1. Introduction

Welcome to the series of E-Learning module on Arithmetic Mean, Geometric mean, Harmonic mean, Weighted Arithmetic mean, Trimmed mean, Corrected mean Part 2.

In the part one session of this topic, we have understood the calculation of mean under various situations. In our today's session, we will be able to:

- Calculate combined mean
- Calculate geometric mean
- Calculate harmonic mean
- Calculate weighted mean
- Calculate truncated mean
- Calculate corrected mean

If we have an arithmetic mean and the number of items of two or more than two related groups, then we can calculate the combined average of these groups by applying the following formula.
Mean of n is equal to the product of the mean of the first group and the number of items in the first group plus the product of the mean of the second group and the number of items in the second group and so on till the product of the mean of the N groups and the number of items in the n group divided by N one plus n two and so on till N n.

The mean age of a combined group of men and women is 30 years. If the mean age of the group of men is 32 and that of group of women is 25, find out the percentage of men and women in the group.

Let us first take a note of the given data. Here, we have mean n is equal to 30, mean one is equal to 32 and mean two is equal to 25. We need to find N one and N two.

Let us denote N1 as men and N2 as women. Using the formula we substitute the values and we get 30 is equal to $32N_1$ plus 25 N_2 divided by N_1 plus N_2, after cross multiplication we will get $30N_1$ plus 30 N_2 is equal to $32N_1$ plus $25N_2$, take the like terms on the same side of the equation we will get 30N2 minus 25N2 is equal to 32N1 minus 30 N1 which is equal to 5N2 is equal to 2 N1. Therefore, the ratio of N1 to N2 is 5 is to 2.

To find the percentage of men we will calculate percentage of men is equal to 5 divided by 7 into 100 is equal to 71.5% and the percentage of women is equal to 2 divided b 7 into 100 is equal to 28.5%.

As we calculate the combined arithmetic mean, we can also calculate the geometric mean for items having a number of sets in them.
If the geometric mean of N items is 6 and then these N items are divided into sets. First containing N1 items and second containing N2 items having G1 and G2 as their respective geometric mean, then Log GM is equal to N1 log G1+ N2 log G2 whole divided by N1 + N2.

In this problem, we have two sets N1 with 5 items having a geometric mean 20 and N2 with

10 items having a geometric mean 35.28. Find the respective geometric mean.

By applying the formula and substituting the given values that is N1=5, N2=10, G1=20, G2=35.28, we find the GM of the entire series 'N'.

N is equal to the total of N1 plus N2. By substituting the values in the equation, N1 that is 5 and N2 is equal to 10 we get, the logarithmic of the geometric mean that is log of 20 is equal to 1.3010 and the log of 35.28 is equal to 1.5475. Substituting the values in the equation, we will get the geometric mean as the antilog of 1.465 and when we find the geometric mean, we get the value as 29.17, which is the antilog of 1.465.

Geometric mean is the 'nth' root of the product of 'n' items of a series. For example, if there are two or three items in a series, then the square root or the cube root of the products of the item in the series is taken.

The annual rate of growth of a factory for 5 years is 7%, 8%, 4%, 6% and 10% respectively. What is the average rate of growth per annum for this period?

If the growth of a company is 100 in the beginning, then in the five years, the growth is given as follows:

Figure 1

Year	X	Log X
1	100+7 = 107	2.0293
2	100+8 = 108	2.0334
3	100+4 = 104	2.0170
4	100+6 = 106	2.0253
5	100+10 =110	2.0413
		$\Sigma \log x = 10.14654$

In the first column of the table, we take year, in the second column we have taken 100 as the beginning growth and added the percentage of increase in the growth and the third column is the logarithm value of the variables. The total of the logarithm of the values is equal to 10.14654

We will now calculate the geometric mean using the following formula:
The Geometric mean is equal to the antilog of the total of logarithm of X variable that is 10.14654 in this case, divided by 5 the total number of the data which is equal to anti log of 2.0293 which is equal to 106.98
Thus, the average rate growth is 106.98 – 100 = 6.98.
Therefore, we can say that the average rate of growth percentage per annum is 6.98%.

2. The Geometric Mean

Geometric mean for discrete frequency distribution
Calculate the geometric mean for the following data.

Figure 2

Wages	20	30	40	50	60	70	80
No. of people	5	2	3	10	3	2	5

In the first step, we will find the logarithms of the X variable as shown in the table. The log of the variable wages 20 is equal to 1.3010, log of the variable wages 30 is 1.4771 and so on.

Figure 3

Wages	Log
20	1.3010
30	1.4771
40	1.6021
50	1.6990
60	1.7782
70	1.8451
80	1.9031

Multiply the log with the frequency that is when we multiply 1.3010 the log value of the variable with the frequency 5 we get, 6.5051. Similarly, the values for the other variables are calculated. We also find the total of the product that is 49.7954.

