Conductive and Convective Heat Transfer

1. Steady-State Heat Transfer

In problems involving heat transfer, we often deal with steady state and unsteady state (or transient) conditions. Steady-state conditions imply that time has no influence on the temperature distribution within an object, although temperature may be different at different locations within the object. Under unsteady-state conditions, the temperature changes with location and time.

1.1 Conductive Heat Transfer in a Rectangular Slab

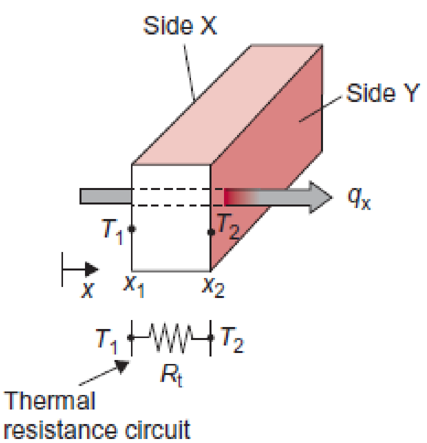


Figure 1: Heat transfer in a wall, also shown with a thermal resistance circuit

Consider a slab of constant cross-sectional area, as shown in Figure 1. The temperature, T_1, on side X is known. We will develop an equation to determine temperature, T_2, on the opposite side Y and at any location inside the slab under steady-state conditions. This problem is solved by first writing Fourier's law,

$$q_x = -k\frac{AdT}{dx} \qquad \text{Eq. (1)}$$

The boundary conditions are
$x = x_1$; $T = T_1$
$x = x_2$; $T = T_2$
Separating variables in Equation (1), we get

$$\frac{q_x}{A}dx = -kdT \qquad \text{Eq. (2)}$$

Integrating Eq. (2)

$$\int_{x_1}^{x_2}\frac{q_x}{A}dx = -\int_{T_1}^{T_2}kdT \qquad \text{Eq. (3)}$$

$$\frac{q_x}{A}(x_2 - x_1) = -k(T_2 - T_1) \qquad \text{Eq. (4)}$$

$$q_x = -kA\frac{(T_2 - T_1)}{(x_2 - x_1)} \qquad \text{Eq. (5)}$$

Temperature on face Y is T_2; thus, rearranging Equation (5),

$$T_2 = T_1 - \frac{q_x}{kA}(x_2 - x_1) \qquad \text{Eq. (6)}$$

To determine temperature, T, at any location, x, within the slab, we may replace T_2 and x_2 with unknown T and distance variable x, respectively, in Equation (6) and obtain,

$$T = T_1 - \frac{q_x}{kA}(x - x_1) \qquad \text{Eq. (7)}$$

If we rearrange Eq. (5)

$$q_x = \frac{(T_1 - T_2)}{\left[\frac{(x_2 - x_1)}{kA}\right]} \qquad \text{Eq. (8)}$$

$$q_x = \frac{(T_1 - T_2)}{R_t} \qquad \text{Eq. (9)}$$

Thermal resistance may be expressed as

$$R_t = \frac{(x_2 - x_1)}{kA} \qquad \text{Eq. (10)}$$

1.2 Conductive Heat Transfer through a Tubular Pipe

Consider a long, hollow cylinder of inner radius r_i, outer radius r_o, and length L, as shown in Figure 2. Let the inside wall temperature be T_i and the outside wall temperature be T_o. We want to calculate the rate of heat transfer along the radial direction in this pipe. Assume thermal conductivity of the metal remains constant with temperature.

