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SOLOW MODEL

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Unit No. & Title:	Unit – 2 Growth Model
Lecture No. & Title:	Lecture – 2 SOLOW MODEL

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1. Introduction

We observed the growth process explained in the Harrod-Domar model. In this lecture we shall see the Solow model of growth which developed as an alternative to the Harrod-Domar model.

Objectives

1. To understand the meaning of technological progress.
2. To understand the difference between endogenous and exogenous growth.
3. To understand the effect of population, capital accumulation and capital - labour ratio on the long term growth of economies.
4. To understand why the growth rate of capital rich economies is lower than that of the developing economies.
5. To understand the meaning of convergence in the long term growth process.

Before we proceed to understand Solow's growth model, we must know the meanings of **technological progress, exogenous and endogenous growth**.

Technological progress is the term used for technical innovation which helps to increase output.

Technological progress can be **'embodied' or 'disembodied'**.

Embodied technical progress means that machines embody inside them the technology which they are made up of. The vintage theory of capital focuses on the fact that a machine embodies the technology which existed on the date of its construction. Hence, with technical progress as new technology is innovated, the old technology becomes obsolete and the machine loses its value. Hence all machines made on different dates have different capacity to produce and hence cannot be aggregated as one single measure of capital. Hence, a separate production function is needed for each vintage (machine with a different date of manufacturing or with a different embedded technology). New machines have new improved technology and thus imply technological progress.

Disembodied technical change is purely organizational which allows more output to be produced from unchanged inputs, without any new investment. It refers to any kind of shift in the production function that leaves the balance between labour and capital undisturbed in the long run.

The production function for such a technical change is written as,

$Q=F(K, L; A)$ which means quantity produced (Q) is a function of capital (K) and labour (L) inputs and of technical change (A).

OR

$Q=A(t)F(K, L)$

Where, A(t) represents technical change which makes the inputs more effective in raising output. This technological change is organizational and does not require new inputs.

Besides, technical progress can be neutral, labour-saving or capital-saving.

Exogenous growth is a term used for growth that occurs owing to factors external to the model of growth. For instance, technological progress which is exogenous factor, in some growth models can lead to growth.

However, the idea of technological progress cannot be easily explained.

Endogenous growth is a term used to explain that growth occurs owing to factors within the model. The Harrod-Domar model is an endogenous growth model as it explains that growth is determined by the rate of savings and the capital-output ratio which are determined within the model. Growth can occur when savings are used to invest in physical or human capital. And, growth can also occur when the capital-output ratio falls. Models based on this idea also explain that investments in human capital have positive spill-over effects and hence by increasing subsidies, by investments in education, investment in research and development etc. growth can increase. These models thus assert that long-run growth can depend upon policy measures which raise investments in certain sectors.

2. SOLOW Model of growth

R. M. Solow's model of growth developed as an alternative to the Harrod-Domar model. Solow's model is based on diminishing returns to individual factors of production.

Solow noted that any increase in Quantity produced could come from one of the three sources as:

1. An increase in Labour - 'L' . However, due to diminishing returns to scale, this would imply a reduction in the Output to Labour ratio **(**or output per worker).

2. An increase in capital An increase in the **stock of capital** would increase both output and **the** Output to Labour ratio **(Q/L); as capital helps to improve labour productivity. 3. An improvement in technology (A)** could also increase **Q / L** or output per worker.

If the production function is a typical Cobb-Douglas production function given by $Y=AK^{\alpha}N^{1-\alpha}$, the change in output

$\frac{\Delta Y}{Y}=\frac{\Delta A}{A}+\alpha\frac{\Delta K}{K} - (1-\alpha)\frac{\Delta N}{N}$ [It must be noted that in this Cobb-Douglas production function, it can be derived that α is the share of capital in aggregate output and 1- α is the share of labour in the aggregate output]

$\frac{\Delta Y}{Y}$ is the rate of growth of output

$\frac{\Delta K}{K}$ is the rate of growth of capital stock

$\frac{\Delta N}{N}$ is the rate of growth of labour force

$\frac{\Delta A}{A}$ is the rate of growth of technological progress.

Technological progress cannot be observed and rate of technological progress (the contribution of technological progress to output) is given as a *residual* after the contribution of labour and capital are subtracted from total output. Hence, the additional growth which is not owing to contribution of labour or capital is the growth owing to technological progress.

$$\frac{\Delta A}{A} = \frac{\Delta Y}{Y} - \alpha \frac{\Delta K}{K} - (1-\alpha)\frac{\Delta N}{N}$$

Long Term Growth in Solow Model

According to Solow, both capital and labour are used together to produce output. And, inputs are subject to diminishing productivity.