Figure 4

Wages	Log	Frequency	F log x
20	1.3010	5	6.5051
30	1.4771	2	2.9542
40	1.6021	3	4.8062
50	1.6990	10	16.9897
60	1.7782	3	5.3345
70	1.8451	2	3.6902
80	1.9031	5	9.5154
		N =30	$\Sigma f \log X$ 49.7954

By applying the following formula, we calculate the geometric mean. That is, geometric mean is equal to antilog of the sigma or total of the product of the frequency with the logarithm value of the X variable divided by N, which is equal to antilog of 49.7954 divided by 30, which is equal to antilog of 1.65985. Therefore, geometric mean is equal to 45.69.

Geometric mean for continuous frequency distribution:
Find the geometric mean for the data given below.

Figure 5

Marks	Frequency	Marks	Frequency
4-8	6	24-28	12
8-12	10	28-32	10
12-16	18	32-36	6
16-20	30	36-40	2
20-24	15		

Let us find the mid value of the classes and then calculate the logarithms. To calculate the mid value for this example, we take the class interval 4-8 and add the upper limit and the lower limit that is 4 plus 8 is equal to 12 divided by 2 to take the average we get 6. Similarly, we calculate the mid values of all the class intervals as shown in the table. Then, we take logarithm of the mid points. In this case, the logarithmic value of 6 is 0.7782, which is further multiplied with the frequency 6, which is equal to 4.66891. Same method is followed for the other values as represented in the table.

Figure 6

Class Intervals	Mid points	Frequency	Logm	flogm
4-8	6	6	0.7782	4.66891
8-12	10	10	1.0000	10.00000
12-16	14	18	1.1461	20.63030
16-20	18	30	1.2553	37.65818
20-24	22	15	1.3424	20.13634
24-28	26	12	1.4150	16.97968
28-32	30	10	1.4771	14.77121
32-36	34	6	1.5315	9.18887
36-40	38	2	1.5798	3.15957
		N=109		$\Sigma f \log m$ =137.19306

Calculation of geometric mean is done using the following formula. The geometric mean is equal to the Antilog of the sigma or total of the product of the frequency with the logarithm value of the m variable divided by N. In this case, the geometric mean is equal to antilog of 137.19306 divided by 109, which is equal to antilog of 1.25865. Thus, the geometric mean is equal to 18.14.

Compound Interest formula:
Compared to the previous year, the overhead expenses went up by 32%. They increased by 40% in the next year and by 50% in the following year. Calculate the average rate of increase in the overhead expenses over the three years.

In average ratios and percentages, geometric mean is more appropriate. Applying geometric mean here:

Figure 7

% Rise	Expenses at the end of the year taking preceding year as 100	LogX
32	100+32=132	2.1206
40	100+40=140	2.1461
50	100+50=150	2.1761
		Σ logx = 6.4428

If the growth for the company is 100 in the beginning, then in the three years, the growth is given as follows.
That is in the first column of the table we take the year, in the second column we have taken 100 as the beginning growth and added the percentage increase in the growth, the third column is the logarithm value of the variables. The total of the logarithm of the values is equal to 6.4428.

We will now calculate the geometric mean using the formula: Geometric mean is equal to the antilog of the total of logarithm of the X variable that is 6.4428 in this case divided by 3 the total number of the data which is equal to anti log of 2.1476 which is equal to 140.5.
Therefore, the average rate of increase in overhead expenses is equal to 140.5 – 100 = 40.5%

Harmonic mean is quotient of “number of the given values” and “sum of the reciprocals of the given values”. The harmonic mean of any series is the reciprocal of the arithmetic mean of the reciprocal of the variate.

3. The Harmonic Mean

Computation of harmonic mean for individual series:
Find the harmonic mean for the following data.

To compute the harmonic mean, we have to first compute the reciprocals of each item. That is for the item 2574 the reciprocal is 1/2574, which is equal to 0.0004, for the item 475 the reciprocal is 1/475 is equal to 0.0021. Similarly, the reciprocal value of the entire item is calculated and shown in the table. Then, the total of the reciprocal is taken that is the sigma of 1/x is equal to 1325.0769.

Figure 8

Size of the item (x)	Reciprocal (1/x)
2574	0.0004
475	0.0021
75	0.0133
5	0.2000
0.8	1.2500
0.08	12.5000
0.005	200.0000
0.0009	1111.1111
N=8	$\Sigma 1/x = 1325.0769$

Now, we calculate the harmonic mean. Harmonic mean is equal to 8 the number of data by 1325.0769, which is equal to 0.006.

Computation of Harmonic Mean of discrete frequency distribution:
From the following data, compute the value of harmonic mean.

