1. Introduction

Welcome to the series of E-Learning module on Arithmetic Mean, Geometric Mean, Harmonic Mean, Weighted Arithmetic Mean, Trimmed Mean, and Corrected Mean (Part I).

At the end of this session, you will be able to:

- Explain the calculations of Arithmetic mean in case of:
- Individual series, Discrete series, Continuous series
- Inclusive series
- Descending series
- Unequal class interval
- Open end class
- Cumulative frequency distribution
- Missing frequency

Arithmetic Mean is the most widely used measure for representing the entire data. It is calculated by adding values of all the items and dividing their total by the number of items. In case of discrete and continuous series, the values of the frequencies are taken into account.

Let us work out a few problems to understand the calculation of Arithmetic Mean under various situations as mentioned above.

Mean in case of individual series:

The calculation of arithmetic mean in case of individual observation is very simple. As discussed in our earlier modules, we will calculate the mean through direct method, shortcut method and step deviation method. Let us now calculate a set of given data using all the three methods.

Problem 1:

The following table gives the daily expenditure of 10 families in a city. Calculate the arithmetic mean of expenditure by (a) direct method, (b) Short cut method (also known as Indirect Method) and (c) step deviation method.

iguie i
Daily Expenditure
(K3.)
30
70
40
20
60
40
30
80
50
90

Figure 1

Solution:

(a) Direct method: In this method, take the total of all the variables. We will get sigma x is equal to 510. Now apply the formula mean is equal to sigma x divided by N where sigma x is equal to 510 and n is equal to 10. Therefore, mean is equal to 510 divided by 10 is equal to 51. Thus, the average daily expenditure is Rs.51.

Figure	2
--------	---

Daily Expenditure				
(Rs.)				
30				
70				
40				
20				
60				
40				
30				
80				
50				
90				
Σx=510				

(b) Short cut method: In this method, we will take the assumed mean (A) and calculate the mean by taking the deviations of the variable from the assumed mean. In this problem, we have taken 50 as the assumed mean, then the deviation of x (dx) is calculated by taking the difference of the variable(x) and the assumed mean (A) we will get 30 minus 50 is equal to minus 20, 70 minus 50 is equal to 20, 40 minus 50 is equal to minus 10 and so on for all the variables as shown in the table. Now we will take the total of deviations called sigma dx, which is equal to 10.

Figure 3

Daily Expenditure (Rs.)	dx = (x - A)
30	30-50 = -20
70	70-50 = 20
40	40-50 = -10
20	20-50 = -30
60	60-50 = 10
40	40-50 = -10
30	30-50 = -20
80	80-50 = 30
50	50-50 = 0
90	90-50 = 40
N - 10	$\Sigma dy = 10$

N = 10Σdx = 10Apply the formula, mean is equal to A plus sigma dxdivided by N. Here A is equal to 50 and sigma dx is equal to 10 and N is equal to 10. Bysubstituting the values we get, mean is equal to 50 plus 10divided by 10 is equal to 50 plus 1is equal to 51. Hence, the average daily expenditure is equal to Rs. 51.

(c) Step-deviation method: In this method we will take the assumed mean (A) and calculate the mean by taking the deviations of the variable from the assumed mean, as done in the previous method and then we calculate the step of the deviation by dividing dx by the common factor 'c'. We get d dash x (dx'). We will get the values minus 2, 2, minus1, minus 3, 1, minus 1, minus 2, 3, zero and 4. Thus, sigma d dash x is equal to 1.

Daily Expenditure (Rs)	dx = (x-A)	dx'= dx/c
30	30-50=-20	-20/10 = -2
70	70-50=20	20/10 = 2
40	40-50=-10	-10/10 = -1
20	20-50=-30	-30/10 = -3
60	60-50=10	10/10 = 1
40	40-50=-10	-10/10 = -1
30	30-50=-20	-20/10 = -2
80	80-50=30	30/10 = 3
50	50-50=0	0/10 = 0
90	90-50=40	40/10 = 4
N=10	Σdx =10	$\overline{\mathbf{x}}\mathbf{d}\mathbf{x}'=1$

Figure 4

Now use the formula, mean is equal to A plus sigma d dash x divided by N multiplied by c. Here, A is equal to 50, sigma d dash x is equal to 1, n is equal to 10 and c is equal to 10. By substituting the value in the formula we will get mean is equal to 50 plus 1 divided by 10 and multiplied by 10 is equal to 51. Hence the average expenditure is Rs. 51.

2. Mean incase of Discrete series

Mean in case of Discrete Series:

In discrete series, arithmetic mean may be calculated applying either direct method, short-cut method or step deviation method.

Problem 2:

Calculate the arithmetic average for the following data using all the three methods.

Figure	5
--------	---

Wages	20	30	40	50	60	70	80
No. of person	5	2	3	10	3	2	5

Solution:

Figure c)	
Wages (x)	No. of People (f)	fx
20	20 5 20x5 = 10	
30	2	30x2 = 60
40	3	40x3 = 120
50	10	$50 \times 10 = 500$
60	3	60x3 = 180
70	2	70x2 = 140
80	5	80x5 = 400
	N or Σf = 30	Σfx = 1500

(a) Direct method:

Let the wages be denoted by x and the no. of people be denoted by f. We will now multiply the variable with frequency to get fx that is 100, 60, 120, 500, 180, 140 and 400. Then, we will take the total of fx which give us sigma fx which is equal to 1500.

Now we will use the formula to calculate the mean. Mean is equal to sigma fx divided by N, here sigma fx is equal to 1500 and N is equal to 30 the total of the frequency. By substituting the value, we will get mean is equal to 1500 divided by 30 is equal to 50. Hence, average wage is equal to Rs. 50.

(b) Short cut method:

According to this method, let us take the assumed mean (A) as 40 and calculate the dx by taking the difference of x and A. We will get the values minus 20, minus 10, zero, 10, 20, 30 and 40. Now multiply the frequency with dx we will get minus 100 minus 20, 0, 100, 60, 60 and 200, take the total we will get sigma fdx, which is equal to 300.

Wages (Rs.) (x)	No. of People (f)	dx=(x-A)	fdx
20	5	20-40=-20	-100
30	2	30-40=-10	-20
40	40 3 40-		0
50	10	50-40=10	100
60	3	60-40=20	60
70	2	70-40=30	60
80	5	80-40=40	200
	N orΣf=30		Σfdx =300

Figure 7

Now, we will calculate the mean using the formula. Mean is equal to A plus sigma fdx divided by N, where A is equal to 40, sigma fdx is equal to 300 and N is equal to 30. By substituting the values we will get mean is equal to 40 plus 300 divided by 30 is equal to 50. Hence, the average wage is equal to Rs. 50.

I IQUIC U

Wages (Rs) (x)	No. of persons (f)	dx=(x-A)	dx′ = dx/c	fdx'
20	5	20-25=-5	-1	-5
30	2	30-25=5	1	2
40	3	40-25=15	3	9
50	10	50-25=25	5	50
60	3	60-25=35	7	21
70	2	70-25=45	9	18
80	5	80-25=55	11	55
	N or Σ f=30			Σfdx'=150

(c) Step deviation method:

In this method, let us take the assumed mean as 25 and calculate the deviation of the

variable from the assumed mean dx and divided it by the common factor c we will get the values for d dash x minus 1, 1, 3, 5, 7, 9, and 11 now multiply the frequency with d dash x we will get minus 5, 2, 9, 50, 21, 18 and 55 taking the total we get sigma fd dash x which is equal to 150.

Now, calculate the mean using the formula. Mean is equal to A plus sigma fd dash x divided by N and multiplied by c, here A is equal to 25, sigma fd dash x is equal to 150, N is equal to 30 and c is equal to 5. By substituting this value we will get mean is equal to 25 plus 150 divided by 30 multiplied 5 is equal to 50. Hence, the average wage is equal to Rs.50.

3. Mean in case of Continuous series

Mean in case of Continuous series:

In continuous series, arithmetic mean may be calculated by applying direct method, short cut method and step deviation method.

Problem 3:

Calculate the mean for the following frequency distribution of marks by direct method, short cut method and step deviation method.

Figure 9

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Students	5	12	30	45	50	37	21

Solution:

Marks (x)	Students (f)	Mid value	fx
0-10	5	5	25
10-20	12	15	180
20-30	30	25	750
30-40	45	35	1575
40-50	50	45	2250
50-60	37	55	2035
60-70	21	65	1365
	N=200		Σfx = 8180

Figure 10

(a) Direct method:

While calculating the mean in case of a continuous series we will first take the mid value of each class interval. The mid value is obtained by adding the upper limit and lower limit of the class interval and dividing it by 2. It may also be noted that the difference between the mid values and the width of the class interval will be the same.

Now in this problem, the mid values will be 10 divided by 2 equal to 5, the second class interval we will get the mid value 15 and so on. Thus, we get 25, 35, 45, 55, 65 and mid value. Then, let us multiply the frequency with the mid value we will get the values fx like 25, 180,

750, 1575, 2250, 2035 and 1365 and the total value of sigma fx is equal the 8180 and the value of N is the total of the frequencies, which is equal to 200.

Let us calculate the mean using the formula, mean is equal to sigma fx divided by N is equal to 8180 divided by 200 is equal to 40.9 or 41.

(b) Short cut method:

In this method of calculation, we will take the deviation from the assumed mean so that we will get dx. Here, we will take 35 as the assumed mean and deduct it from the X value which is equal to minus 30, minus 20, minus 10, zero, 10, 20 and 30. Next we will calculate fdx by multiplying the frequency with the deviation and we will get minus 150, minus 240, minus 300, zero, 500, 740 and 630. We will get the total sigma fdx as 1180 and N as 200.

Marks (x)	Students (f)	Mid value	dx	fdx
0-10	5	5	5-35=-30	-150
10-20	12	15	15-35=-20	-240
20-30	30	25	25-35=-10	-300
30-40	45	35	35-35=0	0
40-50	50	45	45-35=10	500
50-60	37	55	55-35=20	740
60-70	21	65	65-35=30	630
	N=200			Σfdx = 1180

Figure 11

Let us use the formula to calculate the mean. Mean is equal to A the assumed mean plus sigma fdx divided by N. Here the values are A is equal to 35, sigma fdx is equal to 1180, N is equal to 200. By substituting, the value we will get 35 plus 1180 divided by 200 is equal to 40.9. The average marks are equal to 40.9 or 41.

Figure 12

Marks (x)	Students (f)	Mid value	Dx = (x-A)	dx' = dx/c	fdx'
0-10	5	5	5-35=-30	-3	-15
10-20	12	15	15-35=-20	-2	-24
20-30	30	25	25-35=-10	-1	-30
30-40	45	35	35-35=0	0	0
40-50	50	45	45-35=10	1	50
50-60	37	55	55-35=20	2	74
60-70	21	65	65-35=30	3	63
	N=200				Σfdx'=118

(c) Step deviation method:

In this method, we will calculate d dash x by dividing the deviation further by the common factor 'c' to simplify the calculation and later on multiply the result with the common factor. Here, we will get the total value of sigma fd dash x as 118.

Let us use the formula to calculate the mean. Mean is equal to A the assumed mean plus sigma f d dash x divided by N multiplied by c. Where, A is equal to 35, sigma f d dash x is equal to 118, N is equal to 200, and c is equal to 10. The mean value as 35 plus 118 divided by 200 multiplied by 10 is equal to 40.9 or 41.

Note: Arithmetic mean from either direct method, short cut method or step deviation method the result would be the same. However, preference should be always given to step deviation method whenever possible, as it makes further calculations easier.

4. Mean in case of Inclusive series

Mean in case of Inclusive series:

When the data is given in inclusive series form, then it is not necessary to adjust the classes for calculating the arithmetic mean. It is because the mid value remains the same whether the adjustment is made or not.

Problem 4:

Calculate the average wages for the following data:

Figure '	13
----------	----

Wages	10-19	20-29	30-39	40-49	50-59	60-69	70-79
No. of people	5	2	3	10	3	2	5

Solution:

As discussed earlier, we will use the step deviation method to calculate the mean. This is a continuous inclusive series as the upper limit of the preceding class and the lower limit of the succeeding class are not alike. First calculate the mid values of the class interval we will get 14.5, 24.5, 34.5, 44.5, 54.5, 64.5 and 74.5. Take the assumed mean A as 44.5 then calculate the deviation d dash x. By subtracting x minus A divided by c we will get the total sigma fd dash x which is equal to zero and the total of the frequency 30 as the N value.

Figure 14

Wages(x)	No. of people (f)	Mid value	Mid value dx=(x-A)		fdx'
10-19	5	14.5	14.5-44.5=-30	-3	-15
20-29	2	24.5	24.5-44.5=-20	-2	-4
30-39	3	34.5	34.5-44.5=-10	-1	-3
40-49	10	44.5	44.5-44.5=0	0	0
50-59	3	54.5	54.5-44.5=10	1	3
60-69	2	64.5	64.5-44.5=20	2	4
70-79	5	74.5	74.5-44.5=30	3	15
	N=30				Σfdx'=0

By substituting the value in the formula, mean is equal to A plus sigma fd dash x divided by N multiplied by c is equal to 44.5 plus 0 divided by 30 multiplied by 10 is equal to 44.5 plus zero is equal to 44.5 is the average wages.

Mean in case of descending order series.

In this, the data is in a descending order rest of the calculation remains the same.

Problem 5:

Calculate the arithmetic mean for the following data:

Figure 15

x	110-	100	90-	80-	70-	60-	50-	40-	30-	20-	10
	100	-90	80	70	60	50	40	30	20	10	-0
f	4	4	8	16	16	24	18	14	10	2	4

Solution:

Figure 16

x	(f)	Mid value	dx=(x-A)	dx' = dx/c	fdx'
110-100	4	105	105-55=50	5	20
100-90	4	95	95-55=40	4	16
90-80	8	85	85-55=30	3	24
80-70	16	75	75-55=20	2	32
70-60	16	65	65-55=10	1	16
60-50	24	55	55-55=0	0	0
50-40	18	45	45-55=-10	-1	-18
40-30	14	35	35-55=-20	-2	-28
30-20	10	25	25-55=-30	-3	-30
20-10	2	15	15-55 = -40	-4	-8
10-0	4	5	5-55=-50	-5	-20
	N=120				$\Sigma fdx' = 4$

While calculating the mean the series need not to be written in ascending order, we can directly take the data and calculate the mid value of the data that is 105, 95, and so on. Then calculate the deviation d dash x by subtracting A that is 55 from X the mid values and dividing it by c. We get 5, 4, 3 and so on. Now multiply the frequency with d dash x we will get the total of sigma fd dash x as 4.

By substituting the values in the equation, mean is equal to A plus sigma fd dash x divided by n multiplied by c. We get 55 plus 4 divided by 120 multiplied by 10. We will get 55 plus 0.333, which is equal to 55.333. Hence, the average of X is equal to 55.333.

Mean in case of unequal class interval

Sometimes, the class interval of the distribution is unequal. In such cases, we can calculate the mean as per the methods explained without bothering about the inequality of the class interval.

Problem 6:

Calculate the arithmetic mean for the following data:

Figure 17

Wages	0-10	10-30	30-60	60-100	100-150	150-210
No. of workers	4	6	8	10	4	8

Solution:

Figure 18

Wages (x)	No. of workers (f)	Mid value	dx=(x-A)	dx' = dx/c	fdx'
0-10	4	5	5-45=-40	-4	-16
10-30	6	20	20-45=-25	-2.5	-15
30-60	8	45	45-45=0	0	0
60-100	10	80	80-45=35	3.5	35
100-150	4	125	125-45=80	8	32
150-210	8	180	180-45=135	13.5	108
	N=40				Σfdx' =144

As a first step take the mid values of the data, here we get 0 plus 10 divided by 2 is equal to 5 the next we will have 30 plus 10 divided by 2 is equal to 20, next we will get 45, 80, 125 and 180. Then we find the deviation d dash x for simpler calculation purpose we get minus 4, minus 2.5, zero, plus 3.5, 8 and 13.5. Multiply the deviations with frequency we get sigma fd dash x which is equal to 144 and the total of the frequency is equal to 40.

Now by substituting the values in the formula mean is equal to A plus sigma fd dash x divided by N multiplied by 'c'. Where A is equal to 45, sigma fd dash x is equal to 144, N is equal to 40, and c is equal to 10. Mean is equal to 45 plus 144 divided by 40 multiplied by 10 is equal to 45 plus 36 which is equal to 81. Hence, the average wage is equal to Rs.81.

Mean in case of open end class interval:

Open end classes are those in which the lower limit of the first class or the upper limit of the last class is not known. In such cases, arithmetic mean cannot be calculated unless an assumption about the unknown is made. The assumption would depend on the class interval following or succeeding the first class and preceding the last class.

Problem 7:

Calculate the arithmetic mean for the following data:

Figure 19

Income	Below 10	10-30	30-60	60-100	100-150	150 & above
No. of people	4	7	10	16	8	2

Solution:

Figure 20

Income	0 - 10	10-30	30-60	60-100	100-150	150-210
No. of person	4	7	10	16	8	2

Here the class magnitude is unequal. The magnitude of 2nd, 3rd, 4th and 5th is 20,30,40,50 respectively that is it increasing by 10. Therefore, in this situation the lower limit of the first

class will be 10 minus 10 equal to zero and hence the first class interval will be 0-10 and the class limit of the last class will be 150 plus 60 is equal to 210, thus the last class interval will be 150-210.

As a first step, take the mid values of the data. Here we get 0 plus 10 divided by 2 is equal to 5 the next we will have 30 plus 10 divided by 2 is equal to 20, similarly we will get 45, 80, 125 and 180. Then, we find the deviation d dash x

for simpler calculation purpose. We get minus 4, minus 2.5, zero, plus 3.5, 8 and 13.5. Multiply the deviations with frequency we get sigma fd dash x which is equal to 144 and the total of the frequency is equal to 40.

Wages (x)	No. of workers (f)	Mid value	dx=(x-A)	dx' = dx/c	fdx'
0-10	4	5	5-45=-40	-4	-16
10-30	6	20	20-45=-25	-2.5	-15
30-60	8	45	45-45=0	0	0
60-100	10	80	80-45=35	3.5	35
100-150	4	125	125-45=80	8	32
150-210	8	180	180-45=135	13.5	108
	N=40				Σfdx' =144

Figure 21

Now by substituting the values in the formula, mean is equal to A plus sigma fd dash x divided by N multiplied by 'c' which is equal to 45 plus 144 divided by 40 multiplied by 10 is equal to 45 plus 36 which is equal to 81. Hence, the average wage is equal to Rs. 81.

5. Mean in case of Cumulative frequency distribution

Mean in case of Cumulative frequency distribution:

When the data is given in the form of more than or less than, above or below for all items in the series, it is called as cumulative frequency distribution.

Problem 8:

Following are the distribution of marks obtained by the 50 students in Economics. Calculate the Arithmetic mean.

I Iguie ZZ	Fig	ure	22
------------	-----	-----	----

Marks (more than)	10	20	30	40	50	60
No. of students	100	92	80	40	20	6

Solution:

The data is in cumulative form. Let us first convert it into simple frequency distribution. Now from this table we can see that there are 100 students having more than 10 marks and 92 students having more than 20 marks. So the number of students having marks between 10-20 will be 100 minus 92, which is equal to 8. In the same fashion, frequencies of marks between 20-30, 30-40, 40-50, 50-60 and 60-70 can be computed and we will get 12, 40, 20, 14 and 6 respectively. Then, the mid values of the class intervals are calculated as 15, 25, 35, 45, 55 and 65. Take the deviations d dash x and multiply it by f to get fd dash x, so that the total is equal to 38.

Figure 2	23
-----------------	----

Marks	No. of students	Mid value	dx=(x-A)	dx'=dx /c	fdx'
10-20	100-92=8	15	15-35=-20	-2	-16
20-30	92-80=12	25	25-35=-10	-1	-12
30-40	80-40=40	35	35-35=0	0	0
40-50	40-20=20	45	45-35=10	1	20
50-60	20-6=14	55	55-35=20	2	28
60-70	6-0=6	65	65-35=30	3	18
	N=100				Σfdx′ = 38

Now by substituting the values in the formula, mean is equal to A plus sigma fd dash x divided by N multiplied by 'c' which is equal to 35 plus 38 divided by 100 multiplied by 10 is equal to 35 plus 3.8 is equal to 38.8 or 39 marks. Hence, the average mark is equal to 39.

Mean in case of missing frequency:

Problem 9:

Calculate the number of students against the class 30-40 of the following data when mean is equal to 28.

Fia	ure	24

Marks	0-10	10-20	20-30	30-40	40-50	50-60
Students	12	18	27	?	17	6

Solution:

Given that:

Mean is equal to 28, A is equal to 25 and c is equal to 10.

On calculating the mid values of the class intervals, we will get 5, 15, 25, 35, 45 and 55. Take the deviations d dash x and multiply it by f to get fd dash x, so the total is equal to 10+a.

Marks	No. of students	Mid value	dx=(x-A)	dx'=dx /c	fdx'
0-10	12	5	5-25=-20	-2	-24
10-20	18	15	15-25=-10	-1	-18
20-30	27	25	25-25=0	0	0
30-40	а	35	35-25=10	1	а
40-50	17	45	45-25=20	2	34
50-60	6	55	55-25=30	3	18
	80+a				10+a

Figure 25

Now by substituting the values in the formula, mean is equal to A plus sigma fd dash x divided by N multiplied by 'c'. Where, A is equal to 25, sigma fd dash x is equal to 10 plus a, N is equal to 80 plus a, C is equal to 10, and mean is equal to 28.

The mean is equal to 28. Therefore, we write it as 28 is equal to 25 plus 10 plus a divided by 80 plus a multiplied by 10 which is equal to 28 minus 25 is equal to 10plus a divided by 80 plus a into 10 is equal to 3 is equal to 10 plus a divided by 80 plus a into 10. Now by cross multiplication we will get 240 plus 3a equal to 100 plus 10a by equalizing the terms we will get 140 is equal to 7a which is equal to a equal to 140 divided by 7 is equal to 20. Hence, there are 20 students against the 30-40 class intervals.

Here's a summary of our learning in this session, where we have understood:

• The calculation of Arithmetic mean in case of:

- Individual series, Discrete series, Continuous series
- Inclusive series
- Descending series
- Unequal class intervalOpen end class
- Cumulative frequency distribution
- Missing frequency