If there are many labourers and very little capital then capital will help to enhance labour productivity. In other words, productivity of capital will be higher.

If there is less labour and more capital then capital intensive techniques will be adopted. When a lot of capital is used, then its productivity will be lower and hence, the capital-output ratio will be higher.

(**We may recollect here that a higher capital-output ratio means that more units of capital are required to produce**

a unit of output. When productivity of capital is lower, more units will be required to produce a given output).

In this model, the capital-output ratio θ is endogenous.

In the basic Solow model,

$$Y = F(K, L)$$

Where,

Y = total output (total income)

K = capital stock

L = labor supply

(Note: we may also take the Cobb – Douglas production function). If we express the production function in per worker terms (which is similar to income per capita), we obtain,

$$\frac{Y}{L} = F\left(\frac{K}{L}, \frac{L}{L}\right) = F\left(\frac{K}{L}, 1\right) \qquad (1)$$

Now,

- $\frac{Y}{L}$ is nothing but output per worker and we may represent it by *y*.
- $\frac{K}{L}$ is capital per worker (*k*).
- $\frac{L}{L} = 1$

So we get, $y = F(\frac{K}{L}, 1)$ which means that output per worker is a function of capital per worker.

Change in capital stock can be given by,

$\Delta K = \varsigma Y - \delta K$

ΔK = the change in the capital stock

ς= saving rate (fixed fraction of income Y which is saved)

Y = total income (total output)

δ= depreciation rate

K = capital stock

The equation for the change in the capital stock per worker will then be, $\Delta k = \varsigma y - (n + \delta)\, k$

Where,

Δk = change in the capital stock per worker

ςy = saving rate per capita

n = the labor supply growth rate (or the population growth rate)

The capital stock per worker increases due to savings and decreases due to depreciation and an increase in the labor supply

We can use the Harrod-Domar equations to explain this.

Recollect the equation in Harrod-Domar model as,

$K(t+1) = (1-\delta)K(t) + \varsigma Y(t)$ (2)

Let us understand this equation

The stock of capital at the beginning of time period 't+1',

= the stock of depreciated capital carried forward from the previous year 't'

+ savings rate $\varsigma Y(t)$ (which is the constant fraction of income Y which is saved).

Note: Notation S(t) means savings in time period t which can also be expressed as $\varsigma Y(t)$ where, ς is the constant fraction of income Y which is saved. Total savings = fraction of income which is saved. If we assume that population p(t) grows at a constant rate of n then, $p(t+1) = (1+n)P(t)$.

This model assumes that growth of population translates in growth of labour supply (workforce).

Introducing population growth in equation (2), we obtain

$(1+n)P(t)\ K(t+1) =(1-\delta)K(t) + \varsigma Y(t)$ (3)

Dividing equation (3) by P(t), we derive,

$$\frac{(1+n)P(t)\ K(t+1)}{P(t)} = \frac{(1-\delta)K(t)}{P(t)} + \frac{\varsigma Y(t)}{P(t)}$$

Which is, $(1+n)k(t+1) = (1-\delta)k(t) + \varsigma y(t)$ (4)

Where,

$\frac{\mathbf{K(t)}}{\mathbf{P(t)}}$ = capital per capita which is denoted by k(t) and,

$\frac{\mathbf{\varsigma Y(t)}}{\mathbf{P(t)}}$ = savings rate per person which is denoted by ςy(t)

This equation explains that the new per capita capital stock k(t+1) can be obtained by depreciated per capita capital (1-δ)k(t) and per capita savings ςy(t) but population growth drags down the growth of new per capita capital.

The larger the population growth, the lower will be the per capita capital in the subsequent period and hence lower output.

Now if no new savings or investments were made in the previous period, and so depreciation is also taken to be zero then equation (4) which is given as

(1+n)k(t+1) = (1- δ)k(t) + ςy(t) will turn as, (1+n)k(t+1)=(1-0) k(t) +0

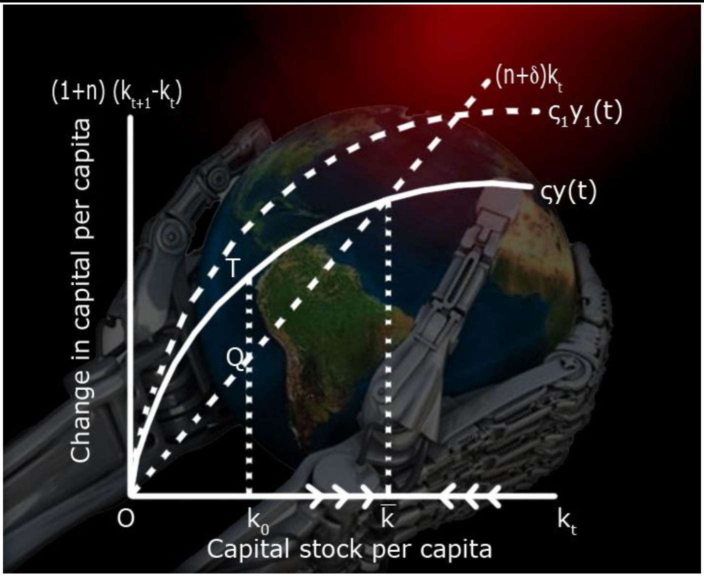
Therefore, (1+n)k(t+1)= k(t)