To compute the harmonic mean, we have to first compute the reciprocals of each item that is the reciprocal of the item 10, 20, 25, 40 and 50. Then, multiply it by the frequency. We will directly take the frequency and divide it with the respective value of the variable. Then, the total of the reciprocal is taken, that is the sigma of frequency multiplied by 1/x is equal to 5.975.

Figure 9

Marks (X)	No. of Students (f)	f/x
10	20	2.000
20	30	1.500
25	50	2.000
40	15	0.375
50	5	0.100
	N = 120	Σ(f/x) = 5.975

Now, we calculate the harmonic mean, which is equal to 120 the total of the frequency divided by 5.975, which is equal to 20.08.

Computation of Harmonic Mean of Continuous frequency distribution:
The following are the salary ranges and the number of employees for a manufacturing firm. Calculate the harmonic mean.

Figure 10

Salary (000) Rs.	50-60	60-70	70-80	80-90	90-100	100-110	110-120	120-130	Total
No. of Employees	12	15	20	44	42	32	32	12	209

To compute the harmonic mean, we have to first compute the mid values of the class intervals 50-60, 60-70, 70-80, 80-90, and so on. That is 50+60/2 = 55. Similarly, we get the mid values of the other class intervals as 65, 75, 85, 95, 105, 115 and125.

Figure 11

Salary	Midpoints (m)	Frequency(f)
50-60	55	12
60-70	65	15
70-80	75	20
80-90	85	44
90-100	95	42
100-110	105	32
110-120	115	32
120-130	125	12

The reciprocal of each item is taken and multiplied by the frequency, but here we will directly take the frequency and divide it with the respective value of the variable. Then, the total of the reciprocal is taken that is the sigma of frequency multiplied by 1/m is equal to 2.354.

Figure 12

Salary	Midpoints (m)	Frequency(f)	f/m
50-60	55	12	0.2181
60-70	65	15	0.2307
70-80	75	20	0.2666
80-90	85	44	0.5176
90-100	95	42	0.4421
100-110	105	32	0.3047
110-120	115	32	0.2782
120-130	125	12	0.096
Total		N = 209	2.354

Now, we calculate the harmonic mean is equal to 209 the total of the frequency divided by 2.354, which is equal to 88.78.

Weighted mean is a mean that is computed with extra weight given to one or more elements of the sample. The term weighted average usually refers to a weighted arithmetic mean, but weighted versions of other means such as the weighted geometric mean and the weighted harmonic mean can also be calculated.

4. Weighted Mean & Truncated Mean

Weighted arithmetic mean:
For the given results of college X and college Y, state which of them is better and why.

Figure 13

Name of the exam	College X		College Y	
	Appeared	Passed	Appeared	Passed
M A	300	250	1000	800
M Com	500	450	1200	950
B A	2000	1500	1000	700
B Com	1200	750	800	500
Total	4000	2950	4000	2950

Let us calculate the weighted arithmetic mean to compare the results of the college. In this case, let us take the appeared as the weights and passed as the x variable. The x variable should be a percentage of the appeared. So, as a first step let us calculate the pass percentages of both the colleges. The pass percentage is calculated by taking the number passed divided by the number appeared into 100, then we will get 83.3, 90, 75 and 62.5 for college X and we will get 80, 79.17, 70 and 62.5 as the pass percentage for college Y.

Figure 14

Name of the exam	College X			College Y		
	Appeared	Passed	Percentage	Appeared	Passed	Percentage
M A	300	250	83.3	1000	800	80
M Com	500	450	90	1200	950	79.17
B A	2000	1500	75	1000	700	70
B Com	1200	750	62.5	800	500	62.5
Total	4000	2950		4000	2950	

In the next step, we will multiply the weights with the pass percentage, then we will get 24990, 45000, 150000, and 75000 for college x and 80000, 95004, 70000 and 50000 for college Y.

Now, take the total of the product of the weights and the variables. In case of college X, the total of the weights is calculated by sigma W as 4000 (W1) and sigma W1x1 as 294990 and in case of college Y, it is calculated by sigma W as 4000 (W2) and sigma W2x2 as 295004.

Figure 15

Name of the exam	College X			College Y		
	Appeared (w1)	Percentage (x1)	W1x1	Appeared (w2)	Percentage (x2)	w2x2
M A	300	83.3	24990	1000	80	80000
M Com	500	90	45000	1200	79.17	95004
B A	2000	75	150000	1000	70	70000
B Com	1200	62.5	75000	800	62.5	50000
Total	$\Sigma w=4000$		$\Sigma w1x1 = 294990$	$\Sigma w=4000$		$\Sigma w2x2 = 295004$

Let us now calculate the weighted arithmetic mean. We will get 73.747 for college X and 73.751 for college Y. As the pass percentage of college Y is greater than college X, we say college Y is better.

