UNIT 6 – Heat and Mass Transfer (Part-3)

In the present module following topics are covered

1. The Diffusion Process
    1.1 Steady-state diffusion of fluids through solids
2. Convective mass transfer
3. Membrane separation
    3.1 Electrodialysis systems
    3.2 Reverse Osmosis Membrane Systems
    3.3 Ultrafiltration Membrane Systems
4. Types of Reverse-Osmosis and Ultrafiltration systems
    4.1 Plate and Frame
    4.2 Tubular
    4.3 Spiral-Wound
    4.4 Hollow-fiber

In food processing, we often create conditions to encourage chemical reactions that produce desirable end-products in the most efficient manner. Mass transfer plays a key role in creating favorable conditions for reactants to physically come together, allowing a reaction to occur. In designing separation processes, an understanding of the mass transfer processes becomes important.

Use of the term mass transfer is restricted to the migration of a constituent of a fluid or a component of a mixture. The migration occurs because of changes in the physical equilibrium of the system caused by the concentration differences.

1. The Diffusion Process

Mass transfer involves both mass diffusion occurring at a molecular scale and bulk transport of mass due to convection flow. The diffusion process can be described mathematically using Fick's law of diffusion, which states that the mass flux per unit area of a component is proportional to its concentration gradient. Thus, for a component B

$$\frac{\dot{m}_B}{A} = -D\frac{\partial c}{\partial x} \qquad \text{Eq. (1)}$$

Where $\dot{m}_B$ is mass flux of component B (kg/s); c is the concentration of component B, mass per unit volume (kg/m^3); D is the mass diffusivity (m^2/s); and A is area (m^2). Mass flux may also be expressed as kg-mole/s, and the concentration of component B will be kg-mole/m^3.

We note that Fick's law is similar to Fourier's law of heat conduction,

$$\frac{q}{A} = -k\frac{\partial T}{\partial x} \qquad \text{Eq. (2)}$$

and Newton's equation for shear-stress strain relationship

$$\sigma = -\mu\frac{\partial u}{\partial y} \qquad \text{Eq. (3)}$$

These similarities between the three transport equations suggest additional analogies among mass transfer, heat transfer, and momentum transfer.

1.1 Steady-State Diffusion of Gases (and Liquids) through Solids

Assuming the mass diffusivity does not depend on concentration, from Eq. (1)

$$\frac{\dot{m}_A}{A} = -D_{AB}\frac{dc_A}{dx} \qquad \text{Eq. (4)}$$

where D_{AB} is mass diffusivity for gas A (or liquid A) in a solid B. Subscript A for $\dot{m}$ and c represents a gas or liquid diffusing through a solid. In reality, D_{AB} represents an effective diffusivity through solids.

By separating variables and integrating

$$\frac{\dot{m}_A}{A}\int_{x_1}^{x_2} dx = -D_{AB}\int_{c_{A1}}^{c_{A2}} dc_A \qquad \text{Eq. (5)}$$

$$\frac{\dot{m}_A}{A} = \frac{D_{AB}(c_{A1} - c_{A2})}{(x_2 - x_1)} \qquad \text{Eq. (6)}$$

For a cylindrical shape, radial coordinates would apply and the following equation is obtained

$$\dot{m}_A = \frac{D_{AB} 2\pi \mathrm{L}(c_{A1} - c_{A2})}{\ln\frac{r_2}{r_1}} \qquad \text{Eq. (7)}$$

2. Convective Mass Transfer

When the transport of a component due to a concentration gradient is enhanced by convection, the mass flux of the component will be higher than would occur by molecular diffusion. Convective mass transfer will occur in liquids and gases, and within the structure of a porous solid.

The convective mass transfer coefficient $\mathbf{k_m}$ is defined as the rate of mass transfer per unit area per unit concentration difference. Thus

$$k_m = \frac{\dot{m}_B}{A(c_{B1} - c_{B2})} \qquad \text{Eq. (8)}$$

where $\dot{m}_B$ is the mass flux (kg/s); c is concentration of component B, mass per unit volume (kg/m^3); A is area (m^2). The units of km are m^3/m^{2}s or m/s. The coefficient represents the volume (m^3) of component B transported across a boundary of one square meter per second.