Or, k(t+1)=$\frac{\boldsymbol{k}\mathbf{(t)}}{\mathbf{1+n}}$

Thus, per capita capital in period t+1 is less than per capita capital in period t.

3. Important conclusion of Solow model equation 4

Let us now draw a graph for equation 4.



- The straight line through the origin shows amount of new capital required per person to keep the amount of capital per person constant. The slope of this line is (1- δ). Since, depreciation and increase in population both reduce the per capita availability of capital, a constant increase in capital (investment) must take place to keep a ***steady state of growth.***
- The curve represented by ςy(t) shows the amount of savings rate per capita income.
- Savings rate is also the investment rate by macroeconomic (product market) identity S=I.
- So the curve ςy(t) represents the investment per worker.
- At, the per capita capital k_0 the investment per worker is

T k_0 which is greater than the investment required to keep the capital per capita constant which is Q k_0.

- Till $\bar{k}$, the investment per worker is greater than the investment required to keep the capital per capita constant.
- At $\bar{k}$, the capital required to maintain a steady state of growth is equal to the capital investment per person.

- If the economy is initially at a point on right hand side of $\bar{k}$, then the investment per worker is lower than the investment required to keep the capital per capita constant.
- This means that the amount of capital per worker goes on declining in the economy. It declines till $\bar{k}$. This movement is shown by leftward arrows.
- Thus $\bar{k}$ represents the steady state of growth where $k\,(t+1) = k\,(t)$.
- In other words, at $\bar{k}$ the capital per capita = capital required to maintain the capital–labour ratio.
- When capital stock per capita is too high, the marginal productivity of capital is very low and hence the output generation is lesser.

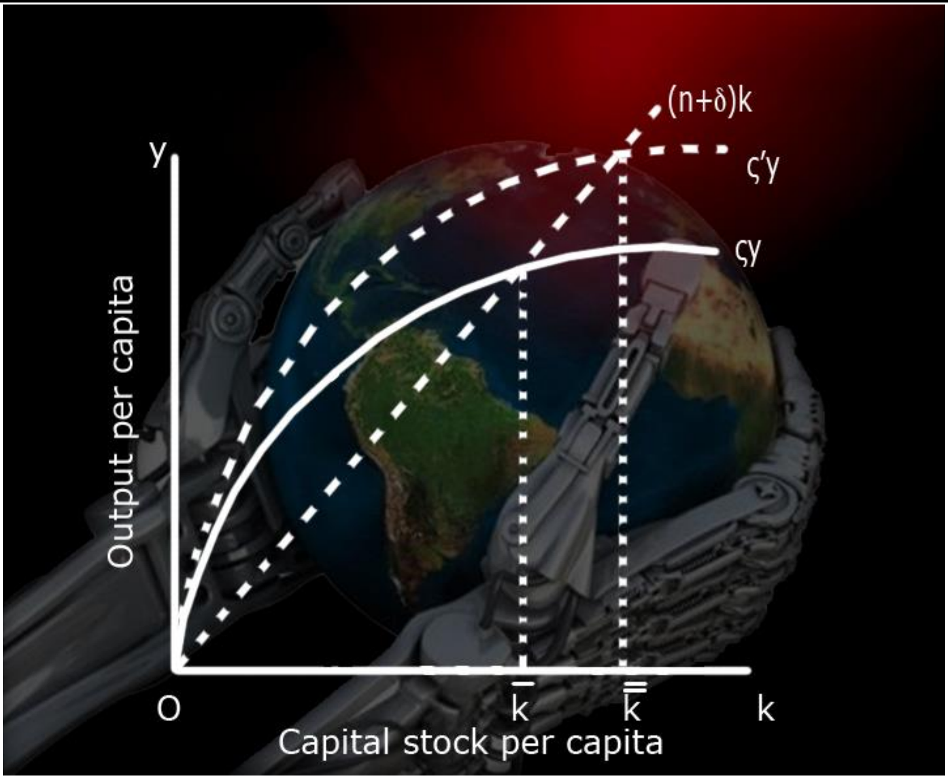
➢ This means when a country is richer and per capita capital stock is higher, the growth rate declines.