Weighted Geometric mean:
Find the weighted geometric mean for the following data.

Figure 16

Group	Index Number	Weights
Food	260	46
Fuel & Lighting	180	10
Clothing	220	8
House Rent	230	20
Education	120	12
Miscellaneous	200	4

In this table, we have taken the given data in the first three columns. Then, we have calculated the logarithm of the variable X in column four, and taken the product of the weights and the logarithms in column 5. We have also taken the total of the weights, which is equal to

100, and the total of the products equal to 233.7706.

Figure 17

Group	Index Number	Weights	Log x	wlogx
Food	260	46	2.4150	111.0900
Fuel & Lighting	180	10	2.2553	22.5530
Clothing	220	8	2.3424	18.7392
House Rent	230	20	2.3617	47.2340
Education	120	12	2.0792	24.9504
Miscellaneous	200	4	2.3010	9.2040
Total		Σw=100		Σwlogx= 233.7706

Calculating the weighted geometric mean by using the formula, Weighted geometric mean is equal to the antilog of sigma of the product of weights and the logarithms of X divided by the sigma of weights which is equal to anti log of 233.7706 divided by 100 is equal to anti log of 2.3377 is equal to 217.6.

Weighted Harmonic mean:
A cyclist covers 5Km at an average speed of 10Km.p.h, another 3Km at 8Km.p.h and the last 2Km at 5Km.p.h. Find the average speed of the entire journey and verify your answer.

Now, we will use the weighted harmonic mean.
The various speeds are 10, 8 & 5. So, their reciprocals are one by ten, one by eight and one by five. The weights are the distance covered in this case, it is 5, 3 and 2. We multiply the reciprocal and the weights we get ½ plus 3/8 plus 2/5 which is equal to 51/40. The total of the weights is equal to 10. Thus, the weighted harmonic mean is equal to 10 divided by 51/40 is equal to 10 into 40 by 51 is equal to 7.84Km.p.h, which is the average speed of the entire journey.

Figure 18

Speed	Reciprocal (x)	Weights (w)	w/x
10	1/10	5	½
8	1/8	3	3/8
5	1/5	2	2/5
Total		10	51/40

Truncated mean
A truncated mean or trimmed mean is a statistical measure of central tendency, much like the mean and median. It involves the calculation of the mean after discarding given parts of a probability distribution or sample at the high and low end, and typically discarding an equal amount of both.

A figure skating competition produces the following scores: 6.0, 8.1, 8.3, 9.1, and 9.9.

A mean trimmed 40% would equal 8.5 ((8.1+8.3+9.1)/3), which is larger than the arithmetic mean of 8.28. To trim the mean by 40%, we remove the lowest 20% and highest 20% of values, eliminating the scores of 6.0 and 9.9. This says that trimming the mean can reduce the effects of outlier bias in a sample.

5. Correction mean

Correction mean:
The mean wage of 100 workers per day was found to be Rs. 80. However, later on, it was found that the wages of two laborers Rs. 93 and Rs. 59 were misread as Rs. 39 and Rs. 95. Find the correct mean per wage.

To calculate the correct mean wage per day, let us see what the given data is. In this problem, we are given the number N as 100; mean as 80, correct items are 93 and 59 and the incorrect items as 39 and 95. We also know that mean is equal to sigma X by N, therefore we can write sigma X is equal to N into mean is equal to 100(80) is equal to 8,000. But, this is the incorrect mean. So, to find the correct mean, we need to take the correct sigma x by N. Let us first find out the correct sigma X which is equal to incorrect sigma X minus wrong items plus correct items. So we will get correct sigma X is equal to 8000minus (39 plus 95) plus (93 plus 59) which is equal to 8000 Plus (minus 134 plus 152) is equal to 8000 plus 18 is equal to 8018. Now, by substituting these values in the equation we get correct mean is equal to correct sigma X 8018 divided by N that is 100 equal to 80.18. Hence, correct mean wage per day is equal to 80.18.

Corrected geometric mean:
The geometric mean of 10 observations was calculated as 28.6. It was later discovered that one of the observations was recorded as 23.4 instead of 32.4. Apply the correction and calculate the correct geometric mean.

n = 10
GM = 28.6
Correct observation = 32.4
Wrong observation = 23.4

Substituting in the equation we get, Correct Geometric mean is equal to the product of the geometric mean 28.6 and the correct value 32.4 divided by the wrong observation 23.4 to the power of one by 10, by adding log on both sides of the equation and calculating we will get the logarithmic value of geometric mean as 1.47053. Thus, the geometric mean is equal to antilog of 1.47053, which is equal to 29.54.

In this session, we have understood the calculation of different types of mean like:

- The Combined mean
- The Geometric mean
- The Harmonic mean
- The Weighted mean
- The Truncated mean
- The Corrected mean