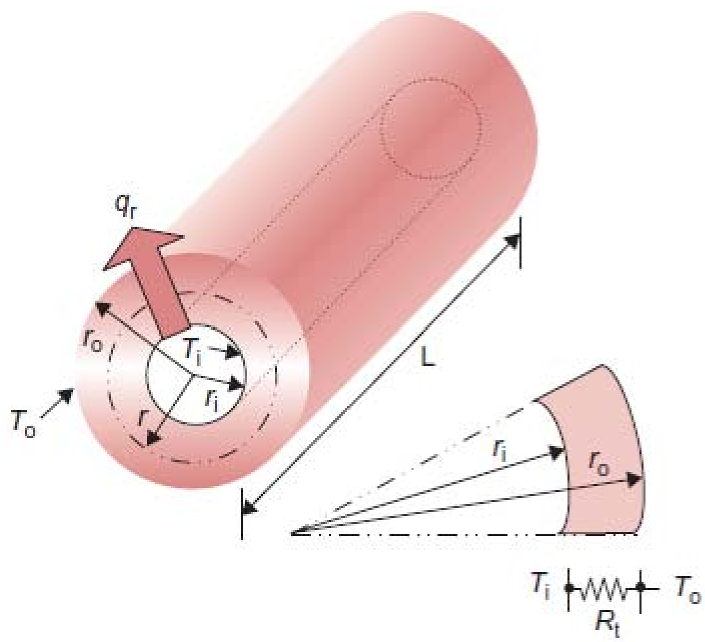


Figure 2: Heat transfer in a radial direction in a pipe, also shown with a thermal resistance circuit.

Fourier's law in cylindrical coordinates may be written as

$$q_r = -kA\frac{dT}{dr} \qquad \text{Eq. (11)}$$

where q_r is the rate of heat transfer in the radial direction. Substituting for circumferential area of the pipe,

$$q_r = -k(2\pi rL)\frac{dT}{dr} \qquad \text{Eq. (12)}$$

The boundary conditions are

$T = T_i$; $r = r_i$
$T = T_o$; $r = r_o$

$$\frac{q_r}{2\pi L}\int_{r_i}^{r_o}\frac{dr}{r} = -k\int_{T_i}^{T_o} dT \qquad \text{Eq. (13)}$$

$$\frac{q_r}{2\pi L}\left|\ln r\right|_{r_i}^{r_o} = -k\left|T\right|_{T_i}^{T_o} \qquad \text{Eq. (14)}$$

$$q_r = \frac{2\pi Lk(T_i - T_o)}{\ln(r_o/r_i)} \qquad \text{Eq. (15)}$$

In terms of thermal resistance

$$q_r = \frac{(T_i - T_o)}{\left[\frac{\ln(r_o/r_i)}{2\pi Lk}\right]} \quad \text{Eq. (16) and} \quad R_t = \left[\frac{\ln(r_o/r_i)}{2\pi Lk}\right] \quad \text{Eq. (17)}$$

1.3 Heat Conduction in Multilayered Systems

1.3.1 Composite Rectangular Wall (in Series)

We will now consider heat transfer through a composite wall made of several materials of different thermal conductivities and thicknesses. An example is a wall of a cold storage, constructed of different layers of materials of different insulating properties. All materials are arranged in series in the direction of heat transfer, as shown in Figure 3.

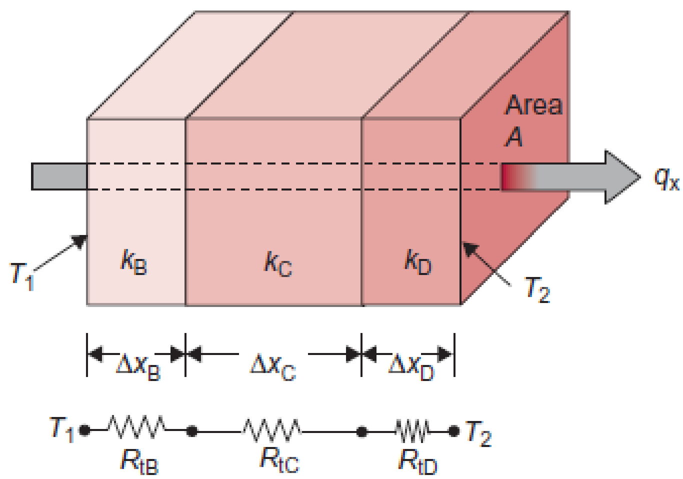


Figure 3: Conductive heat transfer in a composite rectangular wall, also shown with a thermal resistance circuit.