In situations that involve molecular diffusion and mass transfer due to forced convection, the following variables are important: mass diffusivity D_{AB}, for component A in fluid B; the velocity of the fluid, u; the density of the fluid, ρ; the viscosity of the fluid, μ; the characteristic dimension d_c; and the convective mass transfer coefficient k_m. In the case of natural convection, additional important variables include the acceleration due to gravity, g, and the mass density difference $\Delta\rho$. The variables are grouped in the following dimensionless numbers:

$$N_{Sh} = \frac{k_m d_c}{D_{AB}} \qquad \text{Eq. (9)}$$

$$N_{Sc} = \frac{\mu}{\rho D_{AB}} \qquad \text{Eq. (10)}$$

$$N_{\mathrm{Re}} = \frac{\rho u d_c}{\mu} \qquad \text{Eq. (11)}$$

$$N_{Le} = \frac{k}{\rho c_p D_{AB}} \qquad \text{Eq. (12)}$$

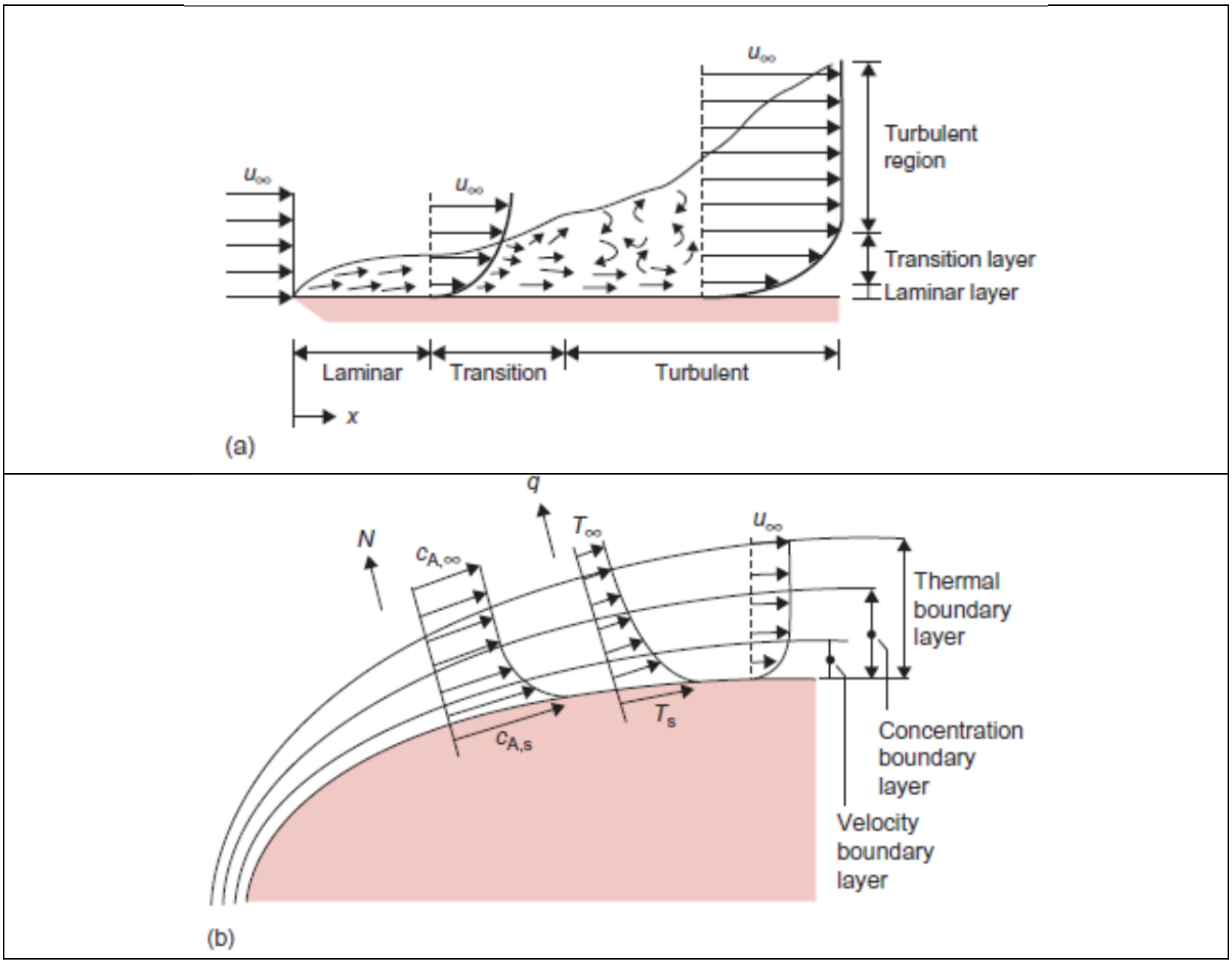


Figure 1: (a) The development of a boundary layer on a flat plate. (b) The development of the thermal, concentration, and velocity boundary layers on a surface.

