five, we add the frequencies in three that is 4 plus 6plus 5 =15, 10 plus 20 plus 22 is equal to 52, and the highest value 52 is encircled.

In column six, we leave the first item and add the frequencies in three that is 6 plus 5 plus 10 = 21, 20 plus 22 plus 24 =66 and encircle the value 66.
In the last column that is in the seventh column, we leave away the first two items and add the remaining frequencies in three that is 5 plus 10 plus 20 =35, 22 plus 24 plus 6 = 52 and encircle the highest value 52. Thus, the grouping table is done.

Next, we will prepare the analysis table.

Table 21

CI	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100	100-110
II							x			
III					X	x				
IV						x	x			
V				x	x	x				
VI					x	x	x			
VII						x	x	x		
				1	3	5	4	1		

Here, we have taken the class interval variables in horizontal and the column values in vertical. In the first row, we will take the highest value of the first column and mark a cross against the corresponding variable 70-80. In the second row, let us take the value of the second column 42, which is corresponding to the variable 50-60, 60-70 and put a cross against these variables. Similarly, we do for all the values of column 3, 4, 5, 6 and 7. Then, take the totals. We can see that 60-70 has the highest total 5. Thus, the mode lies in 60-70.

Now, let us apply the formula for interpolation of the mode.
Z is equal to l1 plus (f1 minus f0) divided by (2f1 minus f0 minus f2) into c
Let us substitute the following values in the equation:
L1 = 60, f1 = 22, f0=20, f2=22 and c=10
Z is equal to 60 plus (22 minus 20) divided by (2 multiplied by 22) minus 20 minus 22) multiplied by 10.
Z is equal to 60 plus 2 divided by 0 multiplied by 10, leaving the mode undefined. Hence formula
We will apply mode, which is equal to lower limit plus delta one divided by delta 1 plus delta2 multiplied by c is applied, where delta 1 is equal to absolute f1 minus f0 is equal to 22-20 is equal to 2 and delta 2 is equal to absolute f1 minus f2 is equal to 22 minus 24 is equal to 2 (ignoring sign). Therefore, mode is equal to 60 plus 2 divided by 2 plus 2 into 10 is equal to 60 plus 5 is equal to 65.

5. Calculation of Mode in Case of Mode less than Lower Limit of Modal Class

Calculation of mode in case of Mode less than lower limit of modal class:
Problem 10
Calculate mode for the following data.

Table 22

Marks	0-10	1440-20	20-30	30-40	40-50	50-60	60-70	70-80
Students	4	44	38	28	6	8	12	2

Solution:
By the grouping & analysis method, we will calculate the mode.
First, let us prepare the grouping table.

Table 23

Marks	**II**	**III**	**IV**	**V**	**VI**	**VII**
0-10	4	48		86		
10-20	44		82		110	
20-30	38	66				72
30-40	28		34	42		
40-50	6	14			26	
50-60	8		20			22
60-70	12	14				
70-80	2					

In this table, we have taken the variable marks 0-10, 10-20, 20-30 and so on in the first column.

The second column has the original frequencies 4, 44, 38,….., and the highest value 44 is encircled. In the third column, we add the frequencies in two that is 4 plus 44=48, 38 plus 28 =66, and so on and encircle the highest value 66.

In column four, we leave the first item that is 4 and then add the frequencies in two that is 44 plus 38 =82, 28 plus 6 = 34 and so on and the highest value 82 is encircled.

In column five, we add the frequencies in threes 4 plus 44 plus 38 =86, 28 plus 6 plus 8 =42, and the highest value 86 is encircled.

In column six, we leave the first item and add the frequencies in three that is 44 plus 38 plus 28 = 110, 6 plus 8 plus 12 =26 and encircle the value 110. In the last column that is seventh column, we leave away the first two items and add the remaining frequencies in three that is 38 plus 28 plus 6 =72, 8 plus 12 plus 2 = 22 and encircle the highest value 72. Thus, the grouping table is done.

Next, we will prepare the analysis table.
Here, we have taken the marks variable in horizontal and the column values in vertical. In the first row, we will take the highest value of the first column and mark a cross against the corresponding variable 10-20. In the second row, let us take the value of the second column 42, which is corresponding to the variable 20-30, 30-40 and put a cross against these variables. Similarly, we do for all the values of column 3, 4, 5, 6 and 7. Then, take the totals. We can see that 20-30 has the highest total 5. Thus, the mode lies in 20-30.

Now, let us apply the formula for interpolation of the mode.
Z is equal to l1 plus (f1 minus f0) divided by (2f1 minus f0 minus f2) into c
Let us substitute the following values in the equation:
L1 = 20, f1 = 38, f0=44, f2=28 and c=10
Z is equal to 20 plus (38 minus 44) divided by (2 multiplied by 38) minus 44 minus 28) multiplied by 10.
Z is equal to 20 plus minus 6 divided by 4 multiplied by 10 is equal to 20 minus 15 is equal to 5. Thus, it is less than the lower limit of modal class. Therefore, it cannot be a mode value. In such a case, the alternative formula can be used.

We will apply mode is equal to lower limit plus f2 divided by f0 minus f2 multiplied c.
Z is equal to 20 plus 28 divided by 44 plus 38 into 10, which is equal to 20 plus 280 divided by 82, which is equal to 23.41.

Here is a summary of our learning in this session, where we have understood:

- The calculation of mode in case of:
    - Individual series
    - discrete series
    - continuous series
    - Inclusive series
    - Cumulative frequency
    - Unequal class interval
    - Bi-modal distribution
    - Undefined mode value
    - Mode less than lower limit of modal class