➢ At lower y, savings and investment will be lower and thus, capital per worker will decline.

➢ But as these countries raise their savings and capital formation, the per capita availability of capital rises and growth tends to rise.

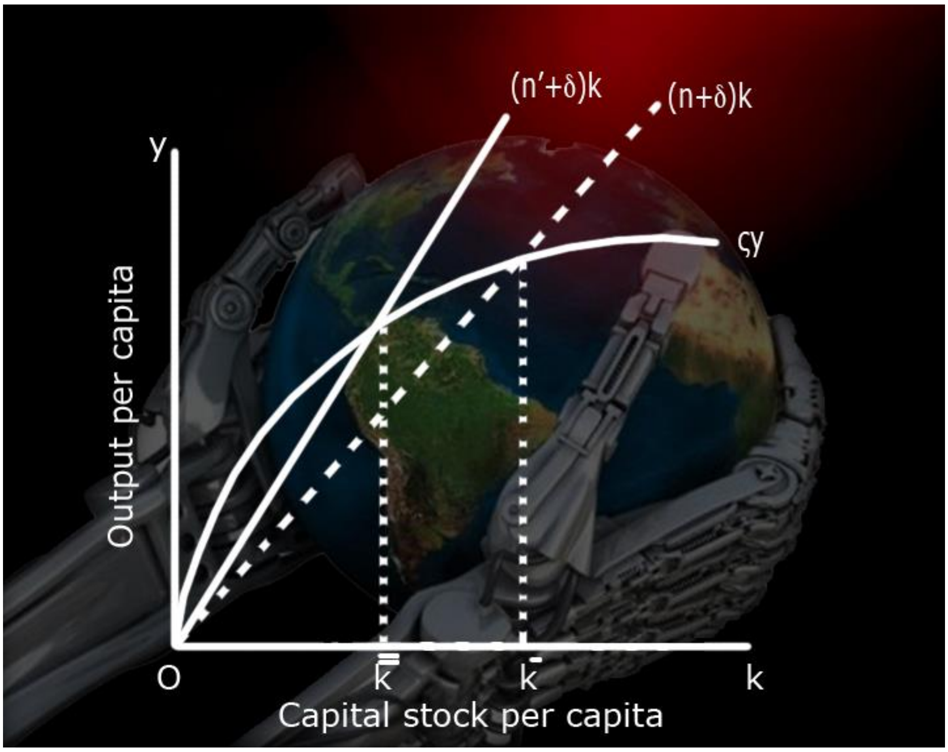
➢ *Hence all economies converge towards the steady state in the very long run.*

- Hence, we can say that each nation requires an appropriate stock of capital to maintain its labour productivity and output.
- Growth will be very strong when countries first begin to accumulate capital, and will slow down as the process of capital accumulation continues. Growth of capital abundant countries was stronger in the 1950s and 1960s than it is now.



Look at this figure now. A higher savings rate ς' increases the investment per worker, and it exceeds the investment required to keep the capital stock per worker constant. Thus the capital stock per worker increases till a new steady state is attained, shown here as $\bar{\bar{k}}$.

Now look at this figure:



As the population rises to n', the investment per capita becomes lower than the investment required to maintain the capital per worker. Thus, the capital stock per worker keeps declining from the steady state $\bar{k}$ to the new steady state $\bar{\bar{k}}$.

4. Summary

Let us summarize that technical change may be embodied or disembodied. Growth can be exogenous or endogenous. Solow model of growth explains that long term growth occurs because of technological progress. Technological progress is exogenous and disembodied in this model. This model is an extension of the Harrod-Domar model.

In the Solow model, any increase in Quantity produced could come from one of the three sources as:

1. An increase in Labour - 'L', which is subject to diminishing returns to scale which imply a reduction in the Output to Labour ratio **(**or output per worker).

2. An increase in capital. An increase in the **stock of capital** would increase both output and **the** Output to Labour ratio **as capital helps to improve labour productivity.**

3. An increase in technology (A) could also increase Output per worker.

If capital per capita is increased then output increases and growth rate rises. But, population increase drags down the availability of capital per capita and hence growth is dragged down. Hence, in long term growth process, technological progress plays an important role, besides growth of capital.

- We must retain the Solow equation
- $(1+n)k(t+1) = (1-\delta)k(t) + \varsigma y(t)$

Which explains an important conclusion of this model that, the new per capita capital stock $k(t+1)$ can be obtained by depreciated per capita capital $(1-\delta)k(t)$ and per capita savings $\varsigma y(t)$ but population growth drags down the growth of new per capita capital.

The larger the population growth, the lower will be the per capita capital in the subsequent period and hence lower output.