Consider a fluid flowing over a flat plate as shown in Figure 1 (a). For the boundary layer from the leading edge of the plate, we can write the following equations for momentum, energy, and concentration, respectively.

$$u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y} = \frac{\mu}{\rho} \frac{\partial^2 u_x}{\partial y^2} \qquad \text{Eq. (13)}$$

$$u_x \frac{\partial T}{\partial x} + u_y \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \qquad \text{Eq. (14)}$$

$$u_x \frac{\partial c_A}{\partial x} + u_y \frac{\partial c_A}{\partial y} = D_{AB} \frac{\partial^2 c_A}{\partial y^2} \qquad \text{Eq. (15)}$$

In Equation (15), c_A represents concentrations of component A at locations within the boundary layer.

Note that

$$\frac{\mu}{\rho\alpha} = \frac{\mu c_p}{k} = N_{\mathrm{Pr}} = \text{Prandtl number} \qquad \text{Eq. (16)}$$

Thus, the Prandtl number provides the link between velocity and temperature profiles.

From Equations (13) and (15), if

$$\frac{\mu}{\rho D_{AB}} = 1 \qquad \text{Eq. (17)}$$

then velocity and concentration profiles have the same shape. The ratio

$$\frac{\mu}{\rho D_{AB}} = N_{Sc} = \text{Schmidt number} \qquad \text{Eq. (18)}$$

The concentration and temperature profiles will have the same shape if

$$\frac{\alpha}{D_{AB}} = 1 \qquad \text{Eq. (19)}$$

The ratio

$$\frac{\alpha}{D_{AB}} = N_{Le} = \text{Lewis number} \qquad \text{Eq. (20)}$$

The functional relationships that correlate these dimensional numbers for forced convection are:

$$\mathrm{N}_{Sh} = f(N_{\mathrm{Re}}, N_{Sc}) \qquad \text{Eq. (21)}$$

If the dimensionless profiles of velocity, temperature, and concentration are assumed to be similar, the Nusselt and Prandtl numbers for heat transfer can be replaced by the Sherwood and Schmidt numbers, respectively, in mass transfer. Thus, it may be deduced that

$$\mathrm{N}_{Sh} = \frac{\text{total mass transferred}}{\text{total mass transferred by molecular diffusion}} \qquad \text{Eq. (22)}$$

$$N_{Sc} = \frac{\text{molecular diffusion of momentum}}{\text{molecular diffusion of mass}} \qquad \text{Eq. (23)}$$

Laminar Flow over a Flat Plate

Laminar flow over a flat plate exists when $N_{\mathrm{Re}} \leq 5 \times 10^5$, and the correlation is

$$N_{Sh_x} = \frac{k_{m,x} x}{D_{AB}} = 0.322 N_{\mathrm{Re}_L}^{1/2} N_{\mathrm{Sc}}^{1/3} \qquad N_{Sc} \geq 0.6 \qquad \text{Eq. (24)}$$

In Equation (24), the convective mass transfer coefficient $k_{m,x}$ in the Sherwood number is at a fixed location; therefore, the $N_{Sh,x}$ is called the local Sherwood number. The characteristic dimension used in the Sherwood and Reynolds numbers is the distance from the leading edge of the plate.

3. Membrane separation

Membrane separation systems have been used extensively in the chemical process industry. Their use in the food industry is now becoming more common. Some of the typical food-related applications include purification of water, and the concentration and clarification of fruit juices, milk products, alcoholic beverages, and wastewater.

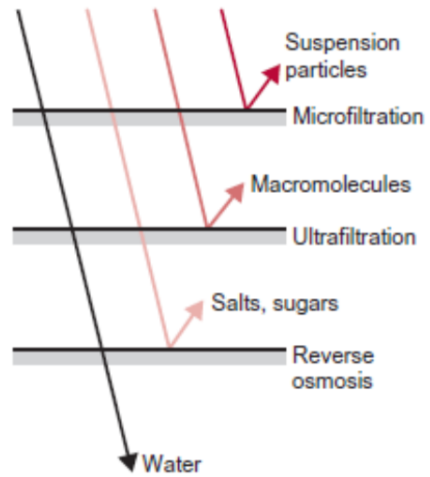


Figure 2:

Use of membrane systems to separate substances of different-sized molecules.

3.1 Electrodialysis Systems

The separation process in an electrodialysis system is based on the selective movement of ions in solution. The membranes in these systems derive their selectivity from the ions (anions or cations) that are allowed to permeate through them. Selected ions are removed from water as they pass through the semipermeable membranes. These membranes do not permit permeation of water.

If a polymeric chain in a membrane has a fixed negative charge, it will repulse any anion that tries to enter the membrane. This is shown schematically in Figure 3. For example, a negatively charged chain attracts cations and allows them to move through.

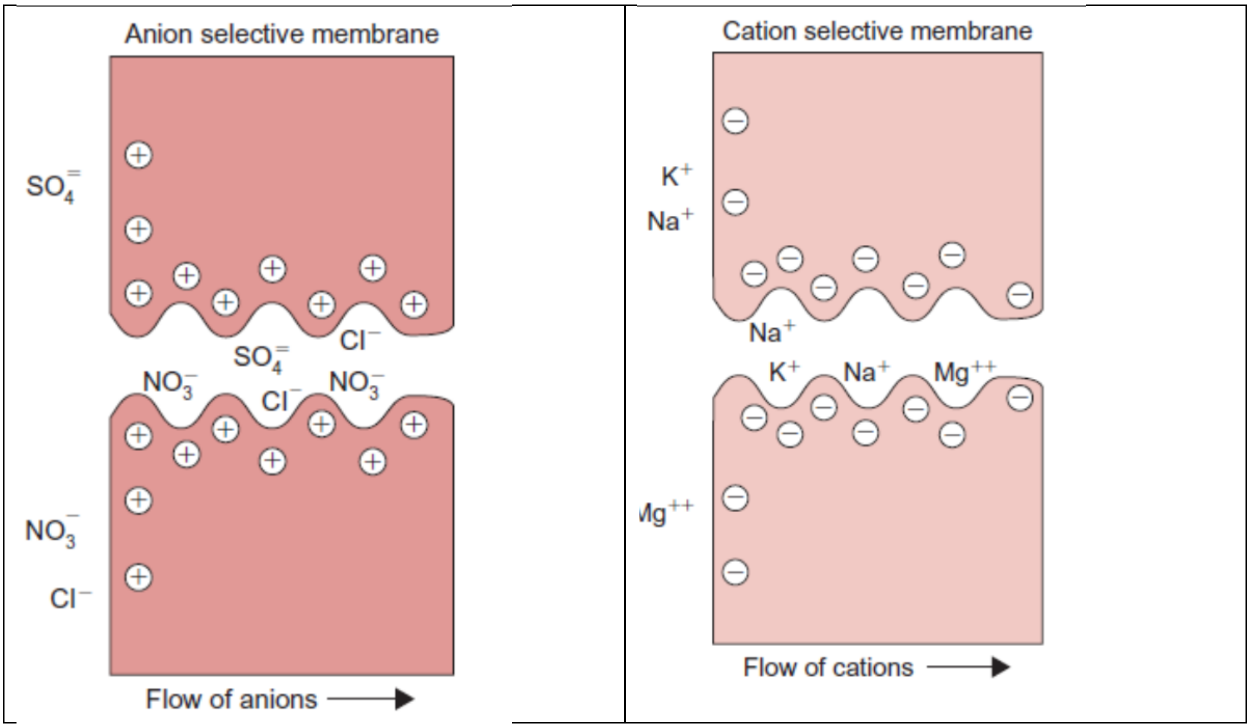


Figure 3: The movement of ions in ionselective membranes.

An ion membrane system used in the electrodialysis system, is shown in Figure 4.

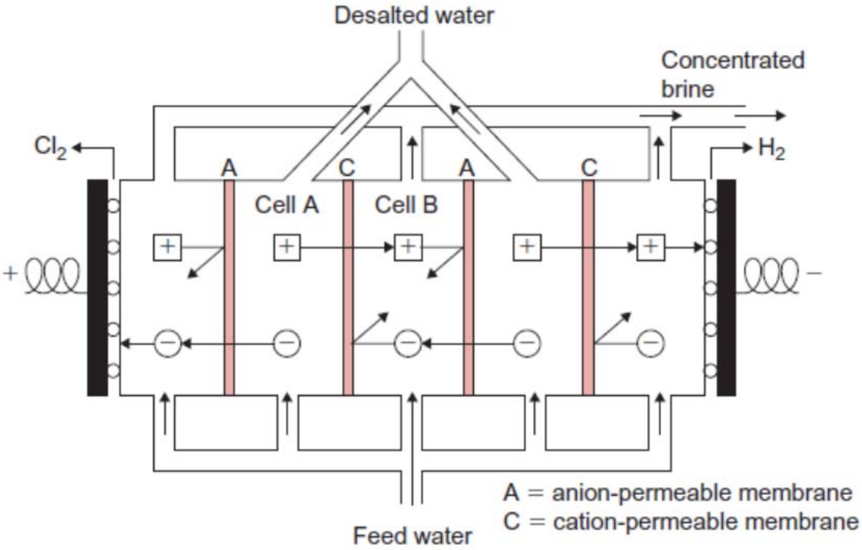


Figure 4: Desalting of water with an electrodialysis system.

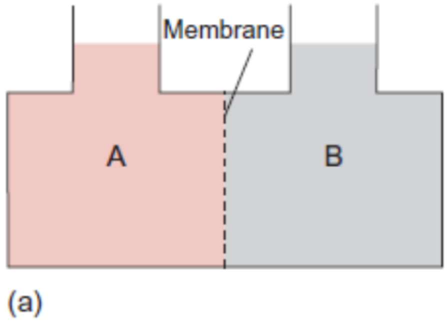
The electrodialysis system uses an electric current to transfer ions through a membrane. The membranes have fixed ionic groups that are chemically bound to the structure of the membrane. As shown in Figure 4, the electrodialysis system involves an array of membranes, alternating between anion-exchange and cation-exchange membranes. In between the membranes are small compartments (0.5 to 1.0 mm thick) that contain the solution.

Electrodialysis has been used extensively in desalting processes employing membranes that are permeable to ions, but impervious to water.

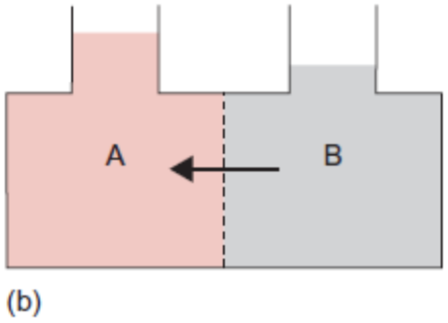
3.2 Reverse Osmosis Membrane Systems

The movement of water occurs from a solution with high concentration of water to a solution with low concentration of water, thus tending to equalize the water concentration on the two sides of the membrane. This movement of water is generally referred to as osmosis.

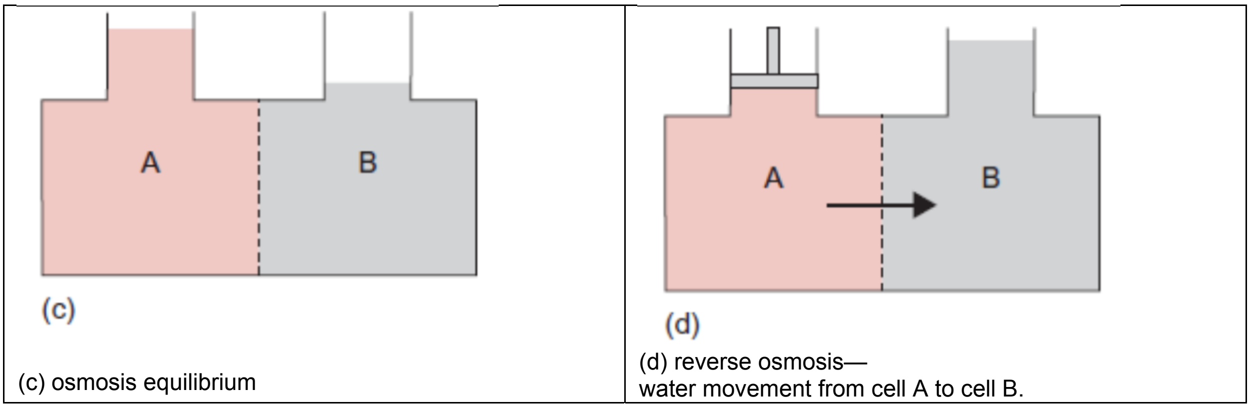
Consider a solution of water containing a solute. In Figure 5(a), a semipermeable membrane separates the solutions of the same solute concentration contained in chambers A and B. Since the chemical potential of the solvent (water) is the same on both sides of the membrane, no net flow of water occurs through the membrane. In Figure 5(b), chamber A contains a solution with a higher solute concentration than chamber B; that is, chamber A has lower water concentration than chamber B. This also means that the chemical potential of the solvent (water) in chamber A will be lower compared with that of chamber B. As a result, water will flow from chamber B to chamber A. As seen in Figure 5(c), this movement of water will cause an increase in the volume of water in chamber A. Once equilibrium is reached, the increased volume represents a change in head, or pressure, which will be equal to the osmotic pressure. If an external pressure greater than the osmotic pressure is then applied to chamber A, as shown in Figure 5(d), the chemical potential of water in chamber A will increase, resulting in water flow from chamber A to chamber B. The reversal in the direction of water flow, obtained by application of an external pressure that exceeds the osmotic pressure, is termed reverse osmosis.



(a) solute concentration same in cells A and B;



b) water movement from cell B to cell A;



(c) osmosis equilibrium

(d) reverse osmosis—
water movement from cell A to cell B.

Figure 5: The reverse-osmosis process

A reverse-osmosis membrane system is used to remove water from a water-solute mixture by application of external pressure. In contrast to electrodialysis, the membrane used in the reverse-osmosis system must be permeable to water.

In a reverse-osmosis system, water is the permeating material referred to as "permeate," and the remaining solution concentrated with the solutes is called "retentate." The osmotic pressure Π of a dilute solution can be obtained by Van't Hoff's equation, which uses colligative properties of dilute solutions.

$$\Pi = \frac{cRT}{M} \qquad \text{Eq. (26)}$$

where Π is osmotic pressure (Pa), c is solute concentration (kg/m^3) of solution, T is absolute temperature (K), R is gas constant, and M is molecular weight.