Using Fourier's law, for materials B, C, and D, we have

$$\Delta T_B = -\frac{q\Delta x_B}{k_B A}; \Delta T_C = -\frac{q\Delta x_C}{k_C A}; \Delta T_D = -\frac{q\Delta x_D}{k_D A} \quad \text{Eq. (18)}$$

$$\Delta T = T_1 - T_2 = \Delta T_B + \Delta T_C + \Delta T_D \quad \text{Eq. (19)}$$

$$T_1 - T_2 = -\left(\frac{q\Delta x_B}{k_B A} + \frac{q\Delta x_C}{k_C A} + \frac{q\Delta x_D}{k_D A}\right) \quad \text{Eq. (20)}$$

$$T_1 - T_2 = -\frac{q}{A}\left(\frac{\Delta x_B}{k_B} + \frac{\Delta x_C}{k_C} + \frac{\Delta x_D}{k_D}\right) \qquad \text{Eq. (21)}$$

$$q = \frac{T_2 - T_1}{\left(\frac{\Delta x_B}{k_B A} + \frac{\Delta x_C}{k_C A} + \frac{\Delta x_D}{k_D A}\right)} \qquad \text{Eq. (22)}$$

or, using thermal resistance values for each layer, we can write Eq. (22) as

$$q = \frac{T_2 - T_1}{R_{tB} + R_{tC} + R_{tD}} \qquad \text{Eq. (23)}$$

1.3.2 Composite Cylindrical Tube (in Series)

Figure 4 shows a composite cylindrical tube made of two layers of materials, A and B. An example is a steel pipe covered with a layer of insulating material. The rate of heat transfer in this composite tube can be calculated as follows.

From Eq. (16) we know that

$$q_r = \frac{(T_i - T_o)}{\left[\frac{\ln(r_o/r_i)}{2\pi L k}\right]} \qquad \text{Eq. (24)}$$

The rate of heat transfer through a composite cylinder using thermal resistances of the two layers is

$$q_r = \frac{(T_1 - T_3)}{R_{tA} + R_{tB}} \qquad \text{Eq. (25)}$$

or, substituting the individual thermal resistance values,

$$q_r = \frac{(T_1 - T_3)}{\frac{\ln(r_2/r_1)}{2\pi L k_A} + \frac{\ln(r_3/r_2)}{2\pi L k_B}} \qquad \text{Eq. (26)}$$

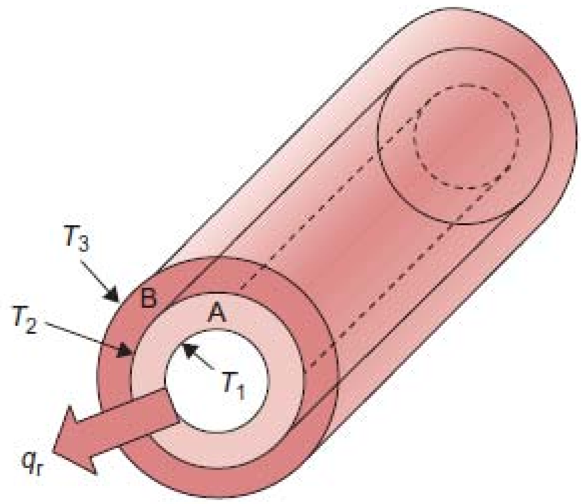
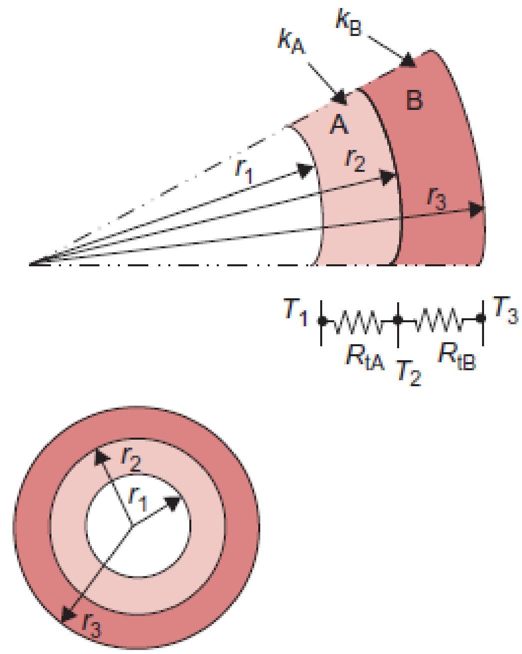


Figure 4: Conductive heat transfer in concentric cylindrical pipes, also shown with a thermal resistance circuit.



2. Estimation of Convective Heat-Transfer Coefficient

Determination of the rate of heat transfer due to convection is complicated because of the presence of fluid motion. In the module of fluid flow in food processing we have studied that a velocity profile develops when a fluid flows over a solid surface because of the viscous properties of the fluid material.

Similar to the velocity profile, a temperature profile develops in a fluid as it flows through a pipe, as shown in Figure 5.

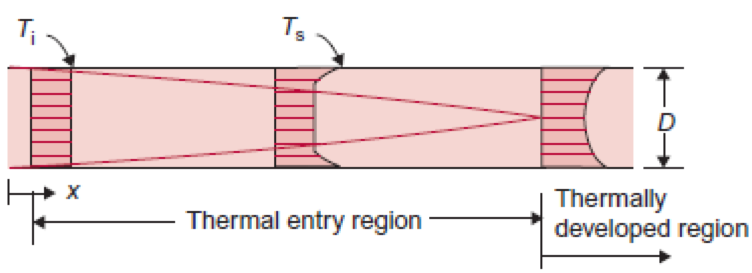


Figure 5: Thermal entry region in fluid flowing in a pipe

Empirical approach

This is widely used to determine the rate of convective heat transfer. A drawback of the empirical approach is that it requires a large number of experiments to obtain the required data.

We overcome this problem and keep the data analysis manageable by using dimensionless numbers. To formulate this approach, first we will identify and review the required dimensionless numbers: Reynolds number, N_{Re}, Nusselt number, N_{Nu}, and Prandtl number, N_{Pr}. Reynolds number has been discussed in the fluid flow module.

The second required dimensionless number for our data analysis is Nusselt number—the dimensionless form of convective heat transfer coefficient, h. Consider a fluid layer of thickness l, as shown in Figure 6. The temperature difference between the top and bottom of the layer is ΔT. If the fluid is stationary, then the rate of heat transfer will be due to conduction, and the rate of heat transfer will be

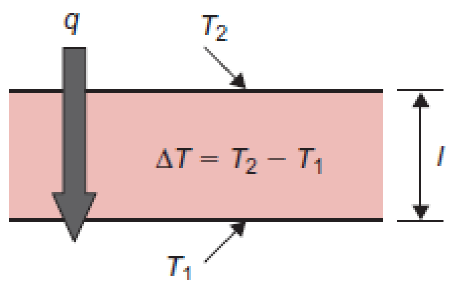


Figure 6: Heat transfer through a fluid layer

$$q_{conduction} = -kA\frac{\Delta T}{l} \qquad \text{Eq. (27)}$$

However, if the fluid layer is moving, then the heat transfer will be due to convection, and the rate of heat transfer using Newton's law of cooling will be

$$q_{convection} = hA\Delta T \qquad \text{Eq. (28)}$$

$$\frac{Eq.(28)}{Eq.(27)} = \frac{q_{convection}}{q_{conduction}} = \frac{hA\Delta T}{kA\frac{\Delta T}{l}} = \frac{hl}{k} = N_{Nu} \qquad \text{Eq. (29)}$$

Replacing thickness l with a more general term for dimension, the characteristic dimension d_C, we get

$$N_{Nu} = \frac{hd_C}{k} \qquad \text{Eq. (30)}$$

Nusselt number may be viewed as an enhancement in the rate of heat transfer caused by convection over the conduction mode. Therefore, if $N_{Nu}=1$, then there is no improvement in the rate of heat transfer due to convection. However, if $N_{Nu}=5$, the rate of convective heat transfer due to fluid motion is five times the rate of heat transfer if the fluid in contact with the solid surface is stagnant.

The third required dimensionless number for the empirical approach to determine convective heat transfer is Prandtl number, N_{Pr}, which describes the thickness of the hydrodynamic boundary layer compared with the thermal boundary layer. It is the ratio between the molecular diffusivity of momentum to the molecular diffusivity of heat. Or,

$$N_{\mathrm{Pr}} = \frac{\text{Molecular diffusivity of momentum}}{\text{molecular diffusivity of heat}} \qquad \text{Eq. (31)}$$

$$N_{\mathrm{Pr}} = \frac{\text{Kinematic viscosity}}{\text{Thermal diffusivity}} = \frac{\nu}{\alpha} = \frac{\mu c_p}{k} \qquad \text{Eq. (32)}$$

If $N_{Pr} = 1$, then the thickness of the hydrodynamic and thermal boundary layers will be exactly the same. On the other hand, if $N_{Pr} << 1$, the molecular diffusivity of heat will be much larger than that of momentum. For gases, N_{Pr} is about 0.7, and for water it is around 10.

For heating of fluid with electrically heated pipe surface, a graphical relationship may be conveniently expressed with an equation as

$$N_{Nu} = C N_{\mathrm{Re}}^{m} N_{\mathrm{Pr}}^{n} \qquad \text{Eq. (33)}$$

where C, m, and n are coefficients.

2.1 Forced Convection

In forced convection, a fluid is forced to move over a solid surface by external mechanical means, such as an electric fan, pump, or a stirrer (Figure 7). The general correlation between the dimensionless numbers is

$$N_{Nu} = \Phi(N_{\mathrm{Re}}, N_{\mathrm{Pr}}) \qquad \text{Eq. (34)}$$

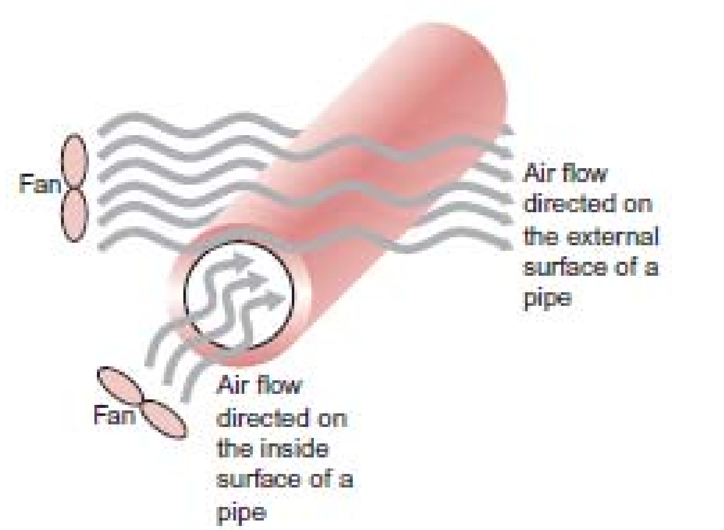


Figure 7:

Forced convective heat transfer from a pipe with flow inside and outside the pipe.

2.1.1 Laminar flow in pipes

1. Fully developed conditions with constant surface temperature of the pipe:

$$N_{Nu} = 3.66 \qquad \text{Eq. (35)}$$

where thermal conductivity of the fluid is obtained at average fluid temperature, T_∞, and d_C is the inside diameter of the pipe.

2. Fully developed conditions with uniform surface heat flux:

$$N_{Nu} = 4.36 \qquad \text{Eq. (36)}$$

where thermal conductivity of the fluid is obtained at average fluid temperature, T_∞, and d_C is the inside diameter of the pipe.

3. For both entry region and fully developed flow conditions:

$$N_{Nu} = 1.86\left(N_{\mathrm{Re}} \times N_{\mathrm{Pr}} \times \frac{d_C}{L}\right)^{0.33}\left(\frac{\mu_b}{\mu_w}\right)^{0.14} \qquad \text{Eq. (37)}$$

where L is the length of pipe (m); characteristic dimension, d_C, is the inside diameter of the pipe; all physical properties are evaluated at the average fluid temperature, T_∞, except μ_w, which is evaluated at the surface temperature of the wall.

2.1.2 Transition flow in pipes

For Reynolds numbers between 2100 and 10,000,

$$N_{Nu} = \frac{(f/8)(N_{\mathrm{Re}} - 1000)N_{\mathrm{Pr}}}{1 + 12.7(f/8)^{1/2}(N_{\mathrm{Pr}}^{2/3} - 1)} \qquad \text{Eq. (38)}$$

where all fluid properties are evaluated at the average fluid temperature, T_∞, and d_C is the inside diameter of the pipe, and the friction factor, f, is obtained for smooth pipes using the following expression:

$$f = \frac{1}{(0.790 \ln N_{\mathrm{Re}} - 1.64)^2} \qquad \text{Eq. (39)}$$

Turbulent flow in pipes

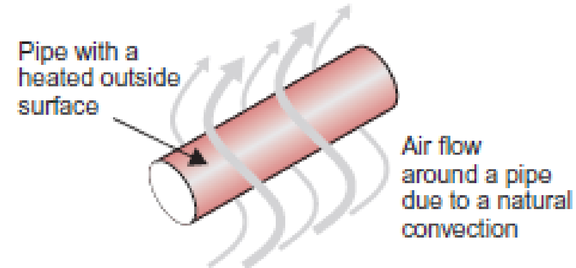
The following equation may be used for Reynolds numbers greater than 10,000:

$$N_{Nu} = 0.023 \times N_{\mathrm{Re}}^{0.8} \times N_{\mathrm{Pr}}^{0.33} \times \left(\frac{\mu_b}{\mu_w}\right)^{0.14} \qquad \text{Eq. (40)}$$

Fluid properties are evaluated at the average film temperature, T_∞, except μ_w, which is evaluated at the wall temperature; dc is the inside diameter of the pipe. Equation (40) is valid both for constant surface temperature and uniform heat flux conditions.

2.2 Free Convection

Free convection occurs because of density differences in fluids as they come into contact with a heated surface (Figure 8). The low density of fluid at a higher temperature causes buoyancy forces, and as a result, heated fluid moves upward and colder fluid takes its place.

	**Figure 8:** Heat transfer from the outside of a heated pipe due to natural convection.

Empirical expressions useful in predicting convective heat-transfer coefficients are of the following form:

$$N_{Nu} = \frac{hd_C}{k} = a(N_{Ra})^m \qquad \text{Eq. (41)}$$

where ***a*** and ***m*** are constants; N_{Ra}, is the Rayleigh number. Rayleigh number is a product of two dimensionless numbers, Grashof number and Prandtl number.

$$N_{Ra} = N_{Gr} \times N_{\Pr} \qquad \text{Eq. (42)}$$

The Grashof number, N_{Gr}, is defined as follows:

$$N_{Gr} = \frac{d_C^3 \rho^2 g \beta \Delta T}{\mu^2} \qquad \text{Eq. (43)}$$

where d_C is characteristic dimension (m); ρ is density (kg/m^3); g is acceleration due to gravity (9.80665 m/s^2); β is coefficient of volumetric expansion (K^{-1}); ΔT is temperature difference between wall and the surrounding bulk (^{o}C); and μ is viscosity (Pa s).

A Grashof number is a ratio between the buoyancy forces and viscous forces. Similar to the Reynolds number, the Grashof number is useful for determining whether a flow over an object is laminar or turbulent. For example, a Grashof number greater than 10^9 for fluid flow over vertical plates signifies a turbulent flow.

2.3 Thermal Resistance in Convective Heat Transfer

A thermal resistance term for convective heat transfer may be defined in a similar manner as in conductive heat transfer

$$q = hA(T_S - T_\infty) \qquad \text{Eq. (44)}$$

$$q = \frac{T_S - T_\infty}{\left(\frac{1}{hA}\right)} \qquad \text{Eq. (45)}$$

where the thermal resistance due to convection $(R_t)_{convection}$ is

$$(R_t)_{convection} = \frac{1}{hA} \qquad \text{Eq. (46)}$$

3. Estimation of Overall Heat-Transfer Coefficient

In many heating/cooling applications, conductive and convective heat transfer may occur simultaneously. An example shown in Figure 9 involves heat transfer in a pipe that carries a fluid at a temperature greater than the temperature of the environment surrounding the outside of the pipe. In this case, heat must first transfer from the inside fluid by forced convection to the inside surface of the pipe, then by conduction through the pipe wall material, and finally by free convection from the outer pipe surface to the surrounding environment. Thus, heat transfer is through three layers in a series.

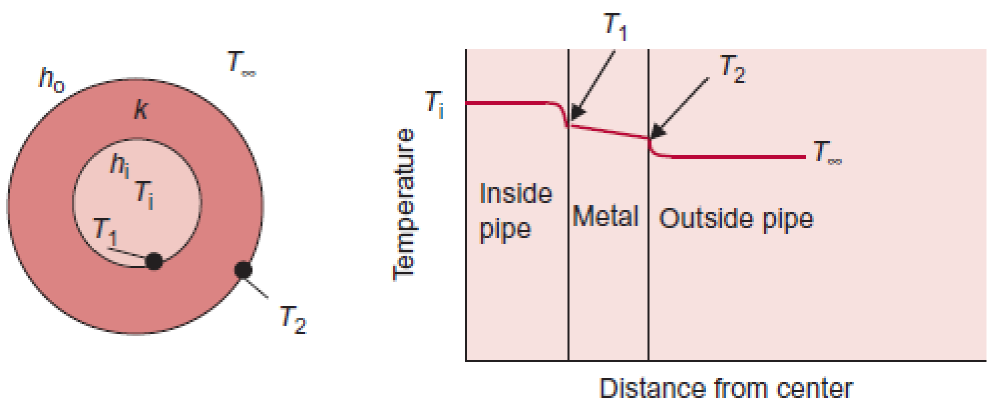


Figure 9: Combined conductive and convective heat transfer.

Using the approach of thermal resistance values, we can write:

$$q = \frac{T_i - T_\infty}{R_t} \qquad \text{Eq. (47)}$$

where $\boldsymbol{R_t}$ is a combination of the thermal resistances in the inside convective layer, the conductive layer in the pipe material, and the outside convective layer, or

$$R_t = (R_t)_{\text{inside convection}} + (R_t)_{\text{conduction}} + (R_t)_{\text{outside convection}} \qquad \text{Eq. (48)}$$

where

$$(R_t)_{\text{inside convection}} = \frac{1}{h_i A_i} \qquad \text{Eq. (49)}$$

where h_i is the inside convective heat transfer coefficient, and A_i is the inside surface area of the pipe.

Resistance to heat transfer in the pipe wall is

$$(R_t)_{\text{conduction}} = \frac{\ln\left(\frac{r_o}{r_i}\right)}{2\pi kL} \qquad \text{Eq. (50)}$$

where k is the thermal conductivity of the pipe material (W/[m K]), r_i is the inside radius (m), and r_o is the outside radius (m). Resistance to heat transfer due to convection at the outside pipe surface is

$$(R_t)_{\text{outside convection}} = \frac{1}{h_o A_o} \qquad \text{Eq. (51)}$$

where h_o is the convective heat transfer coefficient at the outside surface of the pipe (W/[m^2 K]), and A_o is the outside surface area of the pipe. Substituting

$$q = \frac{T_S - T_\infty}{\left(\frac{1}{h_i A_i}\right) + \frac{\ln(r_o/r_i)}{2\pi kL} + \left(\frac{1}{h_o A_o}\right)} \qquad \text{Eq. (52)}$$

We can also write an expression for the overall heat transfer for this example as

$$q = U_i A_i (T_i - T_\infty) \qquad \text{Eq. (53)}$$

where A_i is the inside area of the pipe, and U_i is the overall heat-transfer coefficient based on the inside area of the pipe. From Eq. (53)

$$q = \frac{T_i - T_\infty}{\left(\frac{1}{U_i A_i}\right)} \qquad \text{Eq. (54)}$$

Therefore, $\frac{1}{U_i A_i} = \left(\frac{1}{h_i A_i}\right) + \frac{\ln(r_o/r_i)}{2\pi kL} + \left(\frac{1}{h_o A_o}\right)$ Eq. (55)

Eq. (55) is used to calculate the overall heat-transfer coefficient. The selection of area over which to calculate the overall heat transfer is quite arbitrary.

4. Design of a Tubular Heat Exchanger

One of the key objectives in calculations involving a heat exchanger is to determine the required heat transfer area for a given application. We will use the following assumptions:

1. Heat transfer is under steady-state conditions.
2. The overall heat-transfer coefficient is constant throughout the length of pipe.
3. There is no axial conduction of heat in the metal pipe.
4. The heat exchanger is well insulated. The heat exchange is between the two liquid streams flowing in the heat exchanger.

There is negligible heat loss to the surroundings. The change in heat energy in a fluid stream, if its temperature changes from T_1 to T_2, is expressed as:

$$q = \dot{m} c_p (T_1 - T_2) \qquad \text{Eq. (56)}$$

where $\dot{m}$ is mass flow rate of a fluid (kg/s), c_p is specific heat of a fluid (kJ/[kg ^{o}C]), and the temperature change of a fluid is from some inlet temperature T_1 to an exit temperature T_2.

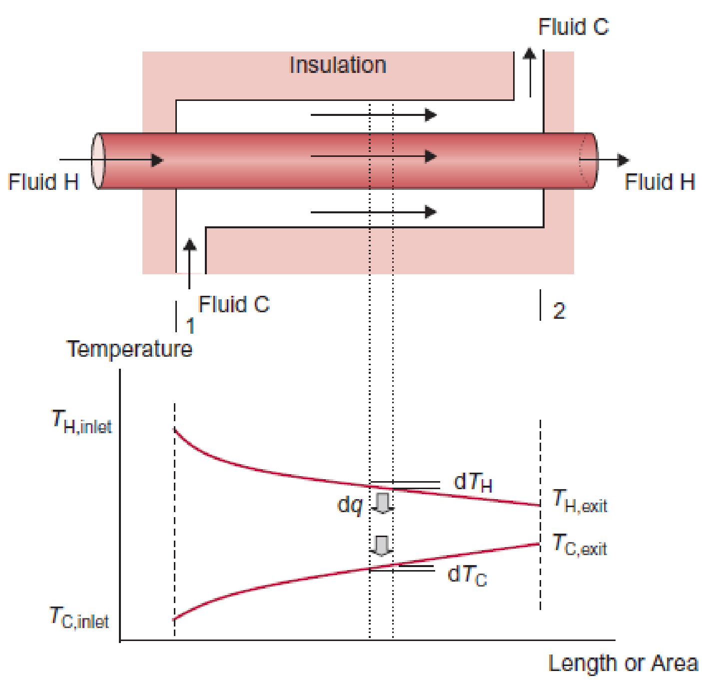


Figure 10: A concurrent flow heat exchanger and temperature plots.

Consider a tubular heat exchanger, as shown in Figure 10. A hot fluid, H, enters the heat exchanger at location (1) and it flows through the inner pipe, exiting at location (2). Its temperature decreases from $T_{H,inlet}$ to $T_{H,exit}$. The second fluid, C, is a cold fluid that enters the annular space between the outer and inner pipes of the tubular heat exchanger at location (1) and exits at location (2). Its temperature increases from $T_{C,inlet}$ to $T_{C,exit}$. The outer pipe of the heat exchanger is covered with an insulation to prevent any heat exchange with the surroundings. Because the heat transfer occurs only between fluids H and C, the decrease in the heat energy of fluid H must equal the increase in the energy of fluid C. Therefore, conducting an energy balance, the rate of heat transfer between the fluids is:

$$q = \dot{m}_H c_{pH} (T_{H,inlet} - T_{H,exit}) = \dot{m}_C c_{pC} (T_{C,exit} - T_{C,inlet}) \qquad \text{Eq. (57)}$$

Eq. (57) is useful if we are interested in determining the inlet and exit temperatures of the two fluid streams. But, this equation does not provide us with any information about the size of the heat exchanger required for accomplishing a desired rate of heat transfer, and we cannot use it to determine how much thermal resistance to heat transfer exists between the two fluid streams.

For those questions, we need to determine heat transfer perpendicular to the flow of the fluid streams.

From Figure 10:

Let, the temperature difference, ΔT, between the two fluids H and C is

$$\Delta T = T_H - T_C \qquad \text{Eq. (58)}$$

where T_H is the temperature of the hot stream and T_C is that of the cold stream.

$$T_{H,inlet} - T_{C,inlet} = \Delta T_1 \qquad \text{Eq. (59)}$$

$$T_{H,exit} - T_{C,exit} = \Delta T_2 \qquad \text{Eq. (60)}$$

$$q = UA \frac{\Delta T_2 - \Delta T_1}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)} \qquad \text{Eq. (61)}$$

$$q = UA(\Delta T_{lm}) \qquad \text{Eq. (62)}$$

$$\Delta T_{lm} = \frac{\Delta T_2 - \Delta T_1}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)} \qquad \text{Eq. (63)}$$

ΔT_{lm} is called the log mean temperature difference (LMTD). Equation (63) is used to design a heat exchanger and determine its area and the overall resistance to heat transfer.

Conclusion

In the present module steady state heat transfer with respect to conduction and convection gas been covered. Emphasis has been given to estimation of convection heat transfer coefficient for forced and free flow cases. Estimation of overall heat transfer coefficient has been detailed for a case involving conduction and convection. Finally equations for design of a tubular heat exchanger has been discussed with the introduction of the term log mean temperature difference.