SANSKRIT

GEOMETRY

Part I

Lecture - 8

EXTRACTION OF SQUARE ROOTS

वर्गे करण्या यदि वा करण्योस्तुल्यानि रूपाण्यथ वा बहूनाम् ।
विशोधयेद्रूपकृतेः पदेन शेषस्य रूपाणि युतोनितानि ॥
पृथक् तदर्धे करणीद्वयं स्यान्मूलेऽथ बह्वी करणी तयोर्या ।
रूपाणि तान्येव कृतानि भूयः शेषाः करण्यो यदि सन्ति वर्गे ॥

Bhaskara gives a method to extract square roots from surds, a surd term is a term that consists of surds and a constant term, so from the constant term converted into a surd subtract the sum of as many surds numerically less than the constant; take the remainder and extract the square root. Add and subtract this from the original numerical term and divide by 2. This is one of the roots, repeat until all terms are exhausted. An example can be used to make this clear;

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Find the square root of $\sqrt{24} + 5$

The constant term here is 5, which is to converted into a surd; $\sqrt{25}$

$25 - 24 = 1$; $\sqrt{1} = 1$

Now, the first rupa is 5 so, add +/- 1 to it you get 6 and 4.

Divide the terms by 2 and you get $\sqrt{3}$ and $\sqrt{2}$

The second example given by Bhaskara is, find the square $\sqrt{10} + \sqrt{24} + \sqrt{40} + \sqrt{60}$. Convert 10 into a surd, which is $\sqrt{100}$.

So $100 - 24 + 40$ which is $100 - 64 = 36$

$\sqrt{36} = 6$

$10 +/- 6 = 16, 4$

So the two roots are, dividing these by 2 you get 8 and 2. $\sqrt{2}$ being the smallest is one of the roots reserve the 8 for doing the process again. So, 8 square is 64 and $64 - 60 = 4$.

$\sqrt{4} = 2$

$8 +/- 2 = 10,6$, Divide both by 2

So the two roots are $\sqrt{5}$ and $\sqrt{2}$. Hence the square root of the given term is $\sqrt{2} + \sqrt{3} + \sqrt{5}$

Extracting the square of surds with a negative sign, Bhaskara gives another Sutra;

- ऋणात्मिका चेत् करणी कृतौ स्याद्धनात्मिकां तां प्रकल्प्य ।
- मूलं करण्यावनयोरभीष्टा क्षयात्मिकैका सुधियाऽवगम्या ॥

When the surd has a negative sign, the square remains the same. That is, the square of $\sqrt{a} - \sqrt{b}$ and $\sqrt{b} - \sqrt{a}$, are the same. For this the example given by Bhaskara is the following;

त्रिसप्तमित्योर्वद मे करण्योर्विश्लेषवर्गं कृतितः पदं च ।

Find the $\sqrt{3} - \sqrt{7}$ and $\sqrt{7} - \sqrt{3}$.

Square of $(\sqrt{3} - \sqrt{7})$ - $(\sqrt{3} - \sqrt{7})(\sqrt{3} - \sqrt{7})$ =
$\sqrt{9} + \sqrt{49} - \sqrt{21} - \sqrt{21}$
Square of $(\sqrt{7} - \sqrt{3})$ - $(\sqrt{7} - \sqrt{3})(\sqrt{7} - \sqrt{3})$ =
$\sqrt{49} + \sqrt{9} - \sqrt{21} - \sqrt{21}$

Now, $\sqrt{3} - \sqrt{7}$ would be;

It is obvious that the product is same in both the cases.

Now here is another example with three terms,

Find the square of $\sqrt{2} + \sqrt{3} - \sqrt{5}$ and $- \sqrt{2} - \sqrt{3} + \sqrt{5}$

Find the square $\sqrt{17} + \sqrt{40} + \sqrt{80} + \sqrt{200}$

- Square of $\sqrt{2} + \sqrt{3} - \sqrt{5} = 2 + 3 + 5 + \sqrt{24} - \sqrt{60} - \sqrt{40}$
- Square of $-\sqrt{2} - \sqrt{3} + \sqrt{5}$
- $(-\sqrt{2} - \sqrt{3} + \sqrt{5}) \times (-\sqrt{2} - \sqrt{3} + \sqrt{5}) = \sqrt{4} + \sqrt{6} - \sqrt{10} + \sqrt{6} + \sqrt{9} - \sqrt{15} - \sqrt{10} - \sqrt{15} + \sqrt{25}$

$= 2 + 3 + 5 + \sqrt{24} - \sqrt{40} - \sqrt{60}$, same as above.

According to Bhaskara's rule, make 17 a surd = $\sqrt{289}$

$289 - (40 + 200) = 49$

$\sqrt{49} = 7$

$17 \pm 7 = 24, 10$. Dividing by 2, roots are 12, 5

Therefore $\sqrt{5}$ is one root; take the other for further process

$(12)^2 = 144$

$144 - 80 = 64$

$\sqrt{64} = 8$

$12 \pm 8 = 20, 4$. Dividing by 2, roots are $\sqrt{10}$, $\sqrt{2}$

Therefore square root of given expression is $\sqrt{2} + \sqrt{5} + \sqrt{10}$

GEOMETRY

Now, we go on to a new genre of mathematics, namely ***' Geometry '***

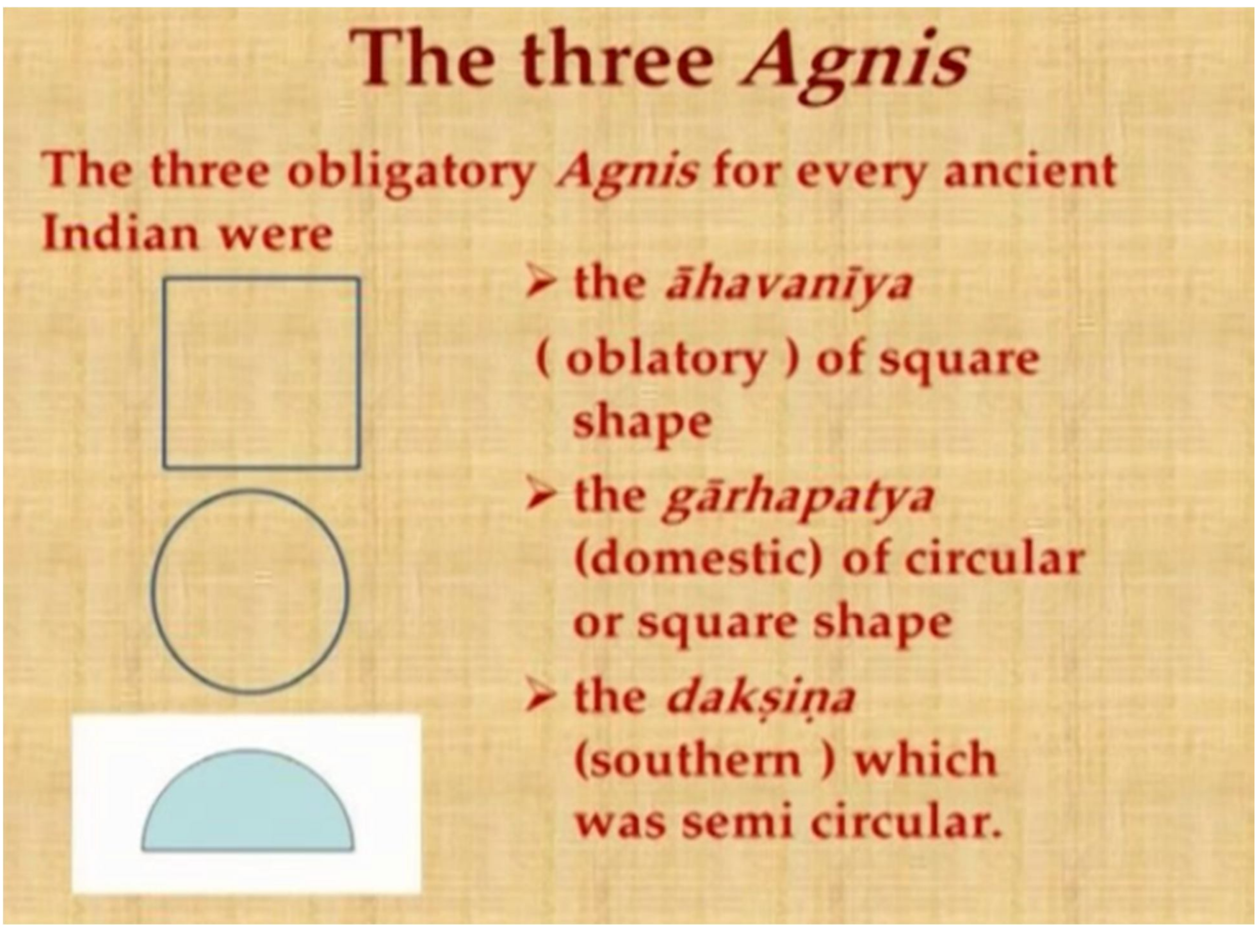
The student of Indian history is struck by the marvelous attainments of the ancient
Indians. The discoveries at Mohenj adaro reveal that as early as 3000 BC the inhabitants of
the land of Indus built brick houses, planned cities, used metals such as gold
silveretc and lived a highly organized life.

The Beginnings of mathematics

It is here that we find the beginnings of the science of mathematics- arithmetic, algebra,
geometry, astronomy etc. The mathematical genius of the ancient Indians was mainly computational and led spectacular achievements in arithmetic and algebra. But the basis and inspiration for the whole of indian mathematics is geometry.

Religion being a prime avocation in ancient India, it is not surprising that all branches of knowledge were subsidiary to it. The religious practices in ancient India required a certain amount of astronomy and Mathematics.

For instance, the vedic man used mathematics to set up the auspicious time and so on and the sulba sutras dealt with mathematics to construct the *vedis*(altars) accurately. The vedic man practiced sacrifice regularly for which they had to know the auspicious time to do the sacrifice and also to construct the sacrifice altars accurately. So for this the sulba sutras gave precise instructions and means to find the auspicious time.



SULBA SUTRAS

The Sulba Sutras are the earliest texts which gives us an idea of ancient Indian geometry. You have many of them, of them the *BaudhayanaSulba Sutra* is the oldest. The contents of Sulba sutra are somewhat similar. Mainly they deal with measurements and constructions. The contents of the Sulba sutras are usually;

Converting a square into a circle.

Dividing a circle into many parts.

Converting a circle into a square

The famous pythagoras theorem.

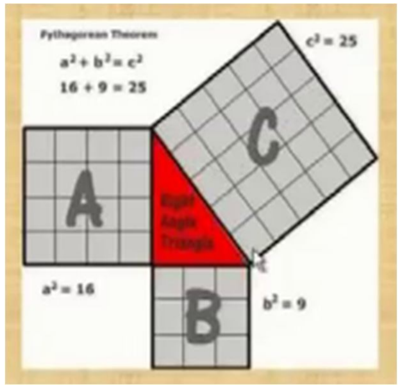
These are some of the concepts dealt with in the sulba sutras.

The Sulba Theorem

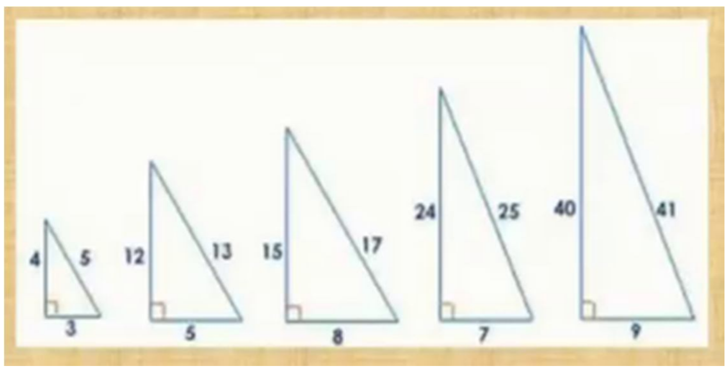
This is the famous pythagoras theorem, now known as pythagoras theorem but it was known in the ancient days, contained in almost all the sulba sutras.

आयाममायामगुणं विस्तारं विस्तरेण तु ।
समस्या वर्गमूलं यत्तत्कर्णं तद्विदो विदुः ॥

The square on the hypotenuse is equal to the sum of the squares on the other two sides. This is already known to the wise.



These are sum of the pythagorean triplets or sulba triplets as we can

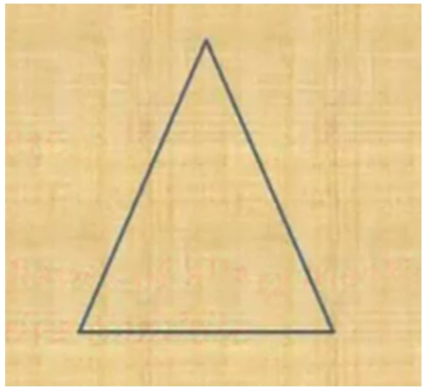


call them now, found in all the sulba sutra texts;
There are many more triplets mentioned in these sulba sutras.
We shall go on to the sum of the different shapes of altars,

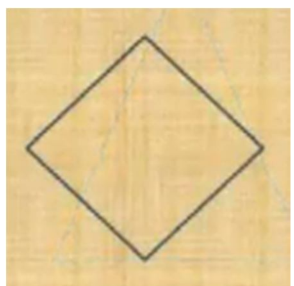
They are the:

>Isosceles triangle
>Rhombus
>Circle
>Syena or falcon
>Trapezium

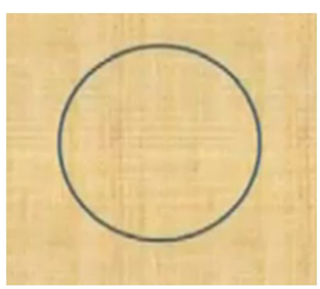
Isosceles triangle



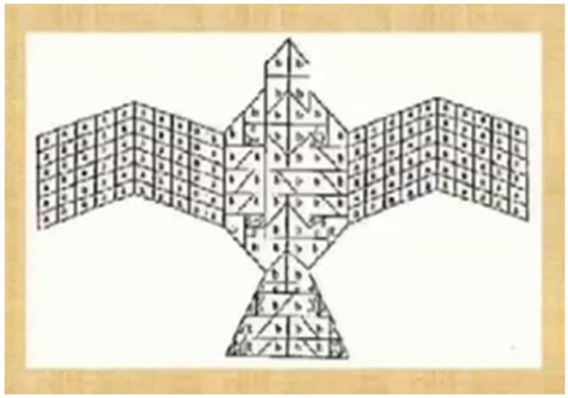
Rhombus



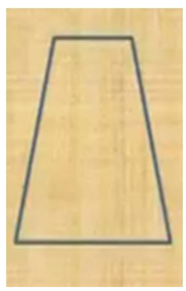
The circle :



Syena or falcon:



Trapezium :

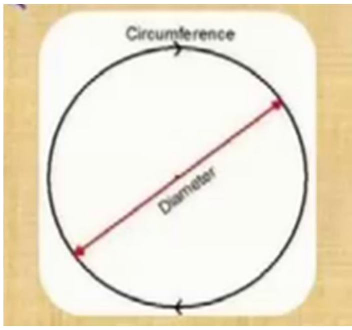


ARYABHATA & BRAHMAGUPTA

Now, we go on to Aryabhata’s geometry. Aryabhata was the earliest astronomer/mathematicians who flourished in the 5th century. He gave an approximate value for pi as the circumference ÷ the diameter’.

चतुरधिकं शतमष्टगुणं द्वाषष्टिस्तथा सहस्राणाम् ।
अयुतद्वयविष्कम्भस्यासन्नो वृत्तपरिणाहः ॥

He says : Add 4 to 100 and multiply by 8 and add this 832 to 62,000 and divide the sum by 20,000. Here, 62,832 is the circumference and 20,000 is the diameter. And when you divide the circumference by the diameter you get what is known as pi. This value 3.1416 is only approximate says Aryabhata. In school geometry, the two values for pi



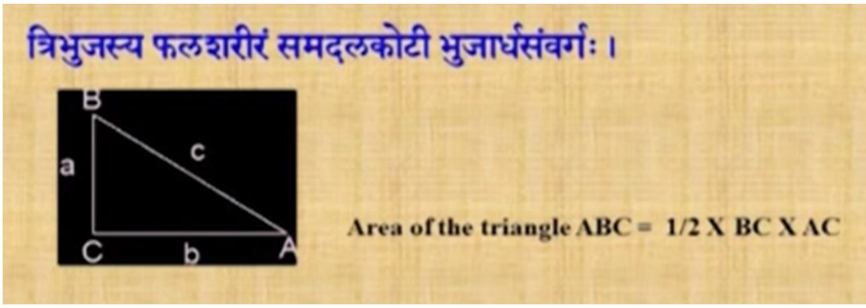
which are very commonly used are 3.14 and 22/7.
Let us take an example,
If the diameter is = 14, what is the circumference ?

Circumference = pi × d
= 14 × 22/7
= 44 cm

Let us take another example, if the circumference = 628 cm, what is the diameter?

Diameter = 628 / pi
= 628 / 3.14 = 200.
So, d = 200.

So, 3.14 and 22/7 are mostly used in school geometry to teach the children to use pi in numerical examples.
Aryabhata has also given the area of the triangle as : the product of the perpendicular and half the base of the triangle gives its area.



Here BC is the altitude or perpendicular and AC is the base

Let us take, base = 20 cm and perpendicular = 36 cm

So, area of the triangle = ½ base × altitude

= ½ 20 × 36 = 360 cm

Another example,

Suppose the area = 624 cm and the base = 13 cm. What is the perpendicular/altitude?

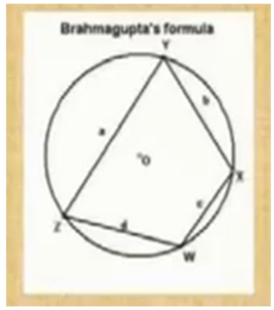
Altitude = area/ ½ base

= 624 × 2 / 13 = 96 cm

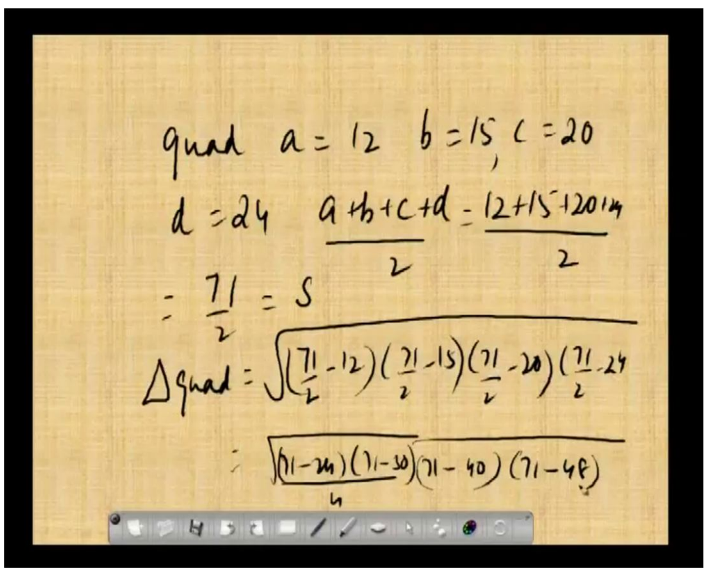
Brahma Gupta in his text, *Brahmasphutasiddhanta* gives the formula for the area of a cyclic quadrilateral. A cyclic quadrilateral is a quadrilateral which can be inscribed in a circle. Brahma Gupta gives the formula to find the area of a cyclic quadrilateral;

भुजयोगार्धचतुष्टय भुजोनघातात् पदं सूक्ष्मम् ।

Now, here WXYZ is a cyclic quadrilateral inscribed in a circle and ABCD are its sides. Then, Brahma Gupta says, if the four sides are ABCD and the semi perimeter = S, Area = √ (s - a) (s - b) (s - c) (s - d)



Let us take an example of a cyclic quadrilateral, if ABCD are the sides of the quadrilateral, S = a + b + c + d ÷ 2.



Area of the quadrilateral = √(s - a) (s - b) (s - c) (s - d)

Anyways, this is the formula given by Brahma Gupta. Brahma Gupta does not say that it is cyclic but he uses the term quadrilateral but we have to assume that he meant a cyclic quadrilateral.

Diagonals of a quadrilateral

Brahma Gupta gives a very neat formula for finding the diagonals of a quadrilateral.