The performance of a membrane system is often described by the "retention factor," R_f.

$$R_f = \frac{\left(c_f - c_p\right)}{c_f} \qquad \text{Eq. (31)}$$

where c_f is the concentration of a solute in the feed stream (kg/m^3) and c_p is the concentration of a solute in the permeate stream (kg/m^3).

3.3 Ultrafiltration Membrane Systems

Ultrafiltration membranes have pore sizes much larger than the reverse osmosis membrane. Ultrafiltration membranes are used primarily for fractionating purposes: that is, to separate high-molecular-weight solutes from those with low molecular weight. Since the ultrafiltration membranes have larger pore sizes, the hydraulic pressures required as a driving force are much smaller when compared with the reverse osmosis membrane systems. Typically, pressures in the range of 70 to 700 kPa are needed for ultrafiltration membrane systems. The pore size of ultrafiltration membrane ranges from 0.001 to 0.02 m, with molecular weight cutoffs from 1000 to 80,000. The flux rate through an ultrafiltration membrane can be obtained from the following equation:

$$N = KA\Delta P$$

where ΔP is pressure difference across the membrane, K is membrane permeability constant (kg/[m^2 kPa s]), and A is membrane surface area (m^2).

Concentration Polarization

In membrane separation processes, when a liquid solution containing salts and particulates is brought next to a semipermeable membrane, some of the molecules accumulate in the boundary layer next to the membrane surface (Figure 6). Thus, the concentration of a retained species will be higher in the boundary layer adjacent to the membrane than in bulk. This phenomenon is called concentration polarization, and it has a major effect on the performance of a membrane system.

Concentration polarization occurs in both reverse-osmosis and ultrafiltration systems. In addition, the causes of concentration polarization are the same, but the consequences are different.

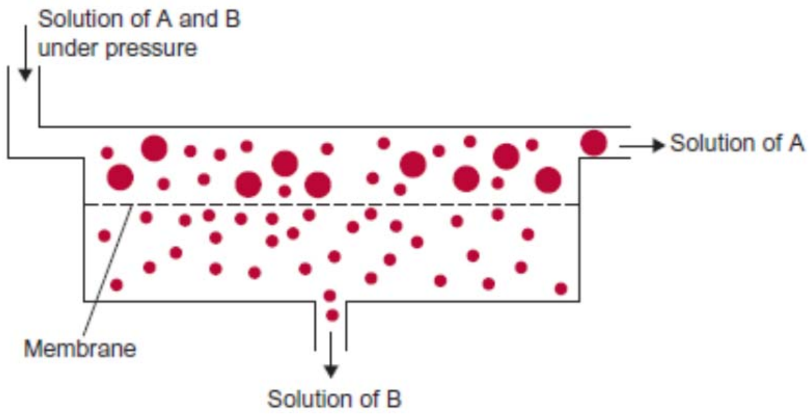


Figure 6: Separation process in a pressure-driven membrane system

4.0 Types of Reverse-Osmosis and Ultrafiltration systems

Four major types of membrane devices are used for reverse-osmosis and ultrafiltration systems: plate-and-frame, tubular, spiral-wound, and hollow-fiber.

Table 1 **Comparison of Process-Related Characteristics for Membrane Module Configurations**

	Module Type			
Characteristic	Plate-and-frame	Spiral-wound	Tube-in-shell	Hollow-fiber
Packing density (m^2/m^3)	200-400	300-900	150-300	9000-30,000
Permeate flux ($m^3/[m^2$ day])	0.3-1.0	0.3-1.0	0.3-1.0	0.004-0.08
Flux density ($m^3/[m^3$ day])	60-400	90-900	45-300	36-2400
Feed channel diameter (mm)	5	1.3	13	0.1
Method of replacement	As sheets	As module assembly	As tubes	As entire module
Replacement labor	High	Medium	High	Medium
Pressure drop				
Product side	Medium	Medium	Low	High
Feed side	Medium	Medium	High	Low
Concentration polarization	High	Medium	High	Low
Suspended solids buildup	Low/medium	Medium/high	Low	High

4.1 Plate-and-Frame

The plate-and-frame membrane systems involve a large number of flat membranes that are sandwiched together with the use of spacers.

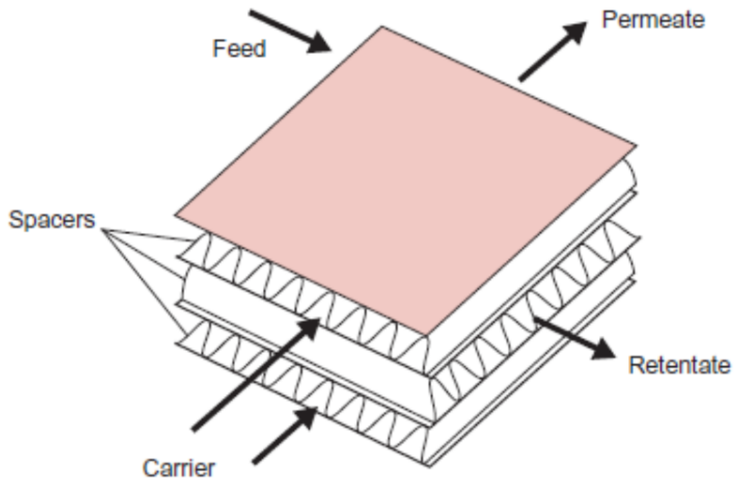


Fig. 7: A module used in a plate and-frame membrane system.

4.2 Tubular

The tubular design was the first commercial design of a reverse osmosis system. It consists of a porous tube coated with the membrane material such as cellulose acetate. Typically, feed solution is pumped into the tube through one end and forced in the radial direction through the porous pipe and the membrane (Fig. 8). Water drips from the outer surface of the membrane while the concentrated stream "retenate" leaves through the outer end of the tube. This type of reverse-osmosis device is expensive to use for high volumetric flow rates, since the membrane area is relatively small.

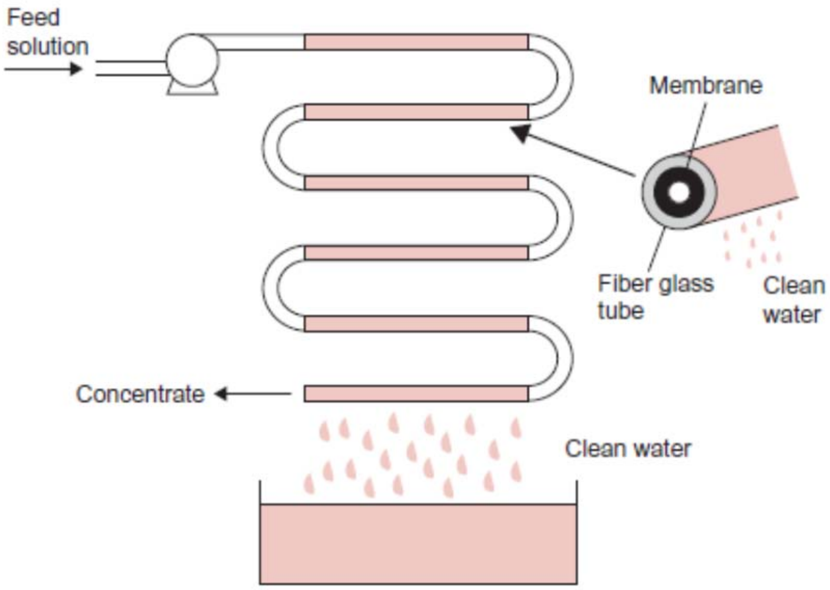


Fig. 8: A tubular membrane system.

4.3 Spiral-Wound

To increase the membrane surface area per unit volume, a spiralwound configuration was a key commercial development following the tubular design. This design, shown in Figure 9, can be visualized as a composite of multilayers. The two layers of membrane are separated by a plastic mesh, and on either side of the membrane is a porous sheet. These five layers are then spirally wound around a perforated tube. The whole spiral assembly is housed in a tubular metal jacket that can withstand applied pressures. Feed is pumped through the perforated tube on one side of the spiral-wound roll. Feed enters inside the plastic mesh, which aids in creating turbulence and minimizing fouling. The feed then permeates the membrane in a radial direction and exits the membrane into porous layers. The permeate (water) transports through the porous sheet in a spiral manner and leaves the assembly through the exit tube, whereas the retentate leaves through the other end of the spiral-wound roll.

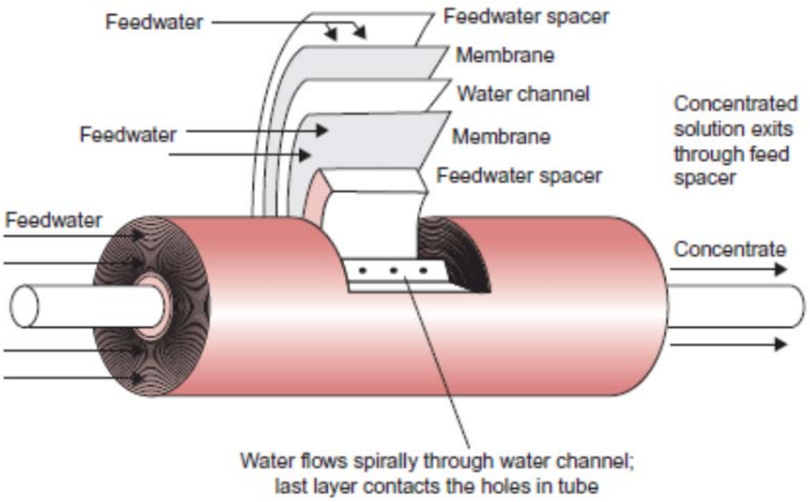
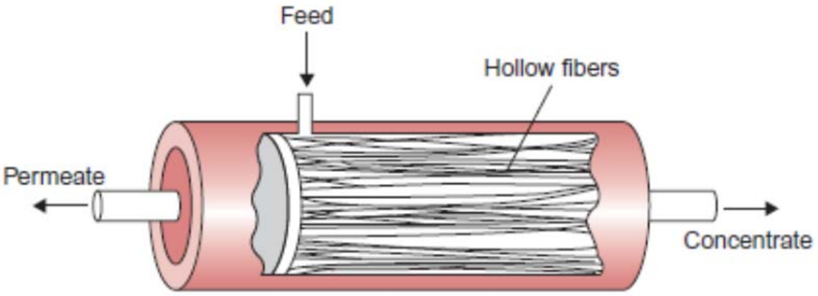


Fig. 9: A spiral-wound membrane system with a cross-sectional view showing the water flow patterns.

4.4 Hollow-fiber

A hollow-fiber, made out of aramid, was first introduced by DuPont in 1970. Hollow-fibers, finer than human hair, have an internal diameter of about 40 µm, and an external diameter of about 85 µm. A large number of hollow-fibers (several millions) are arranged in a bundle around a perforated distributor tube (Fig. 10). Feed water is introduced through the distributor pipe; the permeate flows through the annular space of the fibers into the hollow bore of the fibers and moves to the tube sheet end, discharging from the exit port. The retentate or brine stays on the outside of the fibers and leaves the device from the brine port. Hollow fibers are used mainly to purify water. Liquid foods are difficult to handle in hollow fiber systems because of problems associated with fouling of the fibers.



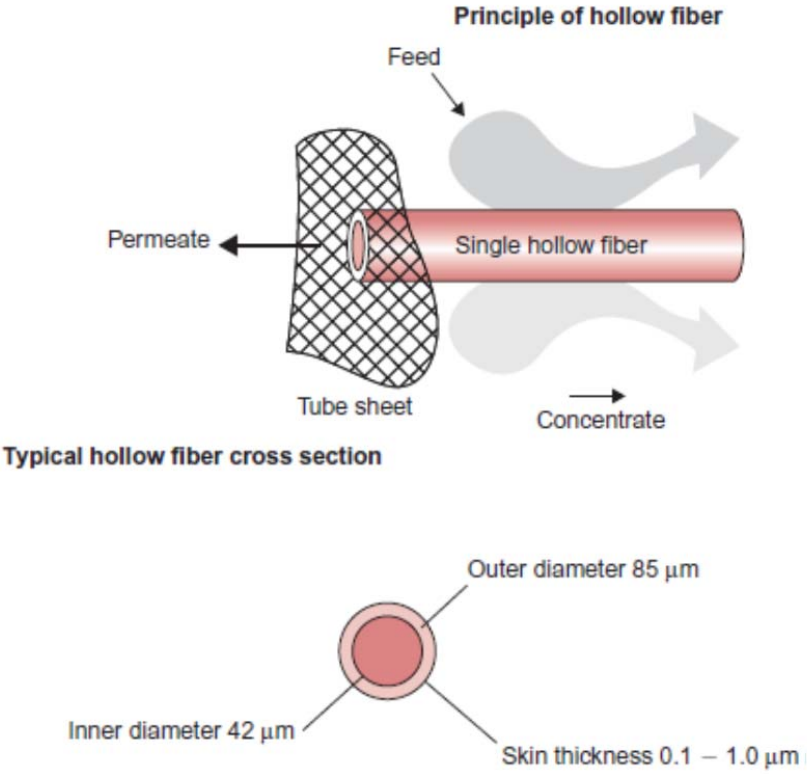


Figure 10: A hollow-fiber membrane system

Most membrane systems in the food industry have been in dairy and fruit juice applications. Other commercial applications include processing of coffee, tea, alcohol, gelatins, eggs, and blood, and corn refining and soybean processing.

Conclusion

In the present module, diffusion and convective mass transfer operations were detailed, which are essential in understanding the designing of reactors and separation processes. Under the membrane- based separation processes, the concepts of reverse-osmosis and ultrafiltration- were emphasized. Finally the membrane systems such as (i) Plate-and Frame (ii) Tubular (iii) Spiral-wound and (iv) and Hollow-fiber systems